Cornell University Library

The original of this book is in the Cornell University Library.

There are no known copyright restrictions in the United States on the use of the text.

http://www.archive.org/details/cu31924003931148

Cornell University Library
TC 179.N52 1875

Hydraulic tables, coefficients, and form

3 1924 003 931 148 engr

HYDRAULIC TABLES,

COEFFICIENTS, AND FORMULÆ,

FOR

FINDING THE DISCHARGE OF WATER

FROM

ORIFICES, NOTCHES, WEIRS, PIPES, AND RIVERS.

By JOHN NEVILLE,
CIVIL ENGINEER, M.R.I.A.; COUNTY SURVEYOR OF LOUTH AND OF
THE COUNTY OF THE TOWN OF DROGHEDA; FELLOW OF
THE ROYAL GEOLOGICAL SOCIETY OF IRELAND.

Third Edition:

WITH ADDITIONS, CONSISTING OF NEW FORMULÆ FOR THE DISCHARGE FROM TIDAL AND FLOOD SLUICES, AND SYPHONS; GENERAL INFORMATION ON RAINFALL, CATCHMENT-BASINS, DRAINAGE, SEWERAGE, WATER SUPPLY FOR TOWNS, AND MILL POWER.

PHILADELPHIA:
HENRY CAREY BAIRD & CO.,
Industrial Publishers and Booksellers,
No. 810 Walnut Street.
1875.

INTRODUCTION

TO THE THIRD EDITION.

In order to render this edition more valuable to the hydraulic engineer the work has been again considerably extended by the insertion of several new formulæ, experimental coefficients, and general estimates of cost. It is hoped that the extent and practical nature of these additions, will render the book still more useful than before, to meet the ever-varying requirements of the profession in connexion with rivers and water-works. The experiments of Mr. Mallet on syphons made in 1843, and printed in Weale's Quarterly Papers on Engineering, have been reduced. New formulæ are given for finding the discharge from syphons, flood-sluices, and tidal-sluices. The practical formulæ for gauging by weirs have been added to. The arrangement of the matter is in some places altered, and some portions of the former introductions transferred, at their proper places, to the text, and others are retained here.

At page 85 the erroneous practice of different engineers in giving only two-thirds of the coefficient of discharge for weirs is noticed. This practice assumes that the theoretical discharge from a notch is the same as if all the particles of water had the same mean theoretical velocity as those undermost, which, being too large by one-third, the experimental coefficient has to be reduced in the same proportion to give a correct result. There is no reason for sanctioning a different coefficient for notches, or orifices at the surface, and for sunk orifices. The coefficients when in thin plates, with large cisterns, have nearly the same general value, ·615 to ·628, for both, and it tends to confusion to adopt in one place a coefficient for a correct formula, and in another a coefficient for an incorrect one; although the final result, by an equality of contrary errors, may be the same. I may here observe how very general the coefficient of *two-thirds,* and thereabouts, is for all orifices and notches; likewise for the useful effect derived from the application of water power; as well also for the relation of the velocity due to the fall and the velocity of water wheels to give a maximum result. The modifications of coefficients, dependent on the position, thickness, form, and approaches of an orifice, are as yet little understood by the profession. The defects in the ordinary formula when the velocity of approach has to be considered are pointed out in pages 88 to 124, and it is to be regretted that the authority of some writers on the subject has misled many as to the correct form. Before the effective power of a water-wheel, or water-engine, can be determined, we must know how to gauge the water supplied to it correctly. This can be done only by the application of formulæ and coefficients varied to suit the circumstances of the case under consideration. From causes, which it is not necessary to enter into here, this has seldom been done, and very little dependence can be placed on results obtained by the formula in common use when applied generally. It is pleasing to follow Mr. Francis and Professor J. Thomson through the steps by which they get the effective power of their wheels, and I have accordingly made considerable use of their labours in Section XIV.

In practice, the irregular, sometimes sloping, broken and jagged crests of most weirs, on larger rivers, render any close estimate of the quantity passing over quite uncertain, especially for lesser depths, unless where the observer has a large scientific experience; and the quantities are generally too large to apply the ordinary notch-gauge. In such cases it is better to measure the flow of the river, stream, or mill-race, from the cross section and the observed mean velocity. Frequently, however, this method presents another difficulty, in an irregular channel, where the depths and velocities vary considerably in the same cross section, and where the cross sections themselves vary in short distances apart.

In such cases I have often found it necessary to divide a selected stretch into two or more longitudinal sections, determining the cross section and mean velocity of each, and taking the sum of the results for the flow: which may be checked off by like admeasurements on a different stretch of the channel.

Long solid stone weirs, with an enlarged weir basin in connexion with mills, to regulate the head and to pond water, above a water-wheel, have especial advantages for this duty; but their application for the purpose of keeping down the head, and to effect drainage for long distances up a river, without a sluice, or falling crest, see page 290, was marvellous—unless for the drainage of capital. In November, 1849, in connexion with a paper on the Benburb Mills and Weir case, page 283, I drew the attention of the Institution of Civil Engineers of Ireland* to the misapplication of such long solid weirs on the Shannon, for navigation and drainage purposes. The "Arterial Drainage Commissioner," on the Board of Works, who was present, "pooh-poohed" the inferences; but the failure of those works—rather the injury they do—has since become patent to all; and after an expenditure of about £600,000, an Act has been passed for the outlay of another £300,000, a moiety of which the riparian proprietors are again expected to contribute. This amount is proposed to be now expended in order to remedy the misapplication of a large portion of the first sum, £300,000 of which had to be paid by the proprietors of the adjacent counties, without having had any control over its expenditure.

---

* This paper, although read at the special request of the President, then Chairman of the Board of Works, and ordered to be printed, did not appear in the *Transactions*, but its substance afterwards became the nucleus of our First Edition. The Commissioner of Drainage was one of the Vice-Presidents, and the author a Member of Council at the time.

Since the Report of the "Commissioners of Inquiry into the Arterial Drainage, in eleven districts in Ireland," see pages 396, 397, and 398, and the removal of the Arterial Drainage Commissioner, Treasury Minute, 24th June, 1853, drainage works have been carried on under a better system. The riparian proprietors of the Shannon, however, if again taxed, should have power to nominate an engineer of their own selection, to consult, act with, or control, if necessary, any engineer selected by the Treasury, or by the Commissioners of Public Works, and to see after and protect their special interests. The system of executing works solely by or under the staff of the Government in Ireland, to the cost of which districts or individuals contribute, has not been successful, and has not given satisfaction. When the Treasury is solicited to lend, or to contribute for State purposes, the control exercised by the Board of Public Works and its engineers, with reference to the plans, estimates, and specifications submitted, is of much value; but the sooner the system adopted for drainage works since 1853 is generally carried out, and persons to be taxed for the Shannon, or for piers, harbours, and other works, are also permitted to have a voice in the engineering plans, and *fair control over their execution*, the better for the State and for all parties immediately interested.

The Tables, from I. to XIV., at the end of the volume are all original, with the exception of Table I., which contains the well-known coefficients of Poncelet and Lesbros; but these are newly arranged, the heads reduced to English inches, and the coefficients for heads measured over and back from the orifice, placed side by side, for more ready comparison. The coefficients in the small Tables throughout the work were all calculated by the author from the original experiments; the formulæ have been carefully investigated, and the continental ones reduced to English measures—some of them, as will be seen, for the first time.

The correction of some of the experimental formulæ, particularly the continental ones, as printed in some English books, cost the author some labour. Even Du Buât's well-known formula is frequently misprinted; and in a hydraulic work, $\sqrt{d} - \cdot 1$, one of the factors, is printed $\sqrt{d - \cdot 1}$ in every page where it is given. It is not always that such mistakes can be avoided, but experimental formulæ are so often copied from one work into another without sufficient examination, that an error of this kind frequently becomes fixed; and when applied to practical purposes erroneous formulæ get the correct ones into disrepute. See note to formula (91), page 218.

The TABLES of velocities and discharges over weirs and notches have been calculated for a great number of coefficients TO MEET DIFFERENT CIRCUMSTANCES OF APPROACH AND OVERFALL, and for various heads from $\frac{1}{4}$th of an inch up to 6 feet. TABLE II. embodies the velocities acquired by falling bodies under the head of "theoretical velocity," and the velocities, suited to various coefficients, for heads up to 40 feet.

The formulæ (45), (46), (45*a*), and (46*a*), for calculating the effects of the velocity of approach to orifices and weirs, and the necessary corrections for the ratio of the channel to the orifice at pages 88 to 114, as well as TABLE V., I believe to be original. They will be found of much value in determining the proper coefficients suited to various ratios. The remarks throughout SECTION IV. are particularly applicable to the proper understanding and use of this TABLE. The hypothesis from which formulæ (44), (45), and (46) are derived, page 96 and 97, and from which TABLE V. is calculated, was framed for the purpose of giving practical results when the orifice A approximates, in size, to the channel C. They are less than those derivable from formulæ (44*a*), (45*a*,), and (46*a*), which give, for every value of the coefficient c_d, infinite results, when A = C; and results much too large in practice as the values of A and C approximate. As the values of the coefficients for (45*a*) and (46*a*) can be immediately deduced from the last two columns of that TABLE, by using the values therein given, for the ratio of the channel to the orifice, as multipliers, for any primary coefficient c_d, as pointed out in the text, the resulting or SECONDARY coefficients for both sets of formulæ can be compared with much advantage.

TABLE VII. of surface and mean velocities will be found to vary from those generally in use, and to be much more correct, and better suited for practical purposes, particularly when applied to finding the mean velocities in rivers.

The TABLES at pages 270 and 271 being for a mean width of 100 feet, will be found perhaps more generally applicable to river channels than TABLES XI. and XII., for a mean width of 70 feet. The TABLES at pages 28, 29, 146, and 199 give similar results for long and short cylindrical pipes. The formula (119A), page 230, for finding the velocity in pipes and rivers, is general in its practical application.

The loss of head from friction in a uniform water channel is $h = \left(\frac{v}{m}\right)^2 \times \frac{l}{r}$ in which the value of m for feet measures may be taken from the Table, page 231, with reference to the velocity v. The loss is therefore directly as the length of the channel, directly as $\left(\frac{v}{m}\right)^2$, and inversely as the hydraulic mean depth

—which is one-fourth of the diameter for a cylindrical pipe. I have known an engineer of considerable practice take it as proportionate to the inner surface of a pipe, which was only correct when the diameter remained constant.

The Statistics of rain-fall, and of catchment-basins have not yet received the full attention which the subjects deserve. The distribution of rain gauges with reference to elevation, contour, temperature, and isothermal lines has not been sufficiently attended to. The connexion of the rain-fall with the discharge generally, for the whole catchment, for the tributary catchments, and their sub-catchments, at the sea in the middle districts and at the sources, noting the geology, must be observed for several years before the questions of supply, discharge, absorption, and evaporation in any climate can be answered. The maximum and minimum discharges in each year and series of years must be observed, as well as the average mean discharges, and the maximums and minimums of these also, before the physical connexion of climate and catchment can be correctly ascertained, and the engineer furnished with reliable data. Heretofore observations, even when of the best, have been partial or limited, and a wide field is here yet open to competent physicists in connexion with our drainage works. Mr. Symons is now, however, reducing the rain-fall in Great Britain and Ireland to a scientific form.

The general items of cost given in SECTION XIII. will be found of use; they are intended, however, more as guides than as standards for estimating other works, the cost of which must depend on their own circumstances. Those who have practical experience of the differences between estimates, cost, and value, and how they are affected by changes of time, locality, quality, and quantity, will estimate for themselves in detail; but the discrepancies between estimates and cost, even under the same circumstances, are too well known to call for any remarks here.

A few words about my publishers. The Messrs. Lockwood and Co. having purchased the copyright of the work, decided on adopting for this edition a more convenient size for the engineer and student than that of the last edition; but printed on paper as good, and with type fully as clear. I have reason to be satisfied with the manner in which the work is again brought out and published, and wish them a full return. I am indebted to Mr. R. Field, Westminster, for some observations and corrections, which, coming after the text was printed, I have only here to thank him for, having made use of them in the errata.

JOHN NEVILLE.

RODEN PLACE, DUNDALK,
February, 1875.

CONTENTS.

PAGES

INTRODUCTION TO THE THIRD EDITION . . . . . iii—xi

SECTION I. APPLICATION AND USE OF THE TABLES, FORMULÆ — EXTRA HORSE-POWER REQUIRED IN PUMPING ENGINES FROM FRICTION IN THE PIPES —TABLE OF HEADS DUE TO FRICTION AND OF DISCHARGES . . . . 1—31

SECTION II. FORMULÆ FOR THE VELOCITY AND DISCHARGE FROM ORIFICES, WEIRS, AND NOTCHES—COEFFICIENTS OF VELOCITY, CONTRACTION, AND DISCHARGE—PRACTICAL REMARKS ON THE USE OF THE FORMULÆ . . . . 31—51

SECTION III. EXPERIMENTAL RESULTS AND FORMULÆ — COEFFICIENTS OF DISCHARGE FOR ORIFICES, NOTCHES, AND WEIRS . . . . 51—86

SECTION IV. VARIATIONS IN THE COEFFICIENTS FROM THE POSITION OF THE ORIFICE—GENERAL AND PARTIAL CONTRACTION—VELOCITY OF APPROACH—CENTRAL AND MEAN VELOCITIES—VARIOUS PRACTICAL FORMULÆ FOR THE DISCHARGE OVER WEIRS AND NOTCHES . . . . 86—134

SECTION V. SUBMERGED ORIFICES AND WEIRS—CONTRACTED RIVER CHANNELS . . . . . 134—144

SECTION VI. SHORT TUBES, MOUTH-PIECES, AND APPROACHES — ALTERATION IN THE COEFFICIENTS FROM FRICTION BY INCREASING THE LENGTH—COEFFICIENTS OF DISCHARGE FOR SIMPLE AND COMPOUND TUBES—SHOOTS . . . . 144—160

PAGES

SECTION VII. LATERAL CONTACT OF THE WATER AND TUBE—ATMOSPHERIC PRESSURE—HEAD MEASURED TO THE DISCHARGING ORIFICE—COEFFICIENT OF RESISTANCE—FORMULA FOR THE DISCHARGE FROM A SHORT TUBE—DIAPHRAGMS—OBLIQUE JUNCTIONS—FORMULA FOR THE TIME OF THE SURFACE SINKING A GIVEN DEPTH—LOCK CHAMBERS—FLOOD AND TIDAL SLUICES . . . . . . 160—188

SECTION VIII. FLOW OF WATER IN UNIFORM CHANNELS—MEAN VELOCITY—MEAN RADII AND HYDRAULIC MEAN DEPTHS — BORDER — TRAIN — HYDRAULIC INCLINATION—EFFECTS OF FRICTION—FORMULÆ FOR CALCULATING THE MEAN VELOCITY—APPLICATION OF THE FORMULÆ AND TABLES TO THE SOLUTIONS OF THREE USEFUL PROBLEMS . . 188—251

SECTION IX. BEST FORMS OF THE CHANNEL—REGIMEN—VELOCITY—EQUALLY DISCHARGING CHANNELS . 251—272

SECTION X. EFFECTS OF ENLARGEMENTS AND CONTRACTIONS—BACKWATER WEIR CASE—LONG AND SHORT WEIRS—THE RIVER SHANNON . . . 272—291

SECTION XI. BENDS AND CURVES—BRANCH PIPES—DIFFERENT LOSSES OF HEAD—GENERAL EQUATION FOR FINDING THE VELOCITY—HYDROSTATIC AND HYDRAULIC PRESSURE—PIEZOMETER—SYPHONS . 291—315

SECTION XII. RAIN-FALL—CATCHMENT BASINS—DISCHARGE INTO CHANNELS — DISCHARGE FROM SEWERS — LOSS FROM EVAPORATION, ETC. . . 315—343

SECTION XIII. WATER SUPPLY FOR TOWNS—STRENGTH OF PIPES — SEWERAGE ESTIMATES AND COST—THOROUGH DRAINAGE—ARTERIAL DRAINAGE . 343—399

SECTION XIV. WATER POWER — DYNAMOMETER, OR FRICTION BRAKE — OVERSHOT, BREAST, AND UNDERSHOT VERTICAL WHEELS—TURBINES AND HORIZONTAL WHEELS—HYDRAULIC RAM—WATER-PRESSURE ENGINE—WORK OF FLOUR MILLS . . 399—435

TABLES NOT EMBODIED IN THE TEXT.

PAGES

TABLE I. *Coefficients of Discharge from Square and Differently proportioned Rectangular Lateral Orifices in Thin Vertical Plates.* (See pp. 60, 61, and 62.) . 436—437

TABLE II. *For finding the Velocities from the Altitudes and the Altitudes from the Velocities* . . . 438—447

TABLE III. *Square Roots for finding the Effects of the Velocity of Approach, when the Orifice is small in proportion to the Head. Also, for finding the Increase in the Discharge from an Increase of Head.* (See pp. 91 to 99.) . . . . 448—449

TABLE IV. *For finding the discharge through Rectangular Orifices; in which* $n = \frac{h}{d}$. *Also, for finding the Effects of the Velocity of Approach to Weirs, and the Depression on the Crest.* (See pp. 91 to 99.) . 450—453

TABLE V. *Coefficients of Discharge for different Ratios of the Channel to the Orifice.* (See pp. 98 to 114.) . 454—459

TABLE VI. *The Discharge over Weirs or Notches of One Foot in Length, in Cubic Feet per Minute.* (See pp. 62 to 86, 90, 96, 109, and 114 to 125.) . . . 460—465

TABLE VII. *For finding the Mean Velocity from the Maximum Velocity at the surface, in Mill-races, Streams, and Rivers with Uniform Channels; and the Maximum Velocity from the Mean Velocity.* (See pp. 188 to 191.) 466

TABLE VIII. *For finding the Mean Velocities of Water flowing in Pipes, Drains, Streams, and Rivers.* (See p. 209.) . . . . 467—479

TABLE IX. *For finding the Discharge in Cubic Feet, per Minute, when the Diameter of a Pipe or Orifice, and the Velocity of Discharge, are known, and* vice versâ 480—481

TABLE X. *For finding the Depths on Weirs of different Lengths, the Quantity discharged over each being supposed constant.* (See p. 289.) 482

PAGES

TABLE XI. *Relative dimensions of Equally-Discharging Trapezoidal Channels, with Slopes from 0 to 1, up to 2 to 1.* (See pp. 267 to 272.) 483

TABLE XII. *Discharges from the Primary Channel in the first column of Table XI.* (See pp. 270 and 271.) 484—485

TABLE XIII. *The Square Roots of the Fifth Powers of Numbers for finding the Diameter of a Pipe, or Dimensions of a Channel from the Discharge, or the reverse; showing the relative Discharging Powers of Pipes of different Diameters, and of any similar Channels whatever, closed or open.* (See pp. 28, 29, 245, *etc.*) 486

TABLE XIV. *English and French Measures* . . . 487—493

TABLE XV. *Weight, Specific Gravity, Ultimate Strengt and Elasticity of Various Materials.* 494

ERRATA.

Page 99, line 13, for c_d within the brackets read c_d^2.

Page 179, second line from top, for "$c_d\,h\,t$," read "$c_d\,a\,h\,t$;" and for "f_1" read "f."

Page 219, lines 4 to 11. This is Dr. Young's modification of Eytelwein. The original formula in the "Handbuch" is equivalent to $92{\cdot}3\,\sqrt{r\,s}$. (Mr. Field.)

Page 222, line one at top, for "v_2," read "v^2."

ON THE

DISCHARGE OF WATER

FROM

ORIFICES, WEIRS, PIPES, AND RIVERS.

SECTION I.

APPLICATION AND USE OF THE TABLES, AND FORMULÆ.

To find the velocity of a falling body from the height fallen, or the height fallen from the velocity.

RULE.—MULTIPLY THE SQUARE ROOT OF THE HEIGHT IN INCHES BY 27·8, AND THE PRODUCT WILL BE THE VELOCITY IN INCHES.* TO FIND THE HEIGHT FROM THE VELOCITY, SQUARE THE VELOCITY IN INCHES AND DIVIDE THE SQUARE BY 772·84, THE QUOTIENT WILL BE THE HEIGHT IN INCHES. See equation (1). TABLE II., column 1, will give the velocity from the height, found in the column of "altitudes," or the height from the velocity, directly.

EXAMPLE 1.—*What is the velocity acquired by a heavy body falling $\frac{1}{8}$th of an inch?*

In the Table opposite to $\frac{1}{8}$th of an inch, found in the column headed "altitudes h," is found 9·829 in column 1, for the required velocity, in inches per second.

---

* The square root of the height in feet multiplied by 8·025 gives the velocity per second in feet; and the square of the velocity in feet divided by 64·4 will give the height in feet. The decimals may be omitted in applications for engineering practice, and the multiplier 8 and divisor 64 only used.

Example 2.—*What is the velocity acquired by a fall of* 11 *feet* 3 *inches?*

Opposite to 11 feet 3 inches, as before, is found 323·007 inches, for the velocity required.

Example 3.—*What height must a heavy body fall through to acquire a velocity of* 40½ *feet per second?*

Here 40½ feet is equal 486 inches, opposite the nearest number to which, found in column 1, is found 25 feet 6 inches for the required fall. In this example, the nearest number to 486 found in the Table is 486·301. The difference ·301 corresponds, very nearly, to ⅜ths of an inch in altitude, and, therefore, the true head according to the rule would be 25′ 5⅝″; but for all practical purposes the difference is immaterial.

By means of Table II., directly, or by simple interpolation, the velocity due to all heights from $\frac{1}{100}$ part of an inch up to 40 feet, can be found, and the heights from the velocities. For a greater height than 40 feet it may be divided by 4, 9, or some square number, n^2, and the velocity found for the quotient, from the Table, multiplied by 2, 3, or n, the square root of the divisor, will give the velocity required.

Example 4.—*What is the velocity acquired by a fall of* 45 *feet?*

$\frac{45}{4} = 11' \; 3''$, the velocity corresponding to which, found from the Table, is 323·007. Hence, $323{\cdot}007 \times \sqrt{4} = 323{\cdot}007 \times 2 = 646''{\cdot}014 = 53' \; 10''{\cdot}014$ is the velocity per second required. The reverse of this example is equally simple.

Columns 3, 4, 5, 6, 7, 8, 9, 10, 11, and 12 in the Table, give the values of $\sqrt{2gh}$ multiplied by the coefficients therein stated. These columns will be found of great practical use in finding the mean velocities in the *vena-contracta*, in the orifice, and in short tubes; and consequently, also, in finding the mechanical force, as well as the discharge. An examination of the coefficients in the small Tables in SECTION III., and also of those in TABLES I. and V., at the end of the work, will show how much they vary; but those most generally useful, and their products by the theoretical velocity due to different heads, up to 40 feet, are given in the columns referred to.

EXAMPLE 5.—*What is the discharge from an orifice* 4 *inches by* 8 *inches, the centre sunk* 20 *feet below the surface of a reservoir?*

From TABLE II., is found 430·676 inches equal 35·89 feet for the theoretical velocity of discharge: hence, $\frac{8 \times 4}{144} \times 35{\cdot}89 = \frac{2}{9} \times 35{\cdot}89 = 7{\cdot}976$ cubic feet per second is the theoretical discharge. If the discharge takes place through a thin plate, or if the inner arrises next the water in the reservoir be *perfectly square*, and the water in flowing out does not fill the passage so as to convert the orifice into a short tube, the coefficient is found from TABLE I. to be ·603. The true discharge then is $7{\cdot}976 \times {\cdot}603 = 4{\cdot}809$ cubic feet per second.

For the determination of the coefficient suited to any particular orifice, and the circumstances of its position, the reader must refer generally to the following pages. If in the example just given, the arrises next the reservoir were rounded into the form of the contracted vein, see Fig. 4, the coefficient would increase from ·603 to ·974 or ·956, for a passage not exceeding a couple of feet in length. With the former the discharge would be 7·976 × ·974 = 7·769 cubic feet, and with the latter 7·976 × ·956 = 7·625 cubic feet. The latter results may be found otherwise from TABLE II. With a head of 20 feet and the coefficient ·974, the velocity is 419·48 inches = 34·957 feet; hence, the discharge is $\frac{2}{9}$ × 34·957 = 7·768 cubic feet. With a coefficient of ·956, the velocity is 411·73 inches = 34·31 feet, and $\frac{2}{9}$ × 34·31 = 7·624, cubic feet. These results are the same, practically, as those previously found.

If the inner arrises be square, and the passage out be from 18 inches to 2 feet long, the orifice will be converted into a short tube, the coefficient for which is ·815. With this coefficient, and a head of 20 feet, find as before, from TABLE II., the mean velocity of discharge 351 inches = 29·25 feet; hence, the discharge now is $\frac{2}{9}$ × 29·25 = 6·5 cubic feet per second.

The velocities in inches per second, given in TABLES II. *and* VIII., *or elsewhere in the following pages, may be converted into velocities in feet per minute, by multiplying by 5; equal* $\frac{60}{12}$.

EXAMPLE 6.—*The discharge from a small orifice having its centre placed* 10 *feet below the surface of a* *reservoir is* 18 *feet per minute, what will be the discharge from the same orifice at a depth of* 17 *feet?*

The discharges will be to each other as $\sqrt{10} : \sqrt{17}$, or as $1 : \sqrt{1{\cdot}7}$; or, from TABLE III., as $1 : 1{\cdot}3038$, whence the discharge sought is equal $1{\cdot}3038 \times 18 = 23{\cdot}4684$ cubic feet.

EXAMPLE 7.—*What is the value of the expression* $c_d \left\{ 1 + \frac{c^2_d}{m^2 - c^2_d} \right\}^{\frac{1}{2}}$ *in equation* (45), *when* $c_d = {\cdot}617$, *and* $m = 2$?

$$\text{Here } \frac{c^2_d}{m^2 - c^2_d} = \frac{{\cdot}617^2}{4 - {\cdot}617^2} = \frac{{\cdot}3807}{3{\cdot}6193} = {\cdot}1052;$$

whence the first expression becomes equal to $\cdot 617 \, (1{\cdot}1052)^{\frac{1}{2}}$ equal, from TABLE III., $\cdot617 \times 1{\cdot}0513 = {\cdot}649$, the value sought. TABLE V. contains the values of this expression for various values of c_d and m, which latter, m, stands for the ratio of the channel to an orifice; and then immediately find from it, opposite 2 in the first column, and under the coefficient ·617 in the sixth column, ·649 the value sought. When the head due to the pressure, and to the velocity of approach, are both known, we can determine the new coefficient of discharge by the above expression, and thence the discharge itself. The coefficient suited to the velocity of approach may however be found directly in TABLE V. The usual methods for finding the effects of the velocity of approach, given by d'Aubuisson and others, are incorrect in principle, see SECTION IV.

EXAMPLE 8.—*What is the discharge from an orifice* 17 *inches long and* 9 *inches deep, having the upper* *edge placed* 4 *inches below the surface, and the lower edge* 13 *inches?*

The expression for the discharge is $\frac{2}{3} \times \text{A}\sqrt{2\,g\,d} \times c_d \left\{\left(1 + \frac{h_t}{d}\right)^{\frac{3}{2}} - \left(\frac{h_t}{d}\right)^{\frac{3}{2}}\right\}$ equation (43), in which take $d = 9$ inches; $h_t = 4$ inches; $\text{A} = 17 \times 9 = 153$ square inches; and $\sqrt{2\,g\,d}$, found from Table II. = 83·4 inches. Also, $\frac{h_t}{d} = \cdot444$, and hence the value of

$$(1\cdot444)^{\frac{3}{2}} - (\cdot444)^{\frac{3}{2}} = \text{(from Table IV.) } 1\cdot44.$$

Assuming the coefficient of discharge to be ·617, then the discharge in cubic inches per second is equal to

$$\frac{2}{3} \times 153 \times 83\cdot4 \times \cdot617 \times 1\cdot44 =$$

$$\frac{2}{3} \times 12760\cdot2 \times \cdot88848 = 7558.$$

Consequently, $\frac{7558}{1728} = 4\cdot374$ is the discharge in cubic feet per second. From equation (6.), the discharge is equal to

$$\frac{2}{3} \times \cdot617 \times 27\cdot8 \times 17 \times \{13^{\frac{3}{2}} - 4^{\frac{3}{2}}\}$$

But $13^{\frac{3}{2}} - 4^{\frac{3}{2}} = 46\cdot872 - 8$, from Table IV., equal to 38·872, whence the discharge is

$\frac{2}{3} \times \cdot617 \times 27\cdot8 \times 17 \times 38\cdot872 = 11\cdot4351 \times 17 \times 38\cdot872 = 194\cdot3967 \times 38\cdot872 = 7557$ cubic inches $= 4\cdot374$ cubic feet, the same as before.

It is shown, equation (31), that by using the mean depth for orifices near the surface, the discharge will approximate very closely to the true discharge, and that even for weirs the error will not exceed

6 per cent. The discharge is then expressed by $\cdot617\sqrt{2g \times 8\frac{1}{2}} \times 9 \times 17$ = (from Table II.) $50\cdot01 \times 153 = 7651\cdot53$ cubic inches $= 4\cdot427$ cubic feet per second. The head to the centre of the orifice is here $8\frac{1}{2}$ inches, and the depth of the orifice 9 inches, therefore, in equation (31), $h = d$ very nearly; and, therefore, this result must be multiplied by ·989, as shown in that equation; then $\cdot989 \times 4\cdot427 = 4\cdot378$ cubic feet, which gives a result differing from those otherwise found, by a very small quantity, which, practically, is of no value. By means of Table VI. the discharge from rectangular orifices near the surface can be found with very great facility.

The discharge from an orifice near the surface may always be found with sufficient accuracy, for practical purposes, by measuring the head to the centre, in the same manner as if the orifice were sunk to a considerable depth; then by applying the corrections given in equation (31); *or if the orifice be circular, those given in equation* (28); *sufficient accuracy, according to the correct formula, is obtainable.*

Example 9.—*What is the discharge from a circular orifice* 4 *inches in diameter, having its centre placed* 4 *inches below the surface, when the coefficient of discharge is* ·617?

The area of the orifice is $4 \times 4 \times \cdot7854 = 12\cdot566$ square inches. The velocity in the orifice at the mean depth of 4 inches, with a coefficient of ·617, is 34·31 inches, whence the discharge is $12\cdot566 \times 34\cdot31 = 431\cdot139$ cubic inches $= \cdot2496$ cubic feet per second, or 14·97 cubic feet per minute. By means of Table IX. the discharge in cubic feet per minute can be found very readily when the velocity (34·31 inches per second) is known. Thus,

	Inches.		Cubic feet.
For a velocity of	30·00	the discharge is	13·089
,, ,,	4·00	,, ,,	1·745
,, ,,	·30	,, ,,	0·131
,, ,,	·01	,, ,,	0·004
,, ,,	34·31	,, ,,	14·969

By applying the coefficient found from equation (28), which is ·992, when the depth at the centre is twice the radius, as it is in this example, ·992 × 14·97 = 14·85 is found for the correct discharge in cubic feet per minute. Here the difference in the results is only 1 in 125.

The application of Table VI. enables us to find the discharge from rectangular orifices near the surface very quickly. Resuming "Example 8," the discharge may be found from this Table for each foot in length of the orifice, as follows. The discharge in cubic feet per minute, when the coefficient is ·617 for a notch 1 foot long and 13 inches deep, is 223·323; and for a notch of 4 inches deep, 38·116; therefore, the discharge from an orifice 9 inches deep, with the upper edge 4 inches below the surface, is 223·323 − 38·116 = 185·207 cubic feet per minute. But as the length of the orifice is 17 inches, this must be multiplied by $\frac{17}{12}$, and the product 262·377 is the discharge in cubic feet per minute; this is equal to a discharge of 4·373 cubic feet per second, and agrees with that before found. This is the simplest way of finding the discharge from rectangular orifices near the surface.

EXAMPLE 10.—*What is the discharge in cubic feet per minute, from an orifice 2 feet 6 inches long and 7 inches deep, the upper edge being 3 inches below the surface, and the coefficient of discharge ·628?*

From TABLE VI. the discharge from a notch 1 foot long and 10 inches deep is found to be 153·353, and for a notch 3 inches deep, 25·199. The difference, or 128·154, multiplied by $2\frac{1}{2}$, will be the discharge required; viz. $2\frac{1}{2} \times 128{\cdot}154 = 320{\cdot}385$ cubic feet per minute.

EXAMPLE 11.—*The size of a channel is 2·75 times the size of an orifice, what is the coefficient of discharge when that for a very large channel in proportion to the orifice is ·628?*

From TABLE V. the coefficient is found to be ·645, when the approaching water suffers full contraction. By attending to the auxiliary Tables in the text, we find for this case, $\frac{\text{orifice}}{\text{channel}} = \frac{1}{2{\cdot}75} = {\cdot}36$. Hence, therefore, multiply 2·75 by ·857, which gives 2·36 for the ratio of the mean velocities in the orifice and in the channel approaching it. With this new value of the ratio of the channel to the orifice, find, as before, the value of the coefficient from TABLE V. which is ·651. The remarks throughout the work, with the auxiliary tables, will be found of much use in determining the coefficients for different ratios of the channel to the orifice, notch, or weir, and the corrections suited to each. If in this example,—other things being the same,—the alteration in the coefficient for a notch, or weir, had to be considered, it would be found from the Table, column 4, to be ·672 instead of ·645 found in column

3, for an orifice sunk some depth below the surface. For the corrections suited to mean and central velocity, and to the nature of the approaches, they may be found in the body of this work and in the auxiliary tables at the end of SECTION IV.

EXAMPLE 12.—*What is the discharge over a weir* 50 *feet long; the circumstances of the overfall, crest, and approaches, being such that the coefficient of discharge is* ·617, *when the head measured from the water in the weir basin,* 6 *feet above the crest, is* 17½ *inches?*

TABLE VI. gives the discharge in cubic feet per minute, over each foot in length of weir, for various depths up to 6 feet. It is divided into two parts; the first for "greater coefficients," viz. ·667 to ·617; and the second for "lesser coefficients," viz. ·606 to ·518. The coefficient assumed being ·617, the discharge over 1 foot in length, with a head of 17½ inches, is found to be 348·799 cubic feet per minute; hence the required discharge is 50 × 348·799 = 17439·95 cubic feet.

The determination of the coefficient suited to the circumstances of the overfall, crest, approaches, and approaching section, will be found discussed elsewhere through this work. The valuable Table derived from Mr. Blackwell's experiments will also be of use; but the heads being taken at a much greater distance back from the crest than is generally usual, the coefficients taken from it for heads greater than 5 or 6 inches, will be found less than the true ones for heads measured immediately at or about 6 feet, above the crest. For heads measured *on* the crest, the small Table of coefficients in SECTION III., applicable to the purpose, will be of use.

EXAMPLE 13.—*What is the mean velocity in a large channel, when the maximum velocity along the central line of the surface is* 31 *inches per second?*

TABLE VII. gives 25·89 inches for the required velocity, and for smaller channels 24·86 inches. In order to find the mean velocity at the surface from the maximum central velocity, the latter must be multiplied by ·914.

The velocity at the surface is best found by means of a floating hollow ball, which just rises out of the water. The velocity at a given depth is best found by means of two hollow balls connected with a link, the lower being made heavier than the upper, and both so weighted by the admission of a certain quantity of water that they shall float along the current, the upper one being in advance but nearly vertical over the other. The velocity of both will then be the velocity at half the depth between them. The velocity at the surface, found by means of a single ball, being also found, the velocity lost at the half depth is had by subtracting the common velocity due to the linked balls from that of the single ball at the surface. The velocity at any given depth is then easily found by a simple proportion; but the result will be most accurate when the given depth is nearly half the distance between the balls, which distance can never exceed the depth of the channel. *Pitot's tube*, *Woltmann's tachometer*, the *hydrometric pendulum*, the *rheometer*, and several other hydrometers, have been used for finding the velocity; but these instruments require certain corrections suited to each separate instrument, as well as each kind of instrument, and are not so correct or simple, for

for measuring the velocity in open channels, as a ball and linked balls. In general the surface maximum velocity can be found by throwing in a rag-weed, or some other plant, and observing the time it is carried over a given distance, say from 30 to 100 feet or more, according to the circumstances of the channel; and multiplying the velocity so found by ·83 to find the mean velocity. If the velocity at the surface be taken at several sections, between the centre and the banks, the multiplier should be increased to ·91.

EXAMPLE 14.—*What is the discharge from a river having a surface inclination of* 18 *inches per mile, or* 1 *in* 3520, 40 *feet wide, with nearly vertical banks, and* 3 *feet deep?*

The area is $40 \times 3 = 120$ feet, and the border $40 + 2 \times 3 = 46$ feet; therefore the hydraulic mean depth is $\frac{120}{46} = 2{\cdot}61$ feet $= 2$ feet $7{\cdot}3$ inches.* With this and the inclination we find from TABLE VIII. $28{\cdot}27 + 2{\cdot}75 \times \frac{1{\cdot}3}{6} = 28{\cdot}87$ inches per second $= 28{\cdot}87 \times 5 = 144{\cdot}35$ feet per minute for the mean velocity; hence $144{\cdot}35 \times 120 = 17{,}322$ cubic feet per minute is the required discharge. For channels with sloping banks divide the border, which is always known, into the area for the hydraulic mean depth, with which, and the surface inclination, find the velocity by TABLE VIII., and thence the discharge. Unless the banks of rivers be protected by stone pavement or otherwise, the slopes will not continue permanent; it is therefore almost useless to give the discharges for channels of particular widths and slide slopes. When the mean velocity is once known, the remaining calculations are those of mere mensuration, and they should be made separately. This example may also be solved, practically, by means of TABLES XI. and XII. A channel 40 × 3 has the same conveying power as one 70 × 2, TABLE XI., which latter, TABLE XII. discharges with a fall of 18 inches in the mile, 17,157 feet; or about one per cent. less than that previously found.

---

* For greater hydraulic depths than 144 inches, the extent of the TABLE, divide by 9, and find the corresponding velocity. This multiplied by 3 will be the velocity sought. Or divide by 4 and multiply by 2.

EXAMPLE 15.—*The diameter of a very long pipe is* 1½ *inch, and the rate of inclination, or whole length of the pipe divided by the whole fall, is* 1 *in* 71½; *what is the discharge in cubic feet per minute?*

The hydraulic mean depth, or mean radius, is $\frac{1{\cdot}5}{4} = {\cdot}375$ inch $= \frac{3}{8}$ inch. Consequently from TABLE VIII. the velocity in inches per second is equal to $25{\cdot}09 - 1{\cdot}92 \times \frac{1{\cdot}5}{10} = 25{\cdot}09 - {\cdot}29 = 24{\cdot}80$. The discharge in cubic feet per minute for a 1½-inch pipe is now found most readily by means of TABLE IX., as follows:—

	Inches.		Cubic feet.
For a velocity of	20·0	the discharge is	1·227
,, ,,	4·0	,, ,,	·245
,, ,,	·8	,, ,,	·049
,, ,,	24·8	,, ,,	1·521

Whence the discharge in cubic feet per minute is 1·521.

For short pipes, of 100 or 200 feet in length, and under, the height due to the velocity and orifice of entry must be deducted from the whole height to find the proper hydraulic inclination, and also the height due to bends, curves, cocks, slides, and erogation. The neglect of these corrections has led some writers into mistakes in applying certain formulæ, and in testing them by experimental results obtained with short pipes. The TABLES shall now be applied to the determination of the discharge from short pipes, and the results compared with experiment, referring generally to equation (153 A) and the remarks preceding it for a correct and direct solution.

EXAMPLE 16.—*What is the discharge in cubic feet per minute from a pipe* 100 *feet long, with a fall or head of* 35 *inches to the lower end, when the diameter is* $1\frac{1}{2}$ *inch? Find also the discharge from pipes* 80 *feet,* 60 *feet,* 40 *feet, and* 20 *feet, of the same diameter and having the same head.*

If the water be admitted by a stop-cock at the upper end, the coefficient due to the orifice of entry will probably be about ·75 or less, ·815 being that for a clear entry to a short cylindrical tube. The approximate inclination is $\frac{100 \times 12}{35} = 1$ in 34·3; but as a portion of the fall must be absorbed by the velocity and orifice of entry, it may be assumed for the present that the inclination is 1 in 35. With this inclination and the mean radius $\frac{1\frac{1}{2}}{4} = \frac{3}{8}$ inch, we find the mean velocity from TABLE VIII. to be 38·06 inches. Now when the coefficient due to the orifice of entry and velocity is ·75, from Table II. the head due to this velocity is $3\frac{3}{8}$ inches nearly, whence $35 - 3\frac{3}{8} = 31\frac{5}{8} = 31·625$ inches is the height due to friction, and $\frac{100 \times 12}{31·625}$ equals 1 in 37·9, the inclination, very nearly. With this new inclination find, as before, from Table VIII. the mean velocity of discharge which is now 36·35 inches; and by repeating the operation the velocity to any degree of accuracy is found in accordance with the table; and the shorter the pipe is, the oftener must it be repeated. The height due to 36·35 inches taken from Table II. as before, with a coefficient of ·750, is $3\frac{1}{8} = 3·125$ inches. The corrected fall due to the friction is now $35 - 3·125 = 31·875$, and $\frac{1200}{31·875}$ equal 1 in 37·6, the corrected inclination. With this inclination the corrected velocity is now 36·53 inches per second. It is not necessary to repeat this operation again. The discharge determined from Table IX. is as follows:—

	Inches.		Cubic feet.
For a velocity of	30·00	the discharge is	1·841
,, ,,	6·00	,, ,,	·368
,, ,,	·50	,, ,,	·031
,, ,,	·03	,, ,,	·002
,, ,,	36·53	,, ,,	2·242

The experimental discharge found by Mr. Provis was 2·264 cubic feet per minute in one experiment, and 2·285 in another. The discharge from the shorter pipes may be found in a similar manner, and the results placed alongside the experimental ones given in the work referred to below * are inserted in the following short table :—

EXPERIMENTAL AND CALCULATED DISCHARGES FROM SHORT PIPES.

Length of pipe, in feet.	Head, in inches.	Observed discharge, in cubic feet.	Velocity per second.	Head due to the orifice and velocity.	Head due to friction.	Hydraulic inclinations.	Calculated velocity.	Calculated discharge.
100	35	2·275	37·082	3⅛	31⅞	37·6	36·53	2·242
80	35	2·500	40·750	3¾	31¼	30·8	41·18	2·521
60	35	2·874	46·846	5	30	24·0	48·02	2·946
40	35	3·504	57·115	7½	27½	17·5	58·50	3·590
20	35	4·528	73·801	12½	22½	10·7	78·61	4·824

The velocities in the fourth column have been calculated by the author from the observed quantities discharged, from which the height due to the orifice of entry and velocity in column 5 is determined, and thence the quantities in the other columns as above shown. The differences between the experimental and calculated results are not large, and had a lesser coefficient than ·750 been used for calculating the reduction of head due to the velocity, stop-cock, and orifice of entry, say ·715, the calculated results, and

---

* "Transactions of the Institution of Civil Engineers," vol. ii. p. 203. "Experiments on the Flow of Water through small Pipes." By W. A. Provis. The small Tables in SECTIONS VI. and VIII. of this edition give at once the coefficient to be multiplied by $\sqrt{2gH}$, or $8\sqrt{H}$, to find the velocity when the ratio of the diameter to the length of the pipe is known. They will be found of great advantage in calculating directly the velocity from short pipes. For long pipes, see the TABLE at the end of this Section.

those in all of Mr. Provis's experiments in the work referred to, would be nearly identical.*

EXAMPLE 17.—*It is proposed to supply a reservoir near the town of Drogheda with water by a long pipe, having an inclination of* 1 *in* 480, *the daily supply to be* 80,000 *cubic feet; what must the diameter of the pipe be?*

The discharge per minute must be $\frac{80,000}{1440} = 56$ † cubic feet, nearly. Assume a pipe whose "mean radius" is 1 inch, or diameter 4 inches, and the velocity per second found from TABLE VIII. will be 14·41 inches. Then from TABLE IX.,

	Inches.		Cubic feet.
For a velocity of	10·00	a discharge of	4·363
,, ,,	4·00	,, ,,	1·745
,, ,,	·40	,, ,,	·175
,, ,,	·01	,, ,,	·004
,, ,,	14·41	,, ,,	6·287

The discharge from a pipe 4 inches in diameter would be therefore 6·287 cubic feet per minute. Then

$$4^{\frac{5}{2}} : d^{\frac{5}{2}} :: 6{\cdot}287 : 56, \text{ or } 1 : d^{\frac{5}{2}} :: {\cdot}196 : 56 :: 1 : 286;$$

therefore $d^{\frac{5}{2}} = 286$, and $d = 9{\cdot}61$ inches, nearly, as may be found from TABLE XIII., &c. This is nearly the required diameter. It is to be observed that the diameters thus found will not always agree exactly with those found from Du Buât's or other formulæ, nor with each other, because the discharges are not strictly as $d^{\frac{5}{2}}$; but in practice the difference is immaterial, and the approximative value thus found can be easily corrected. If a pipe whose diameter is 1, were assumed, the operation would have been more simple; for the velocity would then be, Table VIII., at the given inclination, 6·4 inches; and the discharge ·175 cubic feet, Table IX. Hence $d^{\frac{5}{2}} = \frac{56}{\cdot 175} = 320$, and, therefore, Table XIII., $d = 10$ inches nearly, which differs about half-an-inch from the former value, 9·6 inches, found by assuming a pipe of 4 inches to calculate from. It is necessary to understand that different results must be expected, in working from practical formulæ, for different operations. When once an approximate value is obtained, it can be easily corrected to any required degree of accuracy.

---

* In a late work, "Researches in Hydraulics," the author is led into a series of mistakes as to the accuracy of Du Buât's and several other formulæ, from neglecting to take into consideration the head due to the velocity and orifice of entry when testing them by the experiments above referred to.—*Second Edition.*

† "Hydraulic Tables," Weale, 1854, give at once this discharge for a pipe between 9 and 10 inches in diameter, also the TABLE, p. 28.

Again, *the velocity in inches per second, from a cylindrical pipe* 6 *inches in diameter, is nearly equal to the discharge in cubic feet per minute;* and as $6^{\frac{5}{2}} = 88{\cdot}2$, then $88{\cdot}2 : d$: : the velocity in inches per second from a 6-inch pipe : the discharge per minute from a pipe whose diameter is d. Hence this proposition gives, very nearly, the discharge from the diameter and fall; or the diameter from the discharge and fall by finding the velocity only, due to a 6-inch pipe. See Tables pp. 28 and 29.

Example 18.—*The area of a channel is* 50 *square feet, and the border* 20·6 *feet; the surface has an inclination of* 4 *inches in a mile; what is the mean velocity of discharge?*

$\frac{50}{20{\cdot}6} = 2{\cdot}427$ feet $= 29{\cdot}124$ inches is the hydraulic mean depth; and from TABLE VIII., $12{\cdot}03 - \frac{1{\cdot}30 \times {\cdot}876}{6} = 12{\cdot}03 - {\cdot}19 = 11{\cdot}84$ inches per second is the required velocity. Though this velocity will be found under the true value for straight clear channels, it will yet be more correct for ordinary river courses, with bends and turns, of the dimensions given, than the velocity found from equation (114.). For a straight clear channel of these dimensions, Watt found the mean velocity to be 13·5 to 14 inches; that is to say, 17 at top, 10 at bottom, and 14 in the middle. The author's formula $v = 140\ (r\ s)^{\frac{1}{2}} - 11\ (r\ s)^{\frac{1}{3}}$ gives $v = 1{\cdot}143$ feet, or nearly a mean of these two.

EXAMPLE 19.—*A pipe* 5 *inches in diameter*, 14,637 *feet in length, has a fall of* 44 *feet; what is the discharge in cubic feet per minute?*

The inclination is $\frac{14{,}637}{44} = 332{\cdot}7$, and mean radius $\frac{5}{4} = 1\frac{1}{4}$. Then find from TABLE VIII. the velocity which is equal to $19{\cdot}81 + \frac{{\cdot}41 \times 4{\cdot}8}{12{\cdot}5} = 19{\cdot}81 + {\cdot}16 = 19{\cdot}97$, or 20 inches per second very nearly; and by TABLE IX. the discharge in cubic feet per minute is, as before found to be, 13·635. The TABLE, p. 28, gives, by inspection, 13·6 feet.

EXAMPLE 20.—*What is the velocity of discharge from a pipe or culvert* 4 *feet in diameter, having a fall of* 1 *foot to a mile?*

Here $s = \frac{1}{5280}$, and $r = 1$ foot. Next we find the velocity of discharge from Table VIII. which is 14·09 inches, equal to 1·174 feet per second. By calculating from the different formulæ referred to below, the velocities, when $r\,s = \cdot 0001894$, and $\sqrt{r\,s} = \cdot 01376$, are as follows.

				Velocity in feet.
Reduction of	Du Buât's formula	equation	(81.)	1·174
,,	Girard's do. (Canals with aquatic plants and very slow velocities)	,,	(86.)	·521
,,	Prony's do. (Canals)	,,	(88.)	1·201
,,	Prony's formula (Pipes). . . .	,,	(90.)	1·257
,,	Prony's do. (Pipes and Canals) .	,,	(92.)	1·229
,,	Eytelwein's do. (Rivers) . . .	,,	(94.)	1·200
,,	Eytelwein's do. (Rivers). . . .	,,	(96.)	1·285
,,	Eytelwein's do. (Pipes)	,,	(98.)	1·364
,,	Eytelwein's do. (Pipes)	,,	(99.)	1·350
,,	Dr. Young's do.	,,	(104.)	1·120
,,	*D'Aubuisson's do. (Pipes) . . .	,,	(109.)	1·259
,,	*D'Aubuisson's (Rivers)	,,	(111.)	1·199
,,	The writer's do. (Clear straight Channels with small velocities)	,,	(114.)	1·268
,,	Weïsbach's do. (Pipes)	,,	(119.)	1·285
,,	The author's, for Pipes and Rivers	,,	(119A.)	1·295

This example is calculated from the several formulæ above referred to, whether for pipes or rivers, in order that the results may be more readily compared. The formula from which the velocities and tables for the discharges of rivers are usually calculated is, for measures in feet, $v = 94{\cdot}17\sqrt{r\,s}$. This gives the mean velocity, for the foregoing example, equal to

---

* These two formulæ of D'Aubuisson's are, simply, adoptions of Eytelwein's and Prony's.

1·295 feet per second. This is the same as is found from the author's general formula for all velocities; but the particular expression, $v = 99{\cdot}17 \sqrt{r\,s}$, is only suited for velocities of about 15 inches per second; the results found from it for lesser velocities are too much, and for higher velocities too little, if bends and curves be allowed for separately. For ordinary practical purposes the result of Du Buât's general formula, equation (81), may be safely adopted; and, accordingly, the results in TABLE VIII. calculated for the first edition from it, have been retained, notwithstanding the greater accuracy and simplicity of the general equation (119 A) for the velocity in pipes and rivers, viz., $v = 140\,(r\,s)^{\frac{1}{2}} - 11\,(r\,s)^{\frac{1}{3}}$.

Dr. Young's formula gives lesser results for rivers and large pipes than Du Buât's, but they are too small unless when the curves and bends are numerous and sudden. Girard's formula (86.) is only suited for small velocities in canals containing aquatic plants, and it is entirely inapplicable to rivers or regular channels for conveyance of water. A knowledge of various formulæ, and their comparative results, applied to any particular case, will be found of great value to the hydraulic engineer, and the differences in the results show only an amount of error that may be expected in all practical operations, and which becomes of less importance when it is considered that by increasing the dimensions of a channel every way, by only one-third, its discharging power is more than doubled. See TABLE XIII.

EXAMPLE 21.—*Water flowing down a river rises to a height of* $10\frac{1}{2}$ *inches on a weir* 62 *feet long; to* *what height will the same quantity of water rise, on a weir similarly circumstanced,* 120 *feet long?*

$$\frac{62}{120} = \cdot517, \text{ nearly.}$$

In TABLE X. is found, by inspection, opposite to ·517, the ratio of the lengths, the coefficient ·644, rejecting the fourth place of decimals; whence $10\frac{1}{2} \times \cdot644 = 6\cdot76$ inches, the height required. When the height is given in inches it is not necessary to take out the coefficient to further than two places of decimals.

EXAMPLE 22.—*The head on a weir* 220 *feet long is* 6 *inches; what will the head be on a weir* 60 *feet long, similarly circumstanced, the same quantity of water flowing over each?*

$$\frac{60}{220} = \cdot273.$$

As this lies between ·27 and ·28, from TABLE X. the coëfficent ·4208 is found; hence $\frac{6}{\cdot4208} = 14\cdot26$ inches, the head required.

TABLE X. will be found equally applicable in finding the head above the pass into weir basins, and above contracted water channels. See SECTION X.

EXAMPLE 23.—*A river channel* 40 *feet wide and* 4·5 *feet deep is to be altered and widened to* 70 *feet; what must the depth of the new channel be so that the surface inclination and discharge shall remain unaltered?*

In "TABLE XI., OF EQUALLY DISCHARGING RECTANGULAR CHANNELS," opposite to 4·54, in the column of 40 feet widths, 3 is found in the column of 70 feet widths, which is the depth required in feet.

EXAMPLE 24.—*It is necessary to unwater a river channel* 70 *feet wide and* 1 *foot deep, by a rectangular side cut* 10 *feet wide; what must the depth of the side cut be, the surface inclination remaining the same as in the old channel?*

In TABLE XI. is found 4·5 feet for the required depth. When the width of a channel remains constant, the discharge varies as $\sqrt{r s} \times d$, in which d is the depth; and when the width is very large compared with the depth, the hydraulic mean depth r approximates very closely to the depth d, and therefore $d = r$; consequently the discharge then varies as $d^{\frac{3}{2}} \times s^{\frac{1}{2}}$, and when the discharge is given $d^{\frac{3}{2}}$ must vary inversely as $s^{\frac{1}{2}}$; or more generally $dr^{\frac{1}{2}}$ must vary inversely, as $s^{\frac{1}{2}}$, when the width and discharge remain constant.

In narrow cuts for unwatering, it is prudent to make the depth of the water half the width of the cut very nearly, when local circumstances admit of these proportions; for then a maximum effect will be obtained with the least possible quantity of excavation; but for rivers and permanent channels the proper relation of the depth to the width must be regulated by the principles referred to in SECTION IX.

TABLE XI. is equally applicable, whether the measures be taken in feet, yards, or any other standards whatever.

EXAMPLE 25.—*A new river channel is to have a fall of eighteen inches in a mile, and must discharge* 18,700 *cubic feet per minute; what shall the dimensions be?*

In TABLE XII., in the column of 18 inches per mile, opposite to 18,766, it is seen that a primary channel 70 × 2·125 will be sufficient; and opposite to 2·125 in TABLE XI. it is also seen the equivalent rectangular channels 60 × 2·37; 50 × 2·70; 40 × 3·19; 35 × 3·52; 30 × 3·96; 25 × 4·61; 20 × 5·58; 15 × 7·29; and 10 × 11·37, to select from. If the side have any given slopes, the discharge will not be practically affected as long as the depth and area of the rectangular channel and the one with sloping banks remain the same. See SECTION IX.

EXAMPLE 26.—*A pipe* 100 *feet long and* 1 *inch in diameter has a head of* 150 *feet over the lower end, what will be the discharging velocity?*

Here $r = {\cdot}020833$ in feet, and $s = 1{\cdot}5$, therefore $r\,s = {\cdot}03125$. Hence by formulæ (119A) $v = 140 \times ({\cdot}03125)^{\frac{1}{2}} - 11 \times ({\cdot}03125)^{\frac{1}{3}} = 140 \times {\cdot}1766 - 11 \times {\cdot}315 = 24{\cdot}724 - 3{\cdot}465 = 21{\cdot}259$ feet per second. If allowance is required for the orifice of entry, the velocity is corrected as follows. A square orifice of entry has a coefficient of ·815. The head due to this coefficient for a velocity of about 20½ feet, or 246 inches, is about 10 feet, TABLE II.—The head due to friction is therefore $150 - 10 = 140$ feet, and $s = \frac{100}{140} = 1{\cdot}4$; $r\,s$ now becomes $1{\cdot}4 \times {\cdot}020833 = {\cdot}02917$. Hence $v = 140\sqrt{r\,s} - 11\sqrt[3]{r\,s}$ now becomes $140 \times {\cdot}171 - 11 \times {\cdot}308$ nearly, equal to $23{\cdot}940 - 3{\cdot}388 = 20{\cdot}552$ feet, the velocity for a square junction.

EXAMPLE 27.—*A sewer* 9 *feet in diameter has a fall of* 2 *feet per mile, what will be the velocity and discharge of water flowing through it when full?*

Here $r = 2{\cdot}25$ and $s = \frac{1}{2640}$, therefore $r\,s = {\cdot}0008523$, $(r\,s)^{\frac{1}{2}} = {\cdot}02919$ and $(r\,s)^{\frac{1}{3}} = {\cdot}0948$; and by formula (119A), $v = 140\,(r\,s)^{\frac{1}{2}} - 11\,(r\,s)^{\frac{1}{3}} = 140 \times {\cdot}02919 - 11 \times {\cdot}0948 = 4{\cdot}0866 - 1{\cdot}0428 = 3{\cdot}0438$ feet per second. Hence the discharge per minute is $9^2 \times {\cdot}7854 \times 3{\cdot}0438 \times 60 = 63{\cdot}62 \times 182{\cdot}6 = 11{,}617$ cubic feet nearly. The velocity from a circular pipe or sewer is however greatest when the circumference is open for about 78½ degrees at the top, but the velocity of sewage matter would not be equal to that of water. It would vary according to the dilution in the sewer, and 50 per cent. should be allowed, at least, in deduction, unless the dilution be very considerable.

The TABLE for the values of $r\,s$ and v, calculated from the formula (119 A) SEC. VIII., will give the velocity at once when $r\,s$ is known, and $r\,s$ when the velocity is known, from the latter of which a definite value of r or s can be fixed upon, when the other may be then found, by an operation of simple division.

EXAMPLE 28.—*Water is to be pumped through a pipe* 3000 *feet long and* 2 *feet in diameter, with a velocity not exceeding* 4 *feet per second, what head must be allowed extra for friction in the pipe when calculating horse power?*

From the TABLE of the values of the velocity and products of the hydraulic mean depth and hydraulic inclination, given near the conclusion of SECTION VIII., that for a velocity of 4 feet per second $r\,s = {\cdot}00142$. The diameter of the pipe is 2 feet, therefore $r = {\cdot}5$, whence $s = \frac{\cdot 00142}{\cdot 5} = \cdot 00284$, and as the length of the pipe is 3000 feet we get $3000 \times \cdot 00284 = 8\cdot 52$ feet, the head required. The Table p. 29, would give 9·6 feet nearly, which corresponds with Du Buât's formula. If the velocity in the pipe were 10 feet instead of 4 feet per second, then from the table, $r\,s = \cdot 007576$, and $\frac{r\,s}{r} = s = \frac{\cdot 007576}{\cdot 5} = \cdot 015152$, and therefore, $h = l\,s = 3000 \times \cdot 015152 = 45\cdot 456$ feet, or about six times as much as when the velocity was only 4 feet per second. The great loss of head arising from pumping at high velocities, from friction alone, is therefore apparent. Were the velocity double, or 8 feet per second, the head would be 30 feet nearly, or from the Table, p. 29, 31·6 feet.

For velocities of about 2·1 feet per second, v. is equal to $100\sqrt{r\,s}$, and for velocities of about $5\frac{1}{2}$ feet per second, $v = 110\sqrt{r\,s}$. If l be the length of a pipe, it would be found in the former case that the head, h, in feet due to friction from the formula is $h = \frac{l\,v^2}{10{,}000\,r} = l\,s$; and in the latter $h = \frac{l\,v^2}{12{,}100\,r} = l\,s$.

In questions of this kind, however, the diameter of a pipe, d should be used in preference to the hydraulic mean depth, and as $d = 4\,r$ it will be found in the first case that $h = \frac{l\,v^2}{2500\,d} = l\,s$; and in the second case, $h = \frac{l\,v^2}{3025\,d} = l\,s$.

If it be necessary to substitute the fall per mile for the hydraulic inclination, the first of these will again become $h = \frac{2 \cdot 11\, v^2}{d} = l\, s$ for the loss per mile; and in the second case, $h = \frac{1 \cdot 72\, v^2}{d} = l\, s$ for the loss per mile in feet.

If the velocity were so slow as about 1 foot per second, then $v = 90 \sqrt{r\, s}$, and we should find $h = \frac{l\, v^2}{2025\, d} = l\, s$.

If for the inclination the fall per mile be substituted, this will become $h = \frac{2 \cdot 61\, v^2}{d} = l\, s$; for the loss per mile in feet.

The loss of head varies in the same pipe with the velocity, and must be calculated differently, for small and for high velocities, when using the common formulæ. The TABLE near the end of SECTION VIII. will always give the correct value of $r\, s$, and thence $s = \frac{r\, s}{r}$.

In addition to the loss of head arising from friction, losses also occur from straight or curved bends, from diaphragms, from junctions, and from the orifices of entry and discharge; these must be determined separately for each case, as is shown hereafter, and added together, including the loss of head arising from friction. The sum must then be added to the height the water is to be raised, before the full or total head for determining the power of an engine can be accurately known.

The TABLE on the two following pages will be found

TABLE for finding, very nearly, the velocity and discharge from Cylindrical Water Pipes or Sewers when the diameter and fall are given. Any two of the four quantities, the velocity, discharge diameter, and fall or inclination, being given, the others can be found in THE TABLE *from inspection.* SEE TABLE XIII.

Height in ft. due to friction per mile of length.	Mean hydraulic inclination of pipe or sewer.	The VELOCITY IN INCHES PER SECOND is given in the first horizontal line for each inclination or fall; and the DISCHARGE IN CUBIC FEET PER MINUTE in the next following one.									
		1 in. diam.	2 in. diam.	3 in. diam.	4 in. diam.	5 in. diam.	6 in. diam.	7 in. diam.	8 in. diam.	9 in. diam.	10 in. diam.
	One in										
1	5280	1·7	2·5	3·2	3·8	4·2	4·7	5·1	5·5	5·9	6·2
		·05	·27	·79	1·6	2 9	4·6	6·8	9·6	12·9	16·9
2	2640	2·5	3·8	4·7	5·6	6·3	6·9	7·5	8·1	8·6	9·1
		·07	·41	1·2	2·4	4·3	6·8	10·1	14·2	19·1	24·9
3	1760	3·1	4·7	5·9	7·0	7·8	8·7	9·4	10·2	10·8	11·5
		·08	·51	1·4	3·0	5·3	8·5	12·6	17·7	23·9	31·2
4	1320	3·6	5·5	6·9	8·2	9·2	10·2	11·1	11·9	12·7	13·4
		·10	·60	1·7	3·6	6·3	10·0	14·8	20·8	28·0	36·7
5	1056	4·1	6·2	7·9	9·3	10 4	11·6	12·5	13·5	14·4	15·2
		·11	·68	1·9	4·0	7·1	11·3	16·8	23·6	31·7	41·5
6	880	4·6	6·9	8·7	10·3	11·5	12·8	13·9	15·0	15·9	16·9
		·12	·76	2·2	4·5	7·9	12·6	18·6	26·1	35·1	46·0
7	754	5·0	7·5	9·5	11·2	12·6	14·0	15·1	16·3	17·4	18·4
		·14	·82	2·3	4·9	8·6	13·7	20·2	28·5	38·3	50·2
8	660	5·4	8·1	10·2	12·0	13·5	15·0	16·3	17·6	18·7	20·0
		·15	·89	2·5	5·3	9·2	14·8	21·8	30·6	41·3	54·1
9	587	5·7	8·7	11·0	12·9	14·5	16·1	17·4	18·8	20·0	21·2
		·16	·95	2·7	5·6	9·9	15·8	23·3	32·8	44·1	57·7
10	528	6·1	9·2	11·6	13·7	15·4	17·1	18·5	19·9	21·2	22·5
		·17	1·00	2·9	6·0	9·2	16·7	24·7	34·8	46·8	61·3
11	480	6·4	9·7	12·3	14·4	16·2	18·0	19·5	21·0	22·4	23·7
		·17	1·1	3·0	6·3	11·1	17·7	26·1	36·7	49·4	64·7
12	440	6·7	10·2	12·9	15·2	17·1	18·9	20·5	22·1	23·5	24·9
		·18	1·1	3·2	6·6	11·6	18·6	27·4	38·6	51·9	67·9
13·2	400	7·1	10·8	13·6	16·0	18·0	20·0	21·7	23·3	24·8	26·3
		·19	1·2	3·3	6·9	12·3	19·6	28·9	40·7	54·8	71·7
15·1	350	7·7	11·6	14·7	17·2	19·4	21·6	23·4	25·2	26·8	28·4
		·21	1·3	3·6	7·5	13·2	21·2	31·2	43·9	59·1	77·4
17·6	300	8·4	12·7	16·0	18·8	21·2	23·5	25·5	27·5	29·2	31·0
		·23	1·4	3·9	8·2	14·4	23·1	34·1	48·8	64·6	84·5
21·1	250	9·4	14·1	17·8	20·9	23·5	26·1	28·3	30·5	32·5	34·4
		·26	1·5	4·4	9·2	16·0	25·7	37 8	53·3	71·7	93·8
26·4	200	10·6	16·0	20·2	23·8	26·8	29·7	32·2	34·7	36·9	39·1
		·29	1·7	5·0	10·4	18·2	29·2	43·1	60·6	81·6	106·7
35·2	150	12·5	19·0	23·9	28·1	31·6	35·1	38·1	41·0	43·7	46·3
		·34	2·1	5·9	12·3	21·6	34·5	50·9	71·6	96·4	126·2
52·8	100	15·9	24·1	30·4	35·7	40·1	44·6	48·3	52·1	55·4	58·7
		·43	2·6	7·4	15·6	27·3	43·8	64·6	90·9	122·3	160·1
58·7	90	16·9	25·6	32·3	38·0	42·7	47·4	51·4	55·4	58·9	62·5
		·46	2·8	7·9	16·6	29·1	46·6	68·7	96·7	130·2	170·3
66·	80	18·1	27·5	34·6	40·7	45·8	50·9	55·2	50·4	63·2	67·0
		·49	3·0	8·5	17·8	31·2	49·9	73·7	103·7	139·6	182·7
75·4	70	19·6	30·0	37·5	44·1	49·6	55·1	59·7	64·4	68·4	72·5
		·53	3·2	9·2	19·2	33·8	54·1	79·8	112·3	151·2	197·7
88·	60	21·5	32·6	41·1	48·3	54·4	60·4	65·5	70·6	75·1	79·5
		·59	3·6	10·1	21·1	37·1	59·3	87·5	123·2	165·8	216·9
105·6	50	24·0	36·4	45·8	53·9	60·7	67·4	73·1	78·7	83·7	88·7
		·65	4·0	11·2	23·5	41·3	66·2	97·6	137·4	184·9	242·0
132·	40	27·4	41·6	52·5	61·7	69·4	77·1	83·6	90·1	95·8	101·5
		·75	4·5	12·8	26·9	47·3	75·7	111·7	157·2	211·6	276·8
176	30	32·6	49·5	62·5	73·4	82·6	91·7	99·5	107·2	114·0	120·8
		·89	5·4	15·3	32·0	56·3	90·0	132·9	187·1	251·8	329·4
212·2	25	36·4	55·3	69·8	82·0	92·2	102·4	111·1	119·7	127·3	134·9
		·99	6·0	17·1	35·8	62·9	100·6	148·4	208·9	281·2	367·9
264·1	20	41·7	63·3	79·9	93·8	105·6	117·3	127·2	137·0	145·7	154·4
		1·14	6·9	19·6	40·9	72·0	115·1	169·9	239·2	321·9	421·2
352·	15	49·6	75·3	95·0	111·7	125·6	139·6	151·3	163·1	173·4	183·8
		1·35	8·2	23·3	48·3	85·6	137·0	202·2	284·6	383·1	501·2
528	10	63·3	96·0	121·2	142 4	160·2	178·0	192·9	207·9	221·1	234·3
		1·73	10·5	29·7	62·1	109·2	174·7	257·8	362·9	488·5	639·0

***TABLE for finding, very nearly, the velocity and discharge from Cylindrical Water Pipes or Sewers, when the diameter and fall are given. Any two of the four quantities, the velocity, discharge, diameter, or inclination being given, the others can be found in* THE TABLE *from inspection.* SEE TABLE XIII.**

Height in ft. due to friction per mile of length	Mean hydraulic inclination of pipe or sewer.	The VELOCITY IN INCHES PER SECOND is given in the first horizontal line for each inclination or fall; and the DISCHARGE IN CUBIC FEET PER MINUTE in the next following one.									
	One in	12 in. diam.	14 in. diam.	16 in. diam.	18 in. diam.	20 in. diam.	22 in. diam.	24 in. diam.	26 in. diam.	28 in. diam.	30 in. diam.
1	5280	6·8	7·4	8·0	8·5	8·9	9·4	9·8	10·3	10.7	11·1
		26·9	**40·**	**56·**	**75·**	**98·**	**124·**	**155·**	**189·**	**228·**	**271·**
2	2640	10·1	10·9	11·7	12·5	13·2	13·9	14·5	15·1	15·7	16·3
		40	**58**	**82**	**110**	**144**	**183**	**228**	**278**	**336**	**400**
3	1760	13	14	15	16	17	17	18	19	20	20
		50	**73**	**103**	**138**	**180**	**229**	**285**	**349**	**421**	**500**
4	1320	15	16	17	18	19	20	21	22	23	24
		58	**86**	**120**	**163**	**212**	**269**	**335**	**410**	**494**	**588**
5	1056	17	18	20	21	22	23	24	25	26	27
		66	**97**	**136**	**184**	**240**	**305**	**380**	**464**	**560**	**665**
6	880	19	20	22	23	24	26	27	28	29	30
		73	**108**	**151**	**203**	**265**	**337**	**420**	**514**	**620**	**737**
7	754	20	22	24	25	27	28	29	30	32	33
		80	**118**	**165**	**222**	**289**	**368**	**458**	**560**	**676**	**804**
8	660	22	24	25	27	29	30	31	33	34	35
		86	**127**	**177**	**239**	**312**	**397**	**493**	**604**	**728**	**866**
9	587	23	25	28	29	30	32	33	35	36	38
		92	**135**	**192**	**255**	**333**	**424**	**527**	**645**	**778**	**925**
10	528	25	27	29	31	32	34	36	37	39	40
		97	**144**	**201**	**271**	**354**	**450**	**560**	**685**	**826**	**982**
11	480	26	28	30	32	34	36	38	39	41	42
		103	**152**	**213**	**286**	**373**	**475**	**590**	**723**	**871**	**1036**
12	440	27	30	32	34	36	38	39	41	43	44
		108	**159**	**223**	**300**	**392**	**498**	**620**	**759**	**916**	**1089**
13·2	400	29	31	34	36	38	40	42	43	45	47
		114	**168**	**236**	**317**	**414**	**526**	**655**	**801**	**966**	**1149**
15·1	350	31	34	36	39	41	43	45	47	49	50
		123	**181**	**254**	**342**	**446**	**568**	**707**	**865**	**1043**	**1240**
17·6	300	34	37	40	42	45	47	49	51	53	55
		134	**198**	**278**	**374**	**487**	**620**	**772**	**944**	**1139**	**1354**
21·1	250	38	41	44	47	50	52	55	57	59	61
		149	**219**	**308**	**415**	**541**	**688**	**857**	**1048**	**1264**	**1504**
26·4	200	43	47	50	53	56	59	62	65	67	70
		169	**249**	**350**	**472**	**616**	**783**	**975**	**1192**	**1438**	**1710**
35·2	150	51	55	59	63	67	70	73	76	79	82
		200	**294**	**414**	**558**	**728**	**925**	**1152**	**1409**	**1700**	**2021**
52·8	100	65	70	75	80	85	89	93	97	101	105
		254	**374**	**526**	**708**	**923**	**1174**	**1462**	**1788**	**2157**	**2565**
58·7	90	69	74	80	85	90	95	99			
		270	**398**	**559**	**753**	**982**	**1250**	**1556**			
66	80	74	80	86	91	96	101	106			
		290	**427**	**600**	**808**	**1054**	**1340**	**1669**			
75·4	70	80	86	93	99	105	110	115			
		314	**463**	**650**	**875**	**1141**	**1451**	**1807**			
88	60	88	95	102	109	115	120	126			
		344	**506**	**712**	**959**	**1251**	**1591**	**1981**			
105·6	50	98	106	114	121	128	134	141			
		384	**565**	**795**	**1070**	**1396**	**1775**	**2211**			
132	40	119	124	130	138	146	153	161			
		467	**665**	**908**	**1224**	**1597**	**2031**	**2529**			
176	30	133	144	155	165	174	183	192			
		523	**770**	**1082**	**1456**	**1900**	**2417**	**3010**			
212·2	25	149	161	173	184	195	204	214			
		584	**860**	**1208**	**1627**	**2122**	**2699**	**3362**			
264·1	20	170	184	198	211	223	234	245			
		668	**984**	**1383**	**1862**	**2430**	**3090**	**3848**			
352	15	202	219	236	251	265	278	292			
		795	**1171**	**1646**	**2216**	**2891**	**3677**	**4579**			
528	10	258	279	301	320	338	355	372			
		1014	**1496**	**2098**	**2826**	**3686**	**4688**	**5839**			

Heads in inches due to the velocity and junction.	Velocities in inches, per second, due to heads.		
	Best curved junction.	Square junction.	Pipe at the junction projecting into the cistern.
1	27·8	22·7	20
2	39	32	28
3	48	39	34
4	56	45	40
6	68	56	49
8	79	64	56
10	88	72	63
12	96	78	69
24	136	111	97
36	167	136	119
48	193	157	138
60	215	176	154
72	236	192	169
84	255	208	182
96	272	222	194
108	289	235	207
120	305	248	218

of great practical utility in solving all questions connected with water-pipes and sewers discharging fully-diluted sewage. In using it, interpolate, by inspection, for intermediate diameters or inclinations. For greater diameters, divide those given by 4, and multiply the corresponding velocity found in the table by 2, and the corresponding discharge in the table by 32. If the object be to find the size of the channel, divide greater given velocities by 2, and multiply the diameters or inclinations found from the table by 4; also divide greater discharges by 32, and multiply the diameters found from the table by 4. The small auxiliary table, p. 29, embodied in the larger one, is of great use in making allowance for the velocity and orifice of entry in short pipes, before finding the head due to friction. The table also gives the different diameters and inclinations which, taken together, give the same velocity or discharge; and it enables, from inspection, to select that relation of diameter to declivity which is best suited for other engineering aspects of the question. Taken in connexion with TABLES VIII., XI., XII., and XIII., this table completes the means of finding, by inspection, the dimensions, inclinations, velocities, and discharges of every class of water-channel or sewage-conduit required in engineering practice.

TABLE XIV. gives the comparative values of English and French measures; and TABLE XV. gives the weight, specific gravity, and ultimate strength and elasticity of various materials with which the engineer has to operate.

SECTION II.

FORMULÆ FOR THE VELOCITY, AND DISCHARGE, FROM ORIFICES, WEIRS, AND NOTCHES.—COEFFICIENTS OF VELOCITY, CONTRACTION AND DISCHARGE.—PRACTICAL REMARKS ON THE USE OF THE FORMULÆ.

The quantity of water discharged in a given time through an aperture of a given area in the side or bottom of a vessel, is modified by different circumstances, and varies more or less with the form, position, and depth of the orifice; but the discharge may be easily found, when the velocity and the contraction of the fluid vein has been determined.

VELOCITY.

If g be the velocity acquired by a heavy body falling from a state of rest for one second, *in vacuo*, then it has been shown by writers on mechanics, that the velocity v per second acquired by falling from a height h, will be

$$(1.) \qquad v = \sqrt{2\,g\,h}.$$

The numerical value of g varies with the latitude; then it shall be assumed that $2\,g = 772{\cdot}84$ inches $= 64{\cdot}403$ feet. These will give for measures in inches,

$$v = 27{\cdot}8\,\sqrt{h,}^{*} \text{ and } h = \frac{v^2}{772{\cdot}84} = {\cdot}001293v^2,$$

and for measures in feet,

---

* The velocities for different heights are given in column number 1, TABLE II.

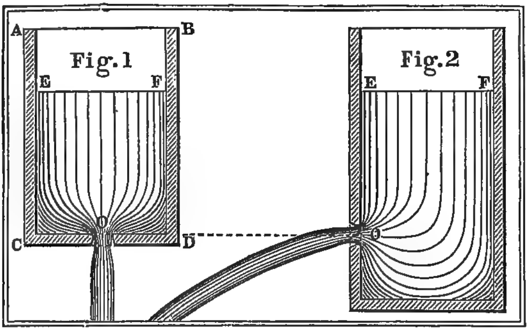
$$v = 8{\cdot}025\ \sqrt{h},\ \text{and}\ h = \frac{v^2}{64{\cdot}403} = {\cdot}01553\ v^2.$$

If v be in feet, and h in inches, then

$$v = 2{\cdot}317\ \sqrt{h},\ \text{and}\ h = \frac{v^2}{5{\cdot}367} = {\cdot}1864\ v^2.\text{*}$$

COEFFICIENT OF VELOCITY.

Let the vessel A B C D, Fig. 1, be filled with water to the level E F: then it has been found, by experiment, that the velocity of discharge through a small orifice O, in a thin plate, at the distance of half the diameter outside it, in the *vena-contracta*, will be very nearly that due to a heavy body falling freely from the height *h*, of the surface of the water E F, above the



---

* The force of gravity increases with the latitude, and decreases with the altitude above the level of the sea, but not to any considerable extent. If λ be the latitude, and h the altitude, in feet, above the mean sea level, then it may be generally taken that

$$g = 32{\cdot}17\ (1 - {\cdot}0029 \cos 2\lambda) \times \left(1 - \frac{2h}{R}\right),$$

in which R, the radius of the earth at the given latitude is equal to

$$20887600\ (1 + {\cdot}0016 \cos 2\lambda).$$

centre of the orifice. The velocity of discharge determined by the equation $v = \sqrt{2\,g\,h}$, for falling bodies, is, therefore, called the "*theoretical velocity.*" If v_d be now put for the actual mean velocity of discharge in the *vena-contracta*, and c_v for its ratio to the theoretical velocity v, we shall get $v_d = c_v\,v$; and by substituting for v, its value $\sqrt{2\,g\,h}$,

(2.) $$v_d = c_v\,\sqrt{2\,g\,h},$$

c_v is termed "*the coefficient of velocity;*" its numerical value, at about half the diameter from the orifice, is about ·974; and, consequently,

$$v_d = \cdot 974\,\sqrt{2\,g\,h}.$$

This for measures in inches becomes

$$v_d = 27\cdot 077\,\sqrt{h},*$$

and for measures in feet

$$v_d = 7\cdot 816\,\sqrt{h}.$$

The orifice o, is termed an *horizontal orifice* in Fig. 1, and in Fig. 2 a *vertical* or *lateral orifice.* When small, each is found to have practically the same velocity of discharge, when the centres of the contracted sections are at the same depth, h, below the surface; but when lateral orifices are large, or rather deep, the velocity at the centre is not, even practically, the mean velocity; and in thick plates and modified forms of adjutage, the mean velocities are found to vary.

---

* The velocities for different heights calculated from this formulæ are given in the column numbered 2, TABLE II. It has been latterly asserted in a *Blue Book* that theoretically $v_d = \frac{2}{3}\sqrt{2\,g\,h}$. It is not necessary here to combat this error, which confounds the discharge with its velocity, and a single practical fact, applicable only to a thin plate, with a theoretical principle. The experimental discharge approximates to $\frac{2}{3}\sqrt{2\,g\,h}$ multiplied by the area of the orifice; but the theoretical velocity $\sqrt{2\,g\,h}$ *always* approximates to the experimental velocity, or ·974 $\sqrt{2\,g\,h}$, obtained immediately outside the orifice in the *vena-contracta*. It would be unnecessary to allude to this theory here if it were not supported and put forward by three engineers whose authority in practical questions may mislead others. *Vide* p. 4 of "Brief observations of Messrs. *Bidder*, *Hawksley*, and *Bazalgette* on the answers of the *Government Referees* on the METROPOLITAN MAIN DRAINAGE, ordered by the House of Commons to be printed 13th July, 1858."

VENA-CONTRACTA AND CONTRACTION.

It has been found that the diameter of a column issuing from a circular orifice in a thin plate, is contracted to very nearly eight-tenths of the whole diameter at the distance of the radius from it, and that at this distance the contraction is greatest. The ratio of the diameter of the orifice to that of the contracted vein, *vena-contracta,* is not always found constant by the same or different experimentalists.

Newton	makes it	1 :	·841,	and, therefore, that of the areas as 1 :	·707
Poleni	,,	1 :	·846 ·788	,, ,, 1 :	·7156 ·622
Borda	,,	1 :	·802	,, ,, 1 :	·6432
Michellotti	,,	1 :	·8	,, ,, 1 :	·64
Bossut	,,	1 :	·81 ·818	,, ,, 1 :	·656 ·669
Du Buât	,,	1 :	·816	,, ,, 1 :	·667
Venturi	,,	1 :	·798	,, ,, 1 :	·637
Eytelwein	,,	1 :	·8	,, ,, 1 :	·64
Bayer	,,	1 :	·7854	,, ,, 1 :	·617

Bayer's value for the contraction has been determined on the hypothesis, that the velocities of the particles of water as they approach the orifice from all sides, are inversely as the squares of their distances from its centre; and the calculations made of the discharge from circular, square, and rectangular orifices, on this hypothesis, coincide pretty closely with experiments.

APPROXIMATE FORM OF THE CONTRACTED VEIN.

Let O R $= d$, Fig. 3, be the diameter of an orifice; then at the distance R $s = \frac{d}{2}$ the contraction is found to be greatest; assuming that the contracted diameter $o\ r = \cdot7854\ d$. Suppose the fluid column between O R and $o\ r$ to be so reduced, that the curve lines R r and O o shall become arcs of circles, then it is easy to show from the properties of the circle, that the radius C r must be equal to $1\cdot22\ d$. The mean velocity in the orifice, O R, is to that in the *vena-contracta*, $o\ r$, as $\cdot617 : 1$; and the mouth piece, R r o O, Fig. 4, in which O $p = \frac{1}{2}$ O R, and $o\ r = \cdot7854 \times$ O R, will give for the velocity of discharge at $o\ r$, the *vena-contracta*,

$$v_d = \cdot974 \sqrt{2\,g\,h} = 7\cdot816 \sqrt{h},$$

in feet very nearly.* In speaking of the velocity of

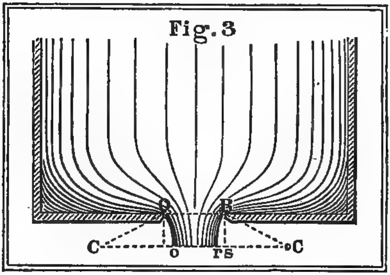


Fig. 3

---

* Correctly the curves O o and R r should perhaps have the line O R continued, for their common tangent. No practically useful result would, however, follow from a more accurate construction than is here given. Running a file round the arris O, R, will effect the object better than a complex construction.

discharge from orifices in thin plates, we always assume it to be the velocity in the *vena-contracta*, and not that in the orifice itself, which varies with the coefficient of discharge, unless in Table II., where the mean velocity in the latter, as representing $c_d \sqrt{2 g h}$, is also given.

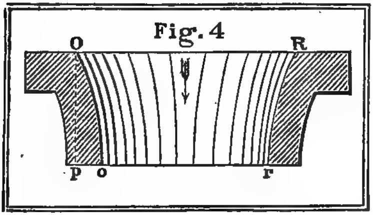


Fig. 4

COEFFICIENTS OF CONTRACTION AND OF DISCHARGE.

Put A for the area of the orifice O R, Fig. 3, and $c_c \times$ A for that of the contracted section at $o\ r$, then c_c is called the "*coefficient of contraction.*" The velocity of discharge v_d is equal to $c_v \sqrt{2 g h}$, equation (2). Multiply this by the area of the contracted section $c_c \times$ A, and there is found for the discharge

$$D = c_v \times c_c \times A \sqrt{2 g h}.$$

It is evident A $\sqrt{2 g h}$ would be the discharge if there were no contraction and no change of velocity due to the height, h; $c_v \times c_c$ is therefore equal to the coefficient of discharge. Call the latter c_d, and there results the equation

$$(3.) \qquad c = c_v \times c_c,$$

and hence the "*coefficient of discharge*" *is equal to the product of the coefficients of velocity and contraction.*

The expression $c_v\, c_c \sqrt{2 g h} = c_d \sqrt{2 g h}$ represents the mean velocity *in the orifice;* the coefficient for this is, therefore, equal to c_d. The values cf the velocity $c_d \sqrt{2 g h}$, for different heights and coefficients, are given in TABLE II.

In the foregoing expression for the discharge D, h must be so taken, that the velocity at that depth shall be the mean velocity in the orifice A. *In full prismatic tubes the coefficients of velocity and discharge are equal to each other.*

MEAN AND CENTRAL VELOCITY.

In order to find the mean velocity of discharge from an orifice, it is, in the first instance, necessary to determine the velocity due to each point in its surface, and the discharge itself; after which, the mean velocity is found by simply dividing the area of the orifice into the discharge. The velocity due to the height of water at the centre of a circular, square, or rectangular orifice, is not strictly the mean velocity, nor is the latter in these, or other figures, that at the centre of gravity. When, however, an orifice is small in proportion to its depth in the water, the velocity of efflux determined for the centre approaches very closely to the mean velocity; and, indeed, at depths exceeding four times the depth of the orifice, the error in assuming the mean velocity to be that at the centre of the orifice is so small as to be of little or no practical consequence, and for lesser depths it never exceeds six per cent. It is, therefore, for greater simplicity, the practice to determine the velocity from the depth h of the centre of the orifice, unless in weirs or notches; and the coefficients of discharge and velocity in the following pages have been calculated from experiments on this assumption, unless it shall be otherwise stated.

DISCHARGES THROUGH ORIFICES OF DIFFERENT FORMS IN THIN PLATES.

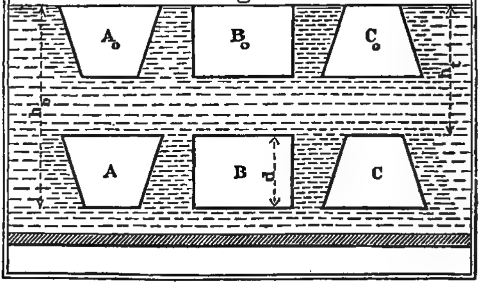
The orifices which the engineer has to deal with in practice are square, rectangular, or circular; and sometimes, perhaps, triangular or quadrangular in form. It will be necessary to give here only the theoretical expressions for the discharge and velocity for each kind of form, but as the demonstrations are unsuited to the purposes of this work they shall be omitted.

TRAPEZOIDAL ORIFICES WITH TWO HORIZONTAL SIDES.

Put *d* for the vertical depth of an orifice, h_t for the altitude of pressure at top, above the upper side, and h_b for the altitude at bottom, above the lower side, then

$$h_b - h_t = d.$$

Fig. 5



Let also the top or upper side of the orifice A or C, Fig. 5, be represented by l_t, and the lower or bottom side by l_b, and put $\frac{l_t + l_b}{2} = l$.

Now, when $l_t = l_b$, the trapezoid becomes a parallelogram whose length is l and depth d; and putting h for the depth to the centre of gravity, there exists the equation

$$h_t + \frac{d}{2} = h_b - \frac{d}{2} = h.$$

The general expression for the discharge, D, through a trapezoidal orifice, A, is then

$$(4.)\quad \mathrm{D} = c_d \sqrt{2\,g} \times \tfrac{2}{3}\left\{l_b h_b^{\frac{3}{2}} - l_t h_t^{\frac{3}{2}} + \tfrac{2}{5}(l_t - l_b)\frac{h_b^{\frac{5}{2}} - h^{\frac{5}{2}}}{d}\right\},$$

in which c_d is the coefficient of discharge; and when the smaller side is uppermost as at C,

$$(5.)\quad \mathrm{D} = c_d \sqrt{2\,g} \times \tfrac{2}{3}\left\{l_b h_b^{\frac{3}{2}} - l_t h_t^{\frac{3}{2}} - \tfrac{2}{5}(l_b - l_t)\frac{h_b^{\frac{5}{2}} - h_t^{\frac{5}{2}}}{d}\right\}.$$

PARALLELOGRAMIC AND RECTANGULAR ORIFICES.

When $l_t = l_b = l$, the orifice becomes a parallelogram, or a rectangle, B, and then the discharge is

$$(6.)\qquad \mathrm{D} = c_d \sqrt{2\,g} \times \tfrac{2}{3}\,l\,\{h_b^{\frac{3}{2}} - h_t^{\frac{3}{2}}\}.$$

NOTCHES.

When the upper sides of the orifices A, B, and C, rise to the surface as at A_0, B_0, and C_0, h_t becomes nothing, and then, as $h_b = d$, for the trapezoidal notch A_0 with the larger side up,

$$(7.)\quad \mathrm{D} = c_d \sqrt{2\,g} \times \tfrac{2}{3}\,d^{\frac{3}{2}}\left\{l_b + \tfrac{2}{5}(l_t - l_b)\right\}$$

$$= \tfrac{2}{15}\,c_d \sqrt{2\,g}\,d^{\frac{3}{2}}(2\,l_t + 3\,l_b).$$

Also for the trapezoidal notch, C$_o$, with the smaller side up,

$$(8.)\quad \mathrm{D} = c_d\sqrt{2\,g} \times \tfrac{2}{3}\,d^{\frac{3}{2}}\left\{ l_d - \tfrac{2}{5}\,(l_b - l_t) \right\}$$

$$= \tfrac{2}{15}\,c_d\sqrt{2\,g}\,d^{\frac{3}{2}}(2\,l_t + 3\,l_b),$$

which is the same in form, but not in value, as the preceding equation. And for a parallelogramic or rectangular notch B$_o$,

$$(9.)\qquad \mathrm{D} = c_d\sqrt{2\,g} \times \tfrac{2}{3}\,l\,d^{\frac{3}{2}} = \tfrac{2}{3}\,c_d\,l\,d^{\frac{3}{2}}\sqrt{2\,g}.$$

It is easy to perceive that the forms of equations (4) and (5), and also of equations (7) and (8), are identical. The values for the discharge in equations (6) and (9) are equally applicable, whether the form of the orifice be a parallelogram or a rectangle, the only difference being in the value of the coefficient of discharge, c_d, which becomes only slightly modified for any form of orifice, and may be taken at ·617 when it is in a thin plate for each.

TRIANGULAR ORIFICES WITH HORIZONTAL BASES, AND RECTILINEAL ORIFICES IN GENERAL.

When the length of the lower side, $l_b = 0$, the orifice becomes a triangle, D, Fig. 6, with the base upwards.

In this case, equation (4) becomes

$$(10.)\quad \mathrm{D} = c_d\sqrt{2\,g} \times \tfrac{2}{3}\,l_t\left\{\tfrac{2}{5} \times \frac{h_b^{\frac{5}{2}} - h_t^{\frac{5}{2}}}{d} - h_t^{\frac{3}{2}}\right\};$$

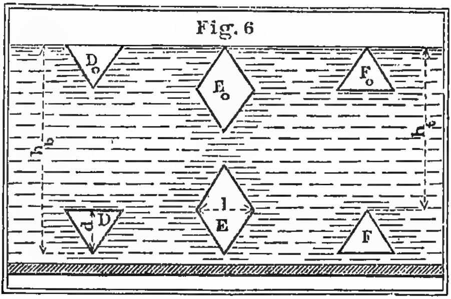
which gives the discharge through the triangular orifice, D.

When $l_t = 0$, in equation (5), the orifice becomes a triangle, F, with the base downwards; in this case, the value of the discharge is

$$(11.)\quad D = c_d \sqrt{2\,g} \times \frac{2}{3}\, l_b \left\{ h_b^{\frac{3}{2}} - \frac{2}{5} \times \frac{h_b^{\frac{5}{2}} - h_t^{\frac{5}{2}}}{d} \right\}.$$

As any triangular orifice whatever can be divided into two others by a line of division through one of the angles parallel to the horizon; and as the discharge from the triangular orifice D or F is the same as for any other on the same base and between the same parallels, by such a division, the discharge from any triangle not having one side parallel to the horizon can easily be found, and thence the discharge from any rectilineal figure whatever by dividing it into triangles.

Fig. 6



If the triangle F be raised so that the base shall be on the same level with the upper side of the triangular orifice D; if, also, the bases be equal, and also the depths, by adding equations (10) and (11), and making the necessary changes indicated by the diagram, there is found

(12.) $$\text{D} = c_{\text{d}}\sqrt{2\,g} \times \frac{4}{15}\frac{l}{d}\left\{h_{\text{b}}^{\frac{5}{2}} + h_{\text{t}}^{\frac{5}{2}} - 2 \times \overline{h_{\text{t}} + d}\,\Big\rceil^{\frac{5}{2}}\right\}$$

for the discharge from a parallelogram E with one diagonal horizontal. Now this is the same as the discharge from any quadrilateral figure whatever, having the same horizontal diagonal, and also having the upper and lower angles on the same parallels, or at the same depths, as those of the parallelogram. If the orifices D, F, and E rise to the surface of the water, as at D_0, E_0, F_0, then for the discharge from the notch D_0 there results

(13.) $$\text{D} = c_{\text{d}}\sqrt{2\,g} \times \tfrac{4}{15}\,l_{\text{t}}d^{\frac{3}{2}};$$

which for a right angled triangle becomes

$$\text{D} = c_{\text{d}}\sqrt{2\,g} \times \tfrac{8}{15}d^{\frac{5}{2}}.\text{*}$$

For the discharge from the notch F_0,

(14.) $$\text{D} = c_{\text{d}}\sqrt{2\,g} \times \tfrac{6}{15}\,l_{\text{b}}\,d^{\frac{3}{2}}:$$

---

* In the Civil Engineer and Architects' Journal, 1858, p. 370, it is stated that Professor Thomson, now of the University of Glasgow, gave at the British Association in Leeds for a right angled triangle, for discharges of from 2 to 10 cubic feet per minute, the expression $\text{Q} = {\cdot}317\,\text{H}^{\frac{5}{2}}$, in which Q is the quantity in cubic feet per minute, and H the head in inches. Now the above equation for a coefficient of ·617 becomes, for inch measures, $\text{D} = 17{\cdot}153 \times \frac{8}{15}\,d^{\frac{5}{2}} = 9{\cdot}15\,d^{\frac{5}{2}}$; or by multiplying by 60, and dividing by 1728, to reduce the discharge to feet per minute, $\text{D} = {\cdot}317\,d^{\frac{5}{2}}$, identically the same as Professor Thomson derived from his experiments. All sections of a triangular notch are similar triangles, and hence the advantage of a triangular-notch-gauge, where it can be used, as, probably, the coefficient remains constant throughout. Professor Thomson first drew attention to this. But the coefficient ·617 gives practically correct results *for all forms of orifices in thin plates.*

and for the discharge through the notch E_0,

$$(15.)\ \text{D} = c_d \sqrt{2g} \times \frac{4l}{15d} \{ h_b^{\frac{5}{2}} - 2d^{\frac{5}{2}} \} = c_d \sqrt{2g} \times \cdot 9752\, l d^{\frac{3}{2}}.$$

When the parallelogram E_0 becomes a square $l = 2\ d$, and hence,

$$(16.)\ \text{D} = c_d \sqrt{2g} \times \cdot 9752 l^{\frac{5}{2}} \times \sqrt{\tfrac{1}{8}} = c_d \sqrt{2gl} \times \cdot 34478\, l^2.$$

The foregoing equations enables the engineer to find an expression for the discharge from any rectilineal orifice whatever; as it can be divided into triangles, the discharge from each of which can be determined as already shown in the remark following equation (11.) The examples which are given will be found to comprehend every form of rectilineal orifice which occurs in practice; but for the greater number of orifices, sunk to any depth below the surface, the discharge will be found with sufficient accuracy by multiplying the area by the velocity due to the centre of gravity.

CIRCULAR AND SEMICIRCULAR ORIFICES.

The discharge through circular and semicircular orifices in thin plates can only be represented by means of infinite series. Represent by s_1 the sum of the series

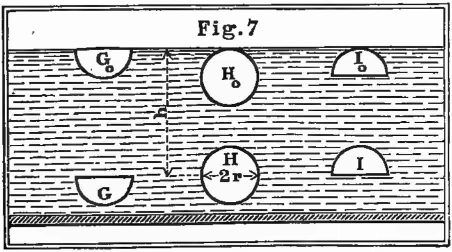


Fig. 7

$$\left\{ \frac{1}{2} - \left(\frac{1}{2}\cdot\frac{1}{4}\right)\left(\frac{1}{2}\cdot\frac{1}{4}\right)\frac{r^2}{h^2} - \left(\frac{1}{2}\cdot\frac{1}{4}\cdot\frac{3}{6}\cdot\frac{5}{8}\right)\left(\frac{1}{2}\cdot\frac{1}{4}\cdot\frac{3}{6}\right)\frac{r^4}{h^4} \right.$$
$$\left. - \left(\frac{1}{2}\cdot\frac{1}{4}\cdot\frac{3}{6}\cdot\frac{5}{8}\cdot\frac{7}{10}\cdot\frac{9}{12}\right)\left(\frac{1}{2}\cdot\frac{1}{4}\cdot\frac{3}{6}\cdot\frac{5}{8}\right)\frac{r^6}{h^6} - \&c. \right\}:$$

Also represent by s_2 the sum of the series

$$\frac{2}{3{\cdot}1416}\left\{ \left(\frac{1}{2}\cdot\frac{1}{3}\right)\frac{r}{h} + \left(\frac{1}{2}\cdot\frac{1}{4}\cdot\frac{3}{6}\right)\left(\frac{1}{3}\cdot\frac{2}{5}\right)\frac{r^3}{h^3} \right.$$
$$\left. + \left(\frac{1}{2}\cdot\frac{1}{4}\cdot\frac{3}{6}\cdot\frac{5}{8}\cdot\frac{7}{10}\right)\left(\frac{1}{3}\cdot\frac{2}{5}\cdot\frac{4}{7}\right)\frac{r^5}{h^5} + \&c. \right\}:$$

then the discharge from the semicircle G, Fig. 7, with the diameter upwards and horizontal, is

(17.) $\text{D} = c_d \sqrt{2\,g\,h} \times 3{\cdot}1416\, r^2\, (s_1 + s_2).$

And the discharge from the semicircle I, with the diameter downwards and horizontal is,

(18.) $\text{D} = c_d \sqrt{2\,g\,h} \times 3{\cdot}1416\, r^2\, (s_1 - s_2).$

If A be put for the area, then also for the discharge from a circle H,

(19.) $\text{D} = c_d \sqrt{2\,g\,h} \times 2\,\text{A}\, s_1.$

In each of these three equations (17), (18), and (19), h is the depth of the centre of the circumference below the surface, and r the radius.

When the orifices rise to the surface, then for the discharge from a semicircular notch G_o, with the diameter horizontal and at the surface,

(20.) $\text{D} = c_d \sqrt{2gr} \times {\cdot}9586\, r^2 = c_d \sqrt{2gr} \times {\cdot}6103\,\text{A}$;

when the circumference of the semicircle is at the surface, and the diameter horizontal, as at I_o,

(21.) $\text{D} = c_d \sqrt{2gr} \times \frac{4}{15}(\sqrt{128} - 7) r^2 = c_d \sqrt{2gr} \times {\cdot}7324\,\text{A}$;

when the horizontal diameter of the semicircle is uppermost, and at the depth r below the surface,

(22.) $\text{D} = c_d \sqrt{2gr} \times 1{\cdot}8667\, r^2 = c_d \sqrt{2gr} \times 1{\cdot}1884\,\text{A}$;

and when the circumference of the entire circle is at the surface, as at H_0,

(23.) $\mathrm{D} = c_d \sqrt{2\,g\,r} \times 3{\cdot}0171 r^2 = c_d \sqrt{2\,g\,r} \times {\cdot}9604\,\mathrm{A}.$

If it be desirable to reduce equations (20), (21), and (22), to others in which the depth h of the centre of gravity from the surface is contained, it is only necessary to substitute $\frac{h}{{\cdot}4244}$ for r in equation (20), and then the discharge from a semicircle with the diameter at the surface is

(24.) $\mathrm{D} = c_d \sqrt{2\,g\,h} \times {\cdot}0367\,\mathrm{A}.$

Also, by substituting $\frac{h}{{\cdot}5756}$ for r in equation (21), the discharge from a semicircle when the circumference is at the surface and the diameter horizontal is

(25.) $\mathrm{D} = c_d \sqrt{2\,g\,h} \times {\cdot}9653\,\mathrm{A};$

and when the horizontal diameter is uppermost, and at the depth r below the surface $r = \frac{h}{1{\cdot}4244}$ and

(26.) $\mathrm{D} = c_d \sqrt{2\,g\,h} \times {\cdot}9957\,\mathrm{A}.$

As A stands for the area of the particular orifice in each of the preceding expressions for the discharge, it must be taken of double the value, in equation (23) for instance, where it stands for the area of a circle, that it is in equations (20), (21), or (23), where it represents only the area of a semicircle.

DIFFERENCE OF "MEAN VELOCITIES." HOW MUCH.

The mean velocity is easily found by dividing the area into the discharge per second given in the preceding equations. For instance, the mean velocity in the example represented in equation (9), is equal $\frac{2}{3} c_d \sqrt{2 g d}$, which is had by dividing the area $l d$ into the discharge; and in like manner the mean velocity in equation (23) is $\cdot 9604\, c_d \sqrt{2 g r}$. The velocity at the centre of an orifice is that generally taken by practical men. This differs very little from the other five circular and rectangular forms. Even for notches at the surface it is only in excess by from four to six per cent.

PRACTICAL REMARKS ON THE DISCHARGE FROM CIRCULAR ORIFICES.

It has been shown, equation (19), that, for the discharge from a circle, we have

$$\text{D} = c_d \sqrt{2 g h} \times 2\, \text{A}\, s_1,$$

in which h is the depth of the centre, A the area, and s_1 the sum of the series

$$\left\{\frac{1}{2} - \left(\frac{1}{2} \cdot \frac{1}{4}\right)\left(\frac{1}{2} \cdot \frac{1}{4}\right)\frac{r^2}{h^2} - \left(\frac{1}{2} \cdot \frac{1}{4} \cdot \frac{3}{6} \cdot \frac{5}{8}\right)\left(\frac{1}{2} \cdot \frac{1}{4} \cdot \frac{3}{6}\right)\frac{r^4}{h^4} - \&c.\right\}:$$

and it has also been shown, equation (23), that, when the circumference touches the surface, this value becomes

$$\text{D} = c_d \sqrt{2 g r} \times \cdot 9604\, \text{A}.$$

Now when h is very large compared with r, it is easy to perceive that $2\, s_1 = 1$, and hence

$$\text{(27.)} \qquad \text{D} = c_d \sqrt{2 g h} \times \text{A}.$$

As this is the formula commonly used for finding the discharge, it is clear, if the coefficient c_d remain constant, that the result obtained from it for D would be too large. The differences, however, for depths greater than three times the diameter, or $6r$, are practically of no importance; for, by calculating the values of the discharge at different depths, it is found, when

$$(28.)\left\{\begin{array}{llll}
h = r, & \text{that } \mathrm{D} = c_d\sqrt{2gh} \times \cdot 960\,\mathrm{A}; \\
h = \frac{5r}{4}, & ,, \quad \mathrm{D} = c_d\sqrt{2gh} \times \cdot 978\,\mathrm{A}; \\
h = \frac{3r}{2}, & ,, \quad \mathrm{D} = c_d\sqrt{2gh} \times \cdot 985\,\mathrm{A}; \\
h = \frac{7r}{4}, & ,, \quad \mathrm{D} = c_d\sqrt{2gh} \times \cdot 989\,\mathrm{A}; \\
h = 2r, & ,, \quad \mathrm{D} = c_d\sqrt{2gh} \times \cdot 992\,\mathrm{A}; \\
h = 3r, & ,, \quad \mathrm{D} = c_d\sqrt{2gh} \times \cdot 996\,\mathrm{A}; \\
h = 4r, & ,, \quad \mathrm{D} = c_d\sqrt{2gh} \times \cdot 998\,\mathrm{A}; \\
h = 5r, & ,, \quad \mathrm{D} = c_d\sqrt{2gh} \times \cdot 9987\,\mathrm{A}; \\
h = 6r, & ,, \quad \mathrm{D} = c_d\sqrt{2gh} \times \cdot 9991\,\mathrm{A}.
\end{array}\right.$$

These results show very clearly that, for circular orifices, the common expression for the discharge $c_d\sqrt{2gh} \times \mathrm{A}$ is abundantly correct for all depths exceeding three times the diameter, and that for lesser depths the extreme error cannot exceed four per cent. in reduction of the quantity found by this formula. Hereafter, when discussing the value of c_d, it will be shown that from the sinking of the surface, and perhaps other causes, the discharge at lesser depths is even larger than that exhibited by the expression $c_d\sqrt{2gh} \times \mathrm{A}$, the value of the coefficient of discharge, c_d, being found to increase near the surface as the depths h decrease. In fact, *the sides and length of the orifice, the* *rounding of the arrises, and the depth and position with reference to the sides of the vessel, and surface of the water, are of far greater practical importance than extreme accuracy in the mathematical formulæ, which when complex may be of little or no practical value.*

PRACTICAL REMARKS ON THE DISCHARGE FROM RECTANGULAR ORIFICES.

It has been shown, equation (6), that the discharge from rectangular orifices, with two sides parallel to the horizon or surface of the water, is expressed by the equation

$$\text{D} = c_{\text{d}} \times \frac{2}{3} \sqrt{2g} \times l \left\{ h_{\text{b}}^{\frac{3}{2}} - h_{\text{t}}^{\frac{3}{2}} \right\},$$

in which l is the horizontal length of the orifice, h_{b} the depth of water on the lower, and h_{t} the depth on the upper, side. As it is desirable in practice to change this form into a more simple one, in which the height h of the centre and depth d of the orifice only shall be included, then $h_{\text{b}} = h + \frac{d}{2}$ and $h_{\text{t}} = h - \frac{d}{2}$. By substituting these values of h_{b} and h_{t} in the foregoing equations, and developing the result into a series, the terms of which, after the third, may be neglected, and putting A for the area $l\,d$, there results

$$(29.)\quad \text{D} = c_{\text{d}} \sqrt{2gh} \times \text{A} \left\{ 1 - \frac{d^2}{96\,h^2} \right\} \text{ very nearly.}$$

Therefore for the accurate theoretical discharge

$$30.)\quad \text{D} = c_{\text{d}} \sqrt{2gh} \times \frac{2}{3} \text{A} \left\{ \frac{(h+\frac{1}{2}d)^{\frac{3}{2}} - (h-\frac{1}{2}d)^{\frac{3}{2}}}{d\,h^{\frac{1}{2}}} \right\};$$

for the approximate discharge

$$\text{D} = c_d \sqrt{2\,g\,h} \times \text{A} \left\{ 1 - \frac{d^2}{96\,h^2} \right\};$$

and for the discharge by the common formula

$$\text{D} = c_d \sqrt{2\,g\,h} \times \text{A}.$$

When the head (h) is large compared with (d), the height of the orifice, each of the three last equations gives the same value for the discharge; but as the common expression $c_d \sqrt{2\,g\,h} \times \text{A}$ is the most simple; and as the greatest possible error in using it for lesser depths does not exceed six per cent., viz. when the orifice rises to the surface and becomes a notch, it is evidently that formula best suited for practical purposes. The following table and equations will show more clearly the differences in the results as obtained from *the true, the approximate, and the common formulæ*, applied to "lesser" heads; and they will also explain, to some extent, why "coefficients" determined from the common formula, and that used by Poncelet and Lesbros, should decrease as the orifice approaches the surface.

	1	2	3
	$h = \frac{d}{2}$,	$\text{D} = c_d \sqrt{2gh} \times \cdot 9428\text{A}.$	$\text{D} = c_d \sqrt{2gh} \times \cdot 9583\,\text{A}.$
	$h = \frac{5\,d}{8}$,	,, ,, ,, $\times \cdot 9693\text{A}.$	,, ,, ,, $\times \cdot 9733\,\text{A}.$
	$h = \frac{3\,d}{4}$,	,, ,, ,, $\times \cdot 9796\text{A}.$	,, ,, ,, $\times \cdot 9815\,\text{A}.$
	$h = \frac{7\,d}{8}$,	,, ,, ,, $\times \cdot 9854\text{A}.$	,, ,, ,, $\times \cdot 9864\,\text{A}.$
(31.)	$h = d$	,, ,, ,, $\times \cdot 9890\text{A}.$	,, ,, ,, $\times \cdot 9896\,\text{A}.$

$$(31.)\left\{\begin{array}{lll} h=\frac{3\,d}{2}, & \mathrm{D}=c_{\mathrm{d}}\sqrt{2gh}\times\cdot 9953\mathrm{A}. & \mathrm{D}=c_{\mathrm{d}}\sqrt{2gh}\times\cdot 9954\,\mathrm{A}. \\ h=2\,d, & ,,\quad ,,\quad ,,\ \times\cdot 9974\mathrm{A}. & ,,\quad ,,\quad ,,\ \times\cdot 9974\,\mathrm{A}. \\ h=\frac{5\,d}{2}, & ,,\quad ,,\quad ,,\ \times\cdot 9983\mathrm{A}. & ,,\quad ,,\quad ,,\ \times\cdot 9983\,\mathrm{A}. \\ h=3\,d, & ,,\quad ,,\quad ,,\ \times\cdot 9988\mathrm{A}. & ,,\quad ,,\quad ,,\ \times\cdot 9988\,\mathrm{A}. \\ h=\frac{7\,d}{2}, & ,,\quad ,,\quad ,,\ \times\cdot 9991\mathrm{A}. & ,,\quad ,,\quad ,,\ \times\cdot 9991\,\mathrm{A}. \\ h=4\,d, & ,,\quad ,,\quad ,,\ \times\cdot 9994\mathrm{A}. & ,,\quad ,,\quad ,,\ \times\cdot 9994\,\mathrm{A}. \\ h\times 10d, & ,,\quad ,,\quad ,,\ \times\cdot 9999\mathrm{A}. & ,,\quad ,,\quad ,,\ \times\cdot 9999\,\mathrm{A}. \end{array}\right.$$

In the foregoing Table the first column contains the head at the centre of the orifice expressed in parts of its height d; the second contains the values of the discharges according to equation (30); and the third column contains the approximate values determined from equation (29), the results in which are something larger than those in column 2, derived from the correct formula. The numerical coefficients of A, at every depth, are less in both than unity, the constant coefficient according to the common formula. This latter, therefore (as in circular orifices), gives results exceeding the true ones, but the excess is inappreciable at greater depths than $h = 3\,d$, and for lesser depths than this the error cannot exceed six per cent. It may be useful to remark here, that when the orifice rises to the surface and becomes a notch, *the "centre of mean velocity" is at four-ninths of the depth, and the centre of gravity at two-thirds of the depth from the surface.* The former fraction is the square of the latter.

SECTION III.

EXPERIMENTAL RESULTS AND FORMULÆ.—COEFFICIENTS OF DISCHARGE FOR ORIFICES AND NOTCHES.

Heretofore the numerical values of the general coefficient of discharge c_d have been only dwelt upon partially. In order to determine its value under different circumstances more particularly, it will be now necessary to consider some of the experiments which have been made from time to time. These do not always give the same results, even when conducted under the same circumstances and by the same parties, and there appears to exist a certain amount of error, more or less, inseparable from the subject. The experiments with orifices in thin plates afford the most consistent results; but even here the differences are sometimes greater than might be expected. In many of the earlier experiments the value of the coefficient c_d comes out too large, which arises, very probably, from the orifices experimented with not being in thin plates, and partaking, more or less, of the nature of short tubes or mouth-pieces with rounded arrises, which, as it shall be seen, give larger coefficients than simple orifices. When an orifice is in the bottom of a vessel, it would appear more correct to measure the head from the surface to the *vena-contracta* than from the surface to the orifice itself; and as any error in measuring the head in any experiment must affect the value of the coefficient derived from such experiment, so as to increase it when the error is to make the head less, and *vice versâ*, it appears that heads measured to an orifice in the bottom of a vessel, and not to the *vena-contracta*, must give larger coefficients from the experimental results than, perhaps, the true ones. The coefficients in the following pages have been almost all arranged and calculated, by the author, from the original experiments.

In 1739 Dr. Bryan Robinson made some experiments on the discharge through small circular orifices, from one-tenth to eight-tenths of an inch in diameter, with heads of two and four feet,* which give the following coefficients.

COEFFICIENTS FROM DR. B. ROBINSON'S EXPERIMENTS.

Heads.	$\frac{1}{10}$ inch diameter.	$\frac{4}{10}$ inch diameter.	$\frac{6}{10}$ inch diameter.	$\frac{8}{10}$ inch diameter.
2 feet head	·768	·767	·761	·728
4 feet head	·768	·774	·765	·742

These results are pretty uniform, and the values from which they are derived are said to be "means taken from five or six experiments;" as values of c_d they are, however, too high. The apparatus made use of is not described; but it is probable, from the results, that the plate containing the hole or orifice was of some thickness, and that the inner arris was slightly rounded. There is here, however, a very perceptible increase in the coefficients for the smaller orifices, but none for the smaller depth.

---

* Helsham's Lectures, p. 390. Dublin, 1739.

In a paper in the Transactions of the Royal Irish Academy, vol. ii. p. 81, read March 1st, 1788, Dr. Matthew Young determines the value of the coefficient for an orifice $\frac{2}{10}$ inch in diameter, with a mean head of 14 inches, to be ·623. The manner in which this value is determined is very elegant; viz., by comparing the observed with the theoretical time of the water, in the vessel, sinking from 16 inches to 12 inches.

The following experiments by Michelotti, with circular orifices from 1 to about 3 inches diameter, and with from 6 to 23 feet heads, give for the mean value $c_d = \cdot 613$; and for square orifices of from 1 to 9 square inches in area, at like depths, the mean value of $c_d = \cdot 628$. The experiments are given in French feet and inches, according to which standard in feet, $\text{D} = 7\cdot 77 \text{ A} \sqrt{h} \times t$; t being the time in seconds.* As the time of discharge in these experiments varies from ten minutes to an hour, and as the depths are considerable, the results must be looked upon as pretty accurate; and it is worthy of

---

* The value of $\sqrt{2 g h}$, equation (1), for measures in French feet, is $7\cdot 77 \sqrt{h}$, and for measures in French inches, $26\cdot 9 \sqrt{h}$; g being equal to 30·2 feet, or 362·4 inches, French measure. One French foot is equal to 1·06578 English feet, and the inches preserve the same proportion. The resulting coefficients must be the same, whatever standards the calculations are made from. Many of the most valuable formulæ and experiments in hydraulics are given in French measures of the old style. As the object, however, in the present section, is to determine from experiment the relation of the experimental to the theoretical discharge, it is not necessary to reduce the experiments to other measures than those in the original; but the value of the force of gravity, g, must, of course, be taken in those measures with which the experiments were made. In the French decimal, or modern style, the

COEFFICIENTS FROM MICHELOTTI'S EXPERIMENTS.

Description and size of orifice, in French inches.	Depth of the centre of the orifices in French feet.	Quantity discharged in cubic feet.	Time of discharge in seconds.	Theoretical time, calculated from $t = \frac{D}{7 \cdot 77 A \sqrt{h}}$	Resulting coefficients of discharge.
Square orifice, 3" × 3"	6·613	463·604	600	371·3	·619
	6·852	566·458	720	445·6	·619
	11·676	516.785	510	311·4	·610
	11·818	612·118	600	366·6	·611
	21·691	415·437	300	183·7	·612
	21·715	499·222	360	220·6	·613
Mean value of the coefficient; square orifice 3" × 3" . .					·614
Square orifice, 2" × 2"	6·625	329·806	900	594·	·660
	11·426	423·465	900	580·4	·645
	21·442	385·333	600	385·7	·643
Mean value of the coefficient; square orifice 2" × 2" . . .					·649
Square orifice, 1" × 1"	6·757	158·549	1800	1585·	·628
	11·889	163·792	1440	880·6	·612
	21·507	562·944	3600	2249·9	·625
Mean value of the coefficient; square orifice 1" × 1" . . .					·621
Circular orifice, 3" diameter	6·694	542·85	900	550·1	·611
	11·590	570·972	720	439·6	·610
	21·611	521·299	480	293·8	·612
Mean value of the coefficient; circular orifice 3" diameter .					·611
Circular orifice, 2" diameter	6·785	488·687	1800	1108·1	·616
	11·722	589·535	1680	1016·4	·605
	21·903	575·486	1200	725·9	·605
Mean value of the coefficient; circular orifice 3" diameter .					·609
Circular orifice, 1" diameter	6·875	247·354	3600	2227·	·619
	11·722	324·11	3600	2233·	·620
	21·903	444·535	3600	2237·2	·621
Mean value of the coefficient; circular orifice 1" diameter .					·620

note that here the coefficients are larger for square than for circular orifices.

It may be remarked here in passing how universal the coefficients ·613 to ·628 are for all forms of orifices in thin plates; or with the outside arrises chamfered. Indeed, the coefficient ·62 may always be used with certainty, for practical purposes, for every orifice of this kind, whether at the surface in the form of a notch, or at the sides or bottom of a vessel, if the section of the approaching water be large in proportion to the area of the discharging orifice or notch. By coefficient of course is here meant that decimal which, multiplied by the theoretical value, gives the practical result; and this is substantially the same for notches and orifices sunk below the surface, as will appear further on. There appears, however, an utter want of accuracy in using the coefficient ·62 or thereabouts in gauging for all orifices, weirs included, no matter what the thickness or form of the orifice or crest of a weir may be, or area of the approaching channel. These will cause the coefficient to vary from ·5 to 1 or more, and hence the necessity for endeavouring to reduce this portion of the subject to rule.

The experiments made by the Abbé Bossut, contained in the following table, give the mean value of c_d, for both circular and square orifices, equal to ·616

---

metre is equal to 3·2809 English feet, or 39·371 inches. The tenth part of a metre is the decimetre, and the tenth part of the decimetre the centimetre, as the names imply. TABLE XIV. contains the weights and measures in general use in Great Britain and France, with their general ratios to each other.

nearly; and it may be perceived that, for the small depth in the last experiment, the coefficient rises so high as ·649. These and other experiments led the Abbé to construct a table of the discharges, at different depths, from a circular orifice 1 inch in diameter, from which the author has determined the following table of coefficients. These increase, as the orifice approaches the surface, from ·617 to ·621; and at lesser depths

COEFFICIENTS FROM BOSSUT'S EXPERIMENTS.

Description, position, and size of orifice, in French inches.	Depth of the centre of the orifice in French inches.	Number of French cubical inches discharged per minute.	Theoretical discharge per minute. $D = 1614 A\sqrt{h}$.	Resulting coefficients.
Horizontal and circular, ½" diameter	140·832	2311	3760·8	·614
Horizontal and circular, 1" diameter	140·832	9281	15043·3	·617
Horizontal and circular, 2" diameter	140·832	37203	60173·1	·618
Horizontal and rectangular, 1" × ¼"	140·832	2933	4788·4	·613
Horizontal and square, 1" × 1" . . .	140·832	11817	19153·7	·617
Horizontal and square, 2" × 2" . .	140·832	47361	76614·6	·617
Lateral and circular, ½" diameter .	108·	2018	3293·3	·613
Lateral and circular, 1" diameter .	108·	8135	13173·3	·617
Lateral and circular, ½" diameter .	48·	1353	2195·5	·616
Lateral and circular, 1" diameter .	48·	5436	8782·2	·616
Lateral and circular, 1" diameter .	0·5833	628	968·	·649

COEFFICIENTS DEDUCED FROM BOSSUT'S EXPERIMENTS.

Heads, in feet.	Coefficients.	Heads, in feet.	Coefficients.	Heads, in feet.	Coefficients.
1	·621	6	·620	11	·619
2	·621	7	·620	12	·618
3	·621	8	·619	13	·618
4	·620	9	·619	14	·618
5	·620	10	·619	15	·617

than 1 foot other experiments show an increase in the coefficient up to ·650. The experiments of Poncelet and Lebros show, however, a reduction in the coefficients for square orifices 8″ × 8″ as they approach the surface from ·601 to ·572.

Brindley and Smeaton's experiments, with an orifice 1 inch square placed at different depths, give a mean value for c_d of ·637. The last experiment, with an orifice only $\frac{1}{2}$ inch by $\frac{1}{2}$ inch, gives so small a coefficient as ·557 placed at a depth of 6 feet!

COEFFICIENTS CALCULATED FROM BRINDLEY AND SMEATON'S EXPERIMENTS.

1 foot head : orifice 1″ × 1″ : coefficient ·639
2 feet head : orifice 1″ × 1″ : coefficient ·635
3 feet head : orifice 1″ × 1″ : coefficient ·648 mean ·637.
4 feet head : orifice 1″ × 1″ : coefficient ·632
5 feet head : orifice 1″ × 1″ : coefficient ·632
6 feet head : orifice $\frac{1}{2}$″ × $\frac{1}{2}$″ : coefficient ·557

For notches 6 inches wide and from 1 to $6\frac{1}{2}$ inches deep, Brindley and Smeaton's experiments give the mean value of $c_d = ·637$. The coefficients of discharge

COEFFICIENTS FOR NOTCHES, CALCULATED FROM BRINDLEY AND SMEATON'S EXPERIMENTS.

Ratio of the length to the depth.	Size of notches in inches.	Coefficients.	Ratio of the length to the depth.	Size of notches in inches.	Coefficients.
·92 to 1	$6 \times 6\frac{1}{2}$	·633	3·7 to 1	$6 \times 1\frac{5}{8}$	·638
1·07 to 1	$6 \times 5\frac{5}{8}$	·571	4·4 to 1	$6 \times 1\frac{3}{8}$	·654
1·2 to 1	6×5	·609	4·8 to 1	$6 \times 1\frac{1}{4}$	·681
1·92 to 1	$6 \times 3\frac{1}{8}$	·602	6· to 1	6×1	·713
2·4 to 1	$6 \times 2\frac{5}{16}$ *	·636	Mean value.		·637

---

* The depth is misprinted $2\frac{5}{10}$ inches in the Encyclopædias, the resulting coefficient for which would be ·568 instead of ·636 as above, for a depth of $2\frac{5}{16}$ inches.

for notches and orifices appear to differ as little from each other as those for either do in themselves. The results also show a general though not uniform increase in the coefficients for smaller depths.

Du Buât's experiments with notches 18·4 inches long, give the mean value of c_a=·632, which differs very little from the mean value determined from Brindley and Smeaton's experiments.

COEFFICIENTS FOR NOTCHES, CALCULATED FROM DU BUAT'S EXPERIMENTS.

Ratio of the length to the depth.	Size of notches in inches.	Coefficients.	Ratio of the length to the depth.	Size of notches in inches.	Coefficients.
2·72 to 1	18·4 × 6·753	·630	5·75 to 1	18·4 × 3·199	·624
3·94 to 1	18·4 × 4·665	·627	10·3 to 1	18·4 × 1·778	·646

Poncelet and Lesbros' experiments give the coefficients in the following table, for notches 8 inches wide; the mean value of all the coefficients in these experiments is ·603. Here the coefficients increase in every instance as the depths decrease, or as the ratio of the length of the notch to its depth increases. It will be necessary to refer to the valuable experi-

COEFFICIENTS FOR NOTCHES, BY PONCELET AND LESBROS.

Ratio of the length to the depth.	Size of notches in inches.	Coefficients.	Ratio of the length to the depth.	Size of notches in inches.	Coefficients.
·9 to 1	8 × 9	·577	3·33 to 1	8 × 2·4	·601
1 to 1	8 × 8	·585	5 to 1	8 × 1·6	·611
1·3 to 1	8 × 6	·590	6·7 to 1	8 × 1·2	·618
2 to 1	8 × 4	·592	10 to 1	8 × 0·8	·625
2·5 to 1	8 × 3·2	·595	20 to 1	8 × 0·4	·636

ments made at Metz, on the discharge from differently-proportioned orifices immediately.

Rennie's experiments for circular orifices at depths from 1 foot to 4 feet, and of diameters from ¼ inch to 1 inch give the following coefficients. Here the increase for the coefficients for lesser orifices and at lesser depths exhibits itself very clearly, notwithstanding a few instances to the contrary. The mean value of the coefficient c_d derived from the whole, is ·635. For small rectilineal orifices the coefficients were as follows :—

COEFFICIENTS FOR CIRCULAR ORIFICES, FROM RENNIE'S EXPERIMENTS.

Heads at the centre of each orifice in feet.	¼ inch diameter.	½ inch diameter.	¾ inch diameter.	1 inch diameter.	Mean values.
1	·671	·634	·644	·633	·645
2	·653	·621	·652	·619	·636
3	·660	·636	·632	·628	·639
4	·662	·626	·614	·584	·621
Means	·661	·629	·635	·616	·635

COEFFICIENTS FOR RECTANGULAR ORIFICES, FROM RENNIE'S EXPERIMENTS.

Heads at the centre of gravity, in feet.	Square orifice, 1 inch × 1 inch.	Rectangular orifice, longer side horizontal, 2″ × ½″.	Rectangular orifice, longer side horizontal, 1⅓″ × ⅝″.	Equilateral triangle of 1 square inch, with base down.	Equilateral triangle of 1 square inch, with base up.
1	·617	·617	·663	,,	·596
2	·634	·635	·668	,,	·577
3	·606	·606	·606	,,	·572
4	·593	·593	·593	·593	·593
Means	·613	·613	·632	·593	·585

The most valuable series of experiments are those made at Metz, by Poncelet and Lesbros. They were made with orifices eight inches wide, nearly, and of different vertical dimensions placed at various depths down to 10 feet. The discrepancies as to any general law in the relation of the different values of the coefficient of discharge c_d to the size and depth of the orifice in the preceding experiments, have been remedied to a great extent by these. They give an increase of the coefficients for the smaller and very oblong orifices as they approach the surface, and a decrease under the same circumstances in those for the larger square and oblong orifices. There are a few depths where maximum and minimum values are obtained: the terms "maximum and minimum values" are used for those which are greater in the one case and less in the other than the coefficients immediately before and after them; and not as being numerically the greatest or least values in the column. These maximum and minimum values are marked with a *, in the arrangement of these coefficients, TABLE I. The heads given in this table were measured to the upper side of the orifices, and by adding half the depth of the orifice to any particular head, the head at the centre will be obtained.

As a perceptible sinking of the surface takes place for heads less than from five to three times the depth of the orifice, the coefficients are arranged in pairs, the first column containing the coefficients for heads measured from the still water surface some distance back from the orifice, and the second obtained when the lesser heads, measured directly at the orifice, were used. A very considerable increase in the value of the coefficients for very oblong and shallow small orifices, may be perceived as they approach the surface, and the mean value for all rectilinear orifices at considerable depths, seems to approach to ·605 or ·606.

It is shown, equation (29), that the discharge is

$$\text{D} = c_d \times \left\{ 1 - \frac{d^2}{96\,h^2} \right\} \times \text{A} \sqrt{2\,g\,h},$$

approximately, in which expression d is the depth of the orifice, and h the head at its centre. Now it is to be observed, that it is not the value of c_d simply, which is given in TABLE I., but the value of $c_d \times \left\{ 1 - \frac{d^2}{96\,h^2} \right\}$, the coefficient of $\text{A} \sqrt{2\,g\,h}$, equation (29). The coefficients in the table are, therefore, less than the coefficients of discharge, strictly so called, by a quantity equal to $\frac{c_d\,d^2}{96\,h^2}$. The value of this expression is in general very small, and it is easy to perceive from the first of the expressions in equation (31), pp. 49 & 50, that it can never exceed 4·2 per cent., or more correctly ·0417 in unity. If it be required to know the discharge from an orifice 4 inches square $= 4'' \times 4''$, with its entire 4 feet below the surface,— which is equivalent to a head of 3 feet 10 inches at the upper side. From the table the value of

$c_d \left\{ 1 - \frac{d}{96\,h^2} \right\}$ is ·601; hence

$$\text{D} = \cdot601 \times \text{A} \sqrt{2\,g\,h} = \cdot601 \times \frac{1}{9} \times 8{\cdot}025 \times 2 =$$

$$\cdot601 \times \frac{1}{9} \times 16{\cdot}05 = \frac{1}{9} \times 9{\cdot}646 = 1{\cdot}072$$

cubic feet per second. In the absence of any experiments with larger orifices, when they occur, it is best to use the coefficients given in this table; and, in order to do so with judgment, it is only necessary to observe the relations of the sides and heads. For example, if the side of an orifice be 16″ × 4″, then seek for the coefficient in that column where the ratio of sides is as four to one, and if the head at the upper side be five times the length of the orifice, the coefficient ·626 will be found, which in this case is the same for depths measured behind, or at the orifice. For lesser orifices, the results obtained from the experiments of *Michelotti* and *Bossut,* pages 54 and 56, are most applicable; and also the coefficients of *Rennie,* p. 59. It is almost needless to observe, that all these coefficients are only applicable to orifices in thin plates, or those having the outside arrises chamfered as in Fig. 8. Very little dependence can be placed on calculations of the quantities of water discharged from other orifices, unless where the coefficients have been already obtained by experiment or correct inference for them. If the inner arris next the water be rounded, the coefficient will be increased.

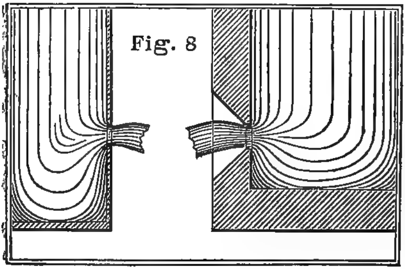


Fig. 8

NOTCHES AND WEIRS.

Some coefficients have been already given at pages

57 and 58, derived from experiments of Du Buât, Brindley and Smeaton, and Poncelet and Lesbros, for finding the discharge over notches in the sides of large vessels; and it does not appear that there is any difference of importance between these and those for orifices sunk some depth below the surface, when the proper formula for finding the discharge for each is used. If Poncelet and Lesbros' coefficients for notches, page 58, be compared with those for an orifice at the surface, Table I., there is little practical difference in the results, the head being measured back from the orifice, unless in the very shallow depths, and where the ratio of the length to the depth exceeds five to one. The depths being in these examples less than an inch, it is probable that the larger coefficients found for the *orifice* at the surface, arise from the upper edge attracting the fluid to it and lessening the effects of vertical contraction, as well as from less lateral contraction. Indeed, the results obtained from experiments with very shallow weirs, or notches, have not been at all uniform, and at small depths the discharge must proportionably be more affected by movements of the air, surface adhesion, and external circumstances than when the depths are considerable. It will be seen that in Mr. Blackwell's experiments the coefficient obtained for depths of 1 and 2 inches was ·676 for a thin plate 3 feet long, while for a thin plate 10 feet long, it increased up to ·805.

The experiments of Castel, with weirs up to about 30 inches long, and with variable heads of from 1 to 8 inches, lead to the coefficient ·497 for notches extending over one-fourth of the side of a reservoir; and to the coefficient ·664 when they extend for the whole width. For lesser widths than one-fourth, the coefficients decrease down to ·584; and for those extending between one-third of, and the whole width, they increase from ·600 to ·665 and ·680. Bidone found $c_d = \cdot620$, and Eytelwein $c_d = \cdot635$. It will be perceived from these and the foregoing results, that the third place of decimals in the value of c_d, and even sometimes the second, is very uncertain; that the coefficient varies with the head and ratio of the notch to the side in which it is placed; and it will soon be shown that the form and size of the weir, weir-basin, and approaches, still further modify its value.

When the sides and edge of a notch increase in thickness, or are extended into a shoot, the coefficients are found to reduce very considerably; and for small heads, to an extent beyond what the increase of resistance, from friction alone, indicates. Poncelet and Lesbros found, *for orifices*, that the addition of a horizontal shoot, 21 inches long, reduced the coefficient from ·604 to ·601, with a head of 4 feet; but for a head of only 4½ inches, the coefficient fell from ·572 to ·483, the orifice being 8″ × 8″. For *notches* 8 inches wide, with a horizontal shoot 9 feet 10 inches long, the coefficient fell from ·582 to ·479, for a head of 8 inches; and from ·622 to ·340, for a head of only 1 inch. Castel found also, for a notch 8 inches wide with a shoot 8 inches long attached and inclined at an angle 4° 18′, that the mean coefficient for heads from 2 to 4½ inches was only ·527. Little dependence can, therefore, be placed on experimental results obtained for shoots which partake of the nature of short pipes, and should be treated in like manner to find the discharge.*

The author has calculated the following table of coefficients from some experiments made by Mr. Ballard, on the river Severn, near Worcester, "with a weir 2 feet long, formed by a board standing perpendicularly across a trough."† *The heads or depths were here measured on the weir,* and hence the coefficients are larger than those found from heads measured back to the surface of still water.

COEFFICIENTS FOR SHORT WEIRS OVER BOARDS.

Heads measured on the crest.

Depths in inches.	Coefficients.	Depths in inches.	Coefficients.	Depths in inches.	Coefficients.
1	·762	3	·801	5	·733
1¼	·662	3¼	·765	5¼	·713
1½	·673	3½	·748	5½	·735
1¾	·692	3¾	·740	5¾	·729
2	·684	4	·759	6	·727
2¼	·702	4¼	·731	7	·716
2½	·756	4½	·744	8	·726
2¾	·786	4¾	·745	Mean	·732

Experiments made at Chew-Magna, in Somersetshire, by Messrs. Blackwell and Simpson, in 1850,‡ give the following coefficients.

"The overfall bar was a cast-iron plate 2 inches thick, with a square top." The length of the over-

---

* Traité Hydraulique, par D'Aubuisson, pp. 46, 94 et 95.

† Civil Engineer and Architect's Journal for 1851, p. 647.

‡ Civil Engineer and Architect's Journal for 1851, pp. 642 and 645.

COEFFICIENTS DERIVED FROM THE EXPERIMENTS OF BLACKWELL AND SIMPSON.

Heads in inches.	Coefficients.	Heads in inches.	Coefficients.	Heads in inches.	Coefficients.
1 to $\frac{7}{8}$	·591	$4\frac{1}{4}$	·743	6	·749
1 to $1\frac{1}{16}$	·626	$4\frac{5}{16}$	·760	$6\frac{3}{16}$	·748
$2\frac{3}{16}$ to $2\frac{1}{4}$	·682	$4\frac{3}{8}$	·741	$6\frac{3}{16}$ to $6\frac{1}{4}$	·747
$2\frac{3}{4}$	·665	$4\frac{7}{16}$	·750	$6\frac{15}{16}$	·772
$2\frac{32}{35}$	·670	$4\frac{1}{2}$	·725	$7\frac{21}{32}$	·717
$2\frac{7}{8}$	·665	5	·780	8	·802
$2\frac{29}{32}$	·653	$5\frac{5}{16}$	·781	8 to $8\frac{13}{16}$	·737
$2\frac{15}{16}$	·654	$5\frac{13}{32}$	·749	$8\frac{15}{16}$	·750
3 to $3\frac{1}{16}$	·725	$5\frac{7}{16}$ to $5\frac{15}{32}$	·751	9	·781
4	·745	$5\frac{15}{16}$	·728	Mean	·723

fall was 10 feet. The heads were measured from still water at the side of the reservoir, and at some distance up in it. The area of the reservoir was 21 statute perches, of an irregular figure, and nearly 4 feet deep on an average. It was supplied from an upper reservoir, by a pipe 2 feet in diameter and of 19 feet fall; the distance between the supply and the weir was about 100 feet. The width of the reservoir as it approached the overfall was about 50 feet, and the plan and section, Fig. 9, of the weir and overfall in connection with it, will give a fair idea of the circumstances attending the experiments. For heads over 5 inches the velocity of approach to the weir was "perceptible to the eye," though its amount was not determined. It will be perceived that the coefficient (derived from two experiments) for a depth of 8 inches is ·802, while the coefficient (derived from three experiments) for a depth of $7\frac{21}{32}$ inches is ·717, and for depths from 8 to $8\frac{15}{16}$ inches the mean coefficient is ·743: as all the attendant circumstances appear the same, these discrepancies and others must arise from some undescribed circumstances of the case: perhaps the supply, and, consequently, the velocity of approach, was increased while making one set of experiments, without affecting the still water near the side where the heads appear to have been taken. By comparing the results with those obtained by one of the same experimenters, Mr. Blackwell, on the Kennet and Avon Canal, we shall immediately perceive that the velocity of approach, and every circumstance which tends to alter and modify it, has a very important effect on the amount of the discharge, and, consequently, on the coefficient.

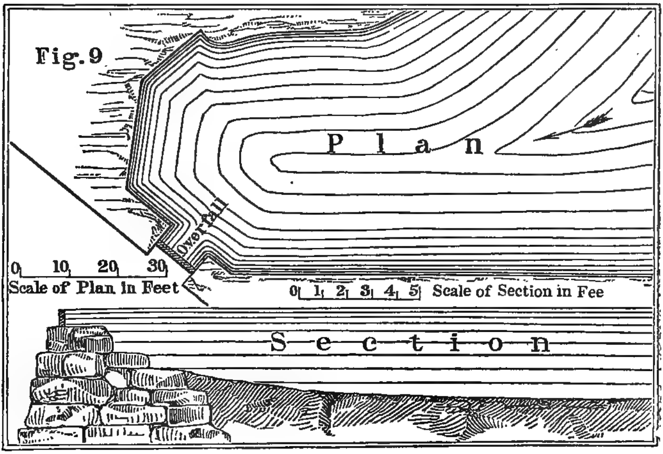


Fig. 9

The experiments made by Mr. Blackwell, on the Kennet and Avon Canal, in 1850,* afford very valuable instruction, as the form and width of the crest were

---

* Civil Engineer and Architect's Journal, 1851, p. 642.

COEFFICIENTS FOR THE DISCHARGE OVER WEIRS, ARRANGED AND DERIVED FROM THE EXPERIMENTS OF MR. BLACKWELL.

*When more than one experiment was made with the same head, and the results were pretty uniform, the resulting coefficients are marked with a *. The effect of the converging wing-boards is very strongly marked.*

NOTE.—Francis' experiments give a coefficient of ·565 for a level crest 3 feet wide, and a head slope of 3½ to 1, see p. 125.

Heads in inches, measured from still water in the reservoir.	Thin plates.		Planks 2 inches thick, square on crest.				Crests 3 feet wide.					
	3 feet long.	10 feet long.	3 feet long.	6 feet long.	10 feet long.	10 feet long, wing-boards making an angle of 60°.	3 feet long, level.	3 feet long, fall 1 in 18.	3 feet long, fall 1 in 12.	6 feet long, level.	10 feet long, level.	10 feet long, fall 1 in 18.
1	·677	·809	·467	·459	·435†	·754	·452	·545	·467	...	·381	·467
2	·675	·803	·509*	·561	·585*	·675	·482	·546	·533	...	·479*	·495*
3	·630	·642*	·563*	·597*	·569*	...	·441	·537	·539	·492*	...	...
4	·617	·656	·549	·575	·602*	·656	·419	·431	·455	·497*	...	·515
5	·602	·650*	·588	·601*	·609*	·671	·479	·516	...	...	·518	...
6	·593	...	·593*	·608*	·576*	...	·501	...	·531	·507	·513	·543
7	...	...	·617*	·608*	·576*	...	·488	·513	·527	·497	...	...
8	...	·581*	·606*	·590*	·548*	...	·470	·491	...	...	·468	·507
9	...	·530	·600	·569*	·558*	...	·476	·492	·498	·480*	·486	...
10	...	...	·614*	·539	·534*	...	...	...	...	·465*	·455	...
12	...	...	...	·525	·534*	...	...	...	...	·467*	...	...
14	...	...	...	·549*	...	...	...	...	...	...	...	...

† The discharge per second varied from ·461 to ·665 cubic feet in two experiments. The coefficient ·435 is derived from the mean value.

varied, and brought to agree more closely with actual weirs across rivers than the thin plates or boards of earlier experimenters. The author has calculated and arranged the coefficients in the following table from these experiments. The variations in the values for different widths of crest, other circumstances being the same, are very considerable; and the differences in the coefficients, at depths of 5 inches and under, for thin plates and crests 2 inches wide, are greater than mere friction can account for; and greater also than the differences at the same depths between the coefficients for crests 2 inches thick, and 3 feet long.

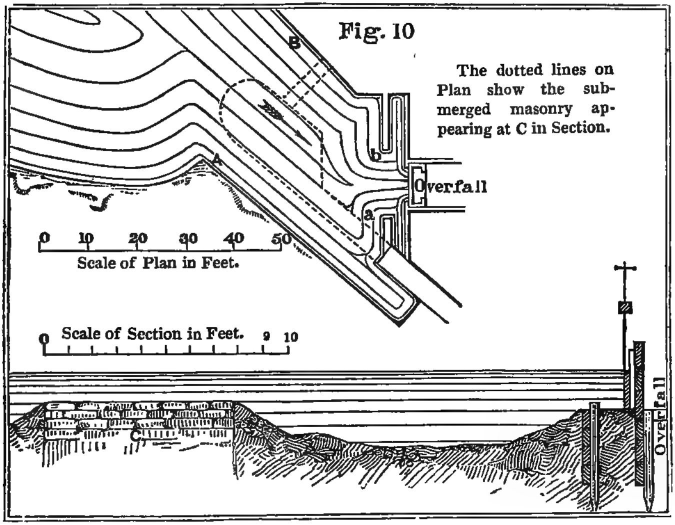


Fig. 10

The dotted lines on Plan show the submerged masonry appearing at C in Section.

The plan and section, Fig. 10, will give a fair idea of the approach to, and nature of the overfall made use of in these experiments. The area of the reservoir was 2A. 1R. 30P., and the head was measured from the surface of the still water in it, which remained unchanged between the beginning and end of each experiment. The width of the approach A B from the reservoir was about 32 feet; the width at *a b* about 13 feet, below which the waterway widened suddenly, and again narrowed to the length of the overfall. The depth in front of the dam appears to have been about 3 feet; the depth on the dam, next the overfall, about 2 feet; and the depth on the sunk masonry in the channel of approach, about 18 inches. Altogether, the circumstances were such as to increase the amount of resistances between the reservoir, from which the head was measured, and the overfall, particularly for the larger heads, and it is accordingly seen that the coefficients become less for heads over six inches, with a few exceptions. The measurements of the quantities discharged appear to have been made very accurately, yet the discharges per second, with the same head and same length of overfall, sometimes vary; for instance, with the plank 2 inches thick and 10 feet long, the discharge per second for 4 inches head varied from 6·098 cubic feet to 6·491 cubic feet, or by about one-sixteenth of the whole quantity. Most of the results, however, are means from several experiments. The quantities discharged varied from one-tenth of a cubic foot to 22 cubic feet per second, and the duration of the experiments from 24 to 420 seconds. If the coefficients for a plank 10 feet long and 2 inches thick in the foregoing table be compared with those for the same overfall at Chew-Magna, it will be seen immediately how much the form of the approaches affects the discharge. Indeed, were the area of the reservoir at Chew-Magna even larger than that for the Kennet and Avon experiments, it would be found, notwithstanding, that the coefficients in the former would still continue the larger, though not fully as large as those found under the particular circumstances.*

The following table gives the mean results of 88 experiments made by Francis, at the Lower Lock, Lowell, Massachusetts, in 1852. The duration of each of these experiments varied from 180 to 822 seconds. The coefficients in column 10 have been calculated by the author, and the other results condensed from the large table given in Francis' Book.† The heads given in the 6th column are those which

---

* There is a very important omission in all the preceding experiments on weirs and notches. In Fig. 10, for instance, it would have been necessary to obtain the heads at A B and *a b* in each experiment, above the crest, and also the head on and a few feet above the crest itself. These are, perhaps, best calculated by means of the observed velocity of approach. They would indicate the resistances at the different passages of approach, and enable us to calculate the coefficients correctly, and thereby render them more generllay applicable to practical purposes. The coefficients in the two previous tables are not as valuable as they otherwise would be from this omission. The level of still water near the banks is below that of the moving water in the current; therefore, heads measured from still water must give larger coefficients than if taken from the centre of the current. This may account, to some extent, for the larger coefficients in the first table, but apart from this, the short contracted channel immediately above the waterfall, Fig. 9, must increase the velocity of approach, and consequently the coefficients.

† Lowell Hydraulic experiments. New York, 1855.

1	2	3	4	5	6	7	8	9	10
Average experiments.	Length of weir (l) in feet.	Observed mean depth over weir in feet (h).	Observed discharge in cubical feet per second.	Observed velocity of approach in feet per second.	Depths (h') corrected for the velocity of approach when $h_a = \frac{v_a^2}{2g}$.	Values of h'' in the formula in column 8.	Value of the formula $D = c\left\{l + \cdot 1\, n\, h''\right\} h''^{\frac{3}{2}}$ in cubic feet per second.	Values of the multiplier c in the formula in column 8.	Corresponding values of the coefficient of discharge c_d.
1	9·997	1·55	62·6	·78	1·56	1·56	62·6	3·32	·621
2	9·997	1·24	45·6	·59	1·25	1·25	45·4	3·33	·623
3	9·997	1·00	33·4	·44	1·00	1·00	32·5	3·32	·621
4	7·997	1·01	26·8	·36	1·02	1·02	26·3	3·36	·628
5	9·997	1·05	36·	·97	1·06	1·06	35·8	3·35	·626
6	9·995	0·98	32·6	·54	0·99	·98	32·4	3·34	·624
7	9·995	1·00	33·5	·55	1·01	1·00	33·3	3·33	·623
8	9·997	0·80	23·5	·33	·80	·80	23·4	3·32	·621
9	9·997	0·82	25·	·75	·83	·83	24·8	3·34	·624
10	9·995	0·80	23·9	·40	·80	·80	23·8	3·34	·624
11	9·997	0·62	16·2	·23	·62	·62	16·0	3·33	·623
12	9·997	0·65	17·5	·53	·65	·65	17·2	3·33	·623
13	7·997	0·68	14·6	·45	·68	·68	14·5	3·34	·623

would give the observed discharge from the formula

$$D = \frac{2}{3} c_d (2g)^{\frac{1}{2}} h'^{\frac{3}{2}}.$$

As also from equation (39)

$$D = \frac{2}{3} c_d\, l\, (2g)^{\frac{1}{2}} \times \{h + h_a)^{\frac{3}{2}} - h_a^{\frac{3}{2}}\},$$

therefore,

$$h' = \{(h + h_a)^{\frac{3}{2}} - h_a^{\frac{3}{2}}\}^{\frac{3}{2}};$$

the values of which are given in column 6. The values of h'' in column 8 are those which would be found by resolving the equation

$$D = c\,(l + \cdot 1\, n\, h'')\, h''^{\frac{3}{2}};$$

n being the number of end contractions, and c a multiplier varying from 3·32 to 3·36.

In this table the theoretical head $\frac{v_a^2}{2g} = \cdot 0155\, v_a^2$ due to the velocity of approach has been used and does not exceed ·02 of a foot. However, this head is much greater, and should be taken $= \frac{v_a^2}{c_d^2 \times 2\,g} = \cdot 04\ v_a^2$ or thereabouts. This would reduce the values of the coefficient of discharge c_d in the 10th column. The differences between h, h', and h'' in columns 3, 6, and 7 are here, practically, of little moment, and the value of c_d in column 10 would be nearly the same derived from either. The crest of the weir experimented upon was 1 inch thick. The weir measuring 10 feet × 13 inches × 1 inch, the top was rounded off at both arrises, leaving the central horizontal portion one quarter of an inch wide. The general result of these experiments verifies the ordinary coefficient for notches in thin plates from ·617 to ·628 for the value of c_d.

Professor Thomson's experiments with right-angled triangular notches, in thin plates, give a mean coefficient of ·617. *Vide* Note p. 42.

HEAD, AND FROM WHENCE MEASURED.

By referring to TABLE I., it is seen that there is a difference in the coefficients as obtained from heads measured on and above the orifice. This difference is greater in notches, or weirs, than in orifices sunk below the surface; and when the crest of a weir is of some width, the depths upon it vary. In the Kennet and Avon experiments, the heads measured from the surface of the water in the reservoir, and the depths at the "outer edge" (by which is understood the lower edge) of the crest were as follows:—

DIFFERENCE OF HEADS MEASURED ON AND ABOVE WEIRS.

Heads from reservoir to crest, in inches.	Heads on crests 2 inches thick.		Heads on crests 3 feet wide.					
	3 feet long.	6 feet long.	3 feet long, crest level.	3 feet long, crest slope 1 in 18.	3 feet long, crest slope 1 in 12.	6 feet long, crest level.	10 feet long, crest level.	10 feet long, crest slope 1 in 18.
1	$\frac{5}{8}$	...	$\frac{7}{16}$	...	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{7}{16}$	$\frac{5}{16}$
2	...	...	$\frac{7}{8}$	...	...	...	$\frac{7}{8}$	$\frac{9}{16}$
3	...	$1\frac{15}{16}$	...	...	...	$1\frac{1}{8}$ to $1\frac{1}{4}$	...	...
4	3 to $2\frac{11}{16}$	$3\frac{1}{2}$	$1\frac{3}{8}$	$1\frac{3}{8}$	$1\frac{1}{2}$	...	...	$1\frac{1}{2}$
5	$3\frac{1}{2}$	$3\frac{5}{9}$	$2\frac{1}{4}$	$1\frac{3}{4}$	...	...	$1\frac{3}{4}$	...
6	$4\frac{3}{8}$	$4\frac{3}{9}$	$2\frac{3}{8}$	...	...	$2\frac{3}{4}$	$2\frac{5}{8}$	$2\frac{1}{4}$
7	...	...	...	$2\frac{1}{8}$	$2\frac{3}{8}$	...	...	...
8	$6\frac{3}{8}$	...	...	...	...	...	$3\frac{5}{8}$	$3\frac{1}{2}$
9	...	...	...	...	...	$4\frac{1}{3}$	$3\frac{1}{2}$	...
10	...	...	...	...	...	...	4	...

No intermediate heads are given, but those registered point out very clearly the great differences which often exist between the heads measured on a weir, or notch, and those measured from the still water above it; and how the form of the weir itself, as well as the nature of the approaches, alters the depth passing over. On a crest 2 feet wide, with $14\frac{1}{2}$ inches depth on the upper edge, we have found that the depth on the lower edge is reduced to $11\frac{1}{2}$ inches, or as 1·26 to 1. The head taken from 3 to 20 feet above the crest, where the plane of the approaching water surface becomes curved, is that in general which is best suited for finding the discharge by means of the common coefficients, but a correct section of the channel and water-line, showing the different depths upon and for some distance above the crest, is necessary in all experiments for determining accurately by calculation the value of the coefficient of discharge c_d.

Du Buât, finding the theoretical expression for the discharge through an orifice of half the depth h,

$$\text{D} = \frac{2}{3}\sqrt{2g} \times l\left\{h^{\frac{3}{2}} - \left(\frac{h}{2}\right)^{\frac{3}{2}}\right\},$$

equation (6)

$$= \frac{2}{3} l\, h \sqrt{2\, g\, h} \times \left\{1 - \left(\frac{1}{2}\right)^{\frac{3}{2}}\right\} = \cdot 646 \times \frac{2}{3}\, l\, h \sqrt{2\, g\, h},$$

to agree pretty closely with his experiments, seems to have assumed that the head h is reduced to $\frac{h}{2}$ in passing over. This is a reduction, however, which never takes place unless with a wide crest and at its lower edge, or where the head h is measured at a considerable distance above the weir, and when a loss of head due to the distance and obstructions in channel takes place. When there is a clear weir basin immediately above the weir, the author has found that, putting h for the head measured from the surface in the weir basin, and h_w for the depth on the upper edge of the weir, that

(32.) $$h - h_w = \cdot 14 \sqrt{h},$$

for measures in feet, and

(33.) $$h - h_w = \cdot 48 \sqrt{h},$$

for measures in inches. The comparative values of h and h_w depend, however, a good deal on the particular circumstances of the case. Dr. Robinson found* $h = 1{\cdot}111\ h_w$, when h was about 5 inches. The expressions given are founded on the hypothesis, that $h - h_w$ is as the velocity of discharge, or as the $\sqrt{h}$ nearly. For small depths, there is a practical difficulty in measuring with sufficient accuracy the relative values of h and h_w. Unless for very small heads the sinking will be found in general to vary from $\frac{h}{10}$ to $\frac{h}{4}$, and in practice it will always be useful to observe the depths on the weir as well as the heads for some distances (and particularly where the widths contract) above it.

In order to convey a more definite idea of the differences between the coefficients for heads measured at the weir, or notch, and at some distance above it, assume the difference of the heads $h - h_w = \frac{h_w}{r}$;

$$\text{then } \frac{h_w}{h-h_w} = r, \text{ and } \frac{1}{r} = \frac{h-h_w}{h_w},$$

$$\text{hence } h = \frac{r+1}{r} h_w \text{ and } h_w = \frac{r}{r+1} h.$$

Now the discharge may be considered as that which would take place through an orifice whose depth is h_w with a head over the upper edge equal to $h - h_w = \frac{h_w}{r}$; hence from equation (6) the discharge is equal to

$$\tfrac{2}{3} l \sqrt{2g} \times c_d \left\{ h^{\frac{3}{2}} - \left(\frac{h_w}{r}\right)^{\frac{3}{2}} \right\},$$

and substituting for $h^{\frac{3}{2}}$ its value $\left(\frac{r+1}{r} h_w\right)^{\frac{3}{2}}$, then the value of the discharge is

---

* Proceedings of the Royal Irish Academy, vol. iv. p. 212.

$$(34.)\quad \mathrm{D} = \tfrac{2}{3} l\, h_w \sqrt{2\, g\, h_w} \times c_d \left\{ \left(1 + \frac{1}{r}\right)^{\frac{3}{2}} - \left(\frac{1}{r}\right)^{\frac{3}{2}} \right\}.$$

As the value would be expressed by

$$\tfrac{2}{3} l\, h_w \sqrt{2\, g\, h_w} \times c_d$$

if the head $h - h_w$ were neglected, it is evident the coefficient is increased, under the circumstances, from c_d to

$$c_d \times \left\{ \left(1 + \frac{1}{r}\right)^{\frac{3}{2}} - \left(\frac{1}{r}\right)^{\frac{3}{2}} \right\};$$

or, more correctly, the common formula has to be multiplied by $\left(1 + \frac{1}{r}\right)^{\frac{3}{2}} - \left(\frac{1}{r}\right)^{\frac{3}{2}}$, to find the true discharge, and the value of this expression for different values of $\frac{1}{r} = n$ will be found in TABLE IV. If it be supposed that

$$h - h_w = \frac{h_w}{10},\ \text{then}\ \frac{1}{r} = \frac{1}{10} = n;$$

and find from the table $\left(1 + \frac{1}{r}\right)^{\frac{3}{2}} - \left(\frac{1}{r}\right)^{\frac{3}{2}} = 1{\cdot}1221$.

Now if the value of c_d be taken, for the full head h, to be ·628, then will 1·1221 × ·628 = ·705, rejecting the latter figures, be the coefficient when the head is measured at the orifice; and if $\frac{1}{r} = \frac{2}{10} = n$, then in the same manner the new coefficient would be found to be 1·2251 × ·628 = ·769 nearly. The increase of the coefficients determined, page 65, from Mr. Ballard's experiments is, therefore, evident from principle, as the heads were taken at the notch; and it is also pretty clear that, *in order to determine the true discharge, the heads both on, at, and above a weir should be taken.* Most of the discrepancies in the coefficients determined from experiment have arisen from imperfect and limited observations of the facts. Amongst these the velocity of approach should never be neglected by observers, as its effect on the discharge is often considerable in increasing the quantity. The effect of the form of the weir and approaches is scarcely ever sufficiently considered by professional men. Most of the discussions which arose with reference to the gaugings on the *Metropolitan* MAIN DRAINAGE QUESTION would have been obviated if the calculators, or engineers, had taken into account the different circumstances attendant on it, instead of applying generally a formula suited to a particular case, namely, a thin crest, a small notch, and a large body of water immediately above it; and applied a correct formula for including the effects of the velocity of approach.

The two following tables have been reduced to English feet measures, from Boileau's experiments; they show the relation of the head to the depth on the crest at the upper arris. The coefficient for the head h being known, that due to h_a on the weir, may be calculated from equation (34).

If the head h_w were used instead of h, to calculate the discharge, then when $\frac{h}{h_w} = 1{\cdot}2$, a coefficient of $\cdot628$ for the head h would become $\cdot769$ for the head h_w in equation (34). For $\frac{1}{r} = \cdot2$, and, therefore, TABLE IV., $\cdot628 \times (1{\cdot}2)^{\frac{3}{2}} - (\cdot2)^{\frac{3}{2}} = \cdot628 \times 1{\cdot}2251 = \cdot769$.

TABLE *showing the ratio of the head, h, to the depth, h_w, on a Plank Weir of the full width of the Channel, immediately at the upper edge, or* $\frac{h}{h_w}$, *see equation* (33), *when the sheet of water is free after passing over, with air under it.*

Head h in feet.	Values of the head h divided by the thickness of the sheet of water passing over the weir immediately at the upper edge; average $\frac{h}{h_w} = \frac{6}{5} = 1\cdot2$ between heads of 3 and 14 inches.			
	Height of weir in feet, ·86′.	Height of weir in feet, 1·07′.	Height of weir in feet, 1·33′.	Height of weir in feet, 1·71′.
·1	1·339	...	...	1·285
·13	1·282	...	1·320	1·250
·16	1·260	...	1·285	1·228
·20	1·234	1·243	1·249	1·214
·23	1·223	1·232	1·231	1·205
·26	1·216	1·232	1·223	1·200
·3	1·212	1·228	1·218	1·199
·33	1·210	1·225	1·217	1·199
·39	1·206	1·221	1·112	1·197
·46	1·202	1·216	1·206	...
·53	1·199	...	1·201	...
·59	1·196	...	1·195	...
·66	1·192	...	1·191	...
·82	1·186	...	...	...
·99	1·184	...	...	...
1·15	1·182	...	...	...

If the head h_w were used instead of h to calculate the discharge, when $\frac{h}{h_w} = 1\cdot25$, TABLE next page, then a coefficient of ·628 for the head h would become ·799 for the head h_w n equation (34). For $\frac{1}{r} = \cdot25$; and, therefore, the value of $c_d\left\{\left(1+\frac{1}{r}\right)^{\frac{3}{2}} - \left(\frac{1}{r}\right)^{\frac{3}{2}}\right\}$, TABLE IV., is $\cdot628 \times (1\cdot25)^{\frac{3}{2}} - (\cdot25)^{\frac{3}{2}} = \cdot628 \times 1\cdot2725 = \cdot799$. And so on we may calculate the value of the coefficient to be applied to the depth h_w on the weir, for any other ratios between h and h_w by means of equation (34).

TABLE *showing the ratio* $\frac{h}{h_w}$, *equation* (33), *when the sheet of water passing over is in contact with the crest and with the water immediately below a Plank Weir.*

Head h in feet.	Values of $\frac{h}{h_w}$ for different heights of weirs and for different heads; mean value for heads between 3 and 14 inches, equal $\frac{5}{4} = 1·25$.		
	Height of weir in feet, 1·07′.	Height of weir in feet, 1·1′.	Height of weir in feet, 1·38′.
·43	...	1·283	...
·46	...	1·275	1·291
·49	1·256	1·266	1·281
·53	1·250	1·258	1·271
·59	1·236	1·245	1·254
·66	1·225	1·232	1·241
·73	1·216	1·223	...
·79	1·208	1·216	...
·86	1·202	1·208	...
·92	1·198	1·203	...
·99	...	1·198	...

Boileau made some valuable experiments at Metz, which were published in 1854. They give the following results for vertical plank weirs extending from side to side of the channel, when the water passed over without adhering to the crest:—

Height of weir over bottom of channel in feet.	Head above.	Mean coefficient.
3·	·2 to 1·6	·645
1·3	·16 to ·5	·622
·6	·15 to ·25	·625

When the water passing over adhered to the crest, and no air between the sheet passing over and the water below the weir, the experiments gave

Height of weir over bottom of channel in feet.	Head above.	Mean coefficient.
2·	1· to 1·6	·694
1·3	·6 to 1·8	·690
·6	·36 to 1·3	·675

When the plank weir leant up-stream 4 inches to a foot, the mean value of c_d was ·620, the height of weir being 1·5 foot, and with heads from ·23 to ·5 foot. When its crest was rounded to a semi-cylinder, the coefficient was, with a head of ·26 foot, ·696, and with a head of ·52 foot, ·843 ; the water adhering to the crest. With a head of ·6 foot the coefficient was ·867, and with a head of ·85 foot, ·840, when the water passed over without air between it and the water below the crest. The following tables give the experimental and reduced coefficients for vertical plank weirs of different heights, and with different heads, when the water passes over in a full sheet, and also when it adheres to the crest and joins it and the lower water. Also for plank weirs suitable for sluices, leaning up-stream with a slope of one-third horizontal to one vertical.

COEFFICIENTS *of Vertical Plank Weirs at right angles to the Channel, when the edge is chamfered at the lower side, and when the water is free and not in contact with the slope, or water below; derived from Boileau's experiments.*

Head h in feet.	Heights of weirs, in feet, over the bottom of the channel, and corresponding values of c_d in the formula $v = c_d \times \frac{2}{3} \sqrt{2gh}.$																			Head h in feet.
	·66′	·82′	·99′	1·15′	1·32′	1·48′	1·65′	1·81′	1·98′	2·14′	2·31′	2·47′	2·64′	2·80′	3·07′	3·13′	3·30′	3·46′	3·60′	
·13′	·631	·637	·639	·636	·627	·616	·612	·607	·603	·603	·606	·610	·619	·630	·637	·634	·627	·619	·612	·13
·16	·628	·633	·634	·633	·628	·622	·613	·606	·598	·595	·597	·603	·613	·628	·634	·631	·624	·616	·609	·16
·20	·624	·630	·633	·630	·621	·610	·606	·600	·597	·597	·600	·606	·615	·628	·633	·630	·624	·616	·609	·20
·23	·627	·630	·633	·630	·622	·613	·606	·600	·597	·597	·600	·606	·615	·628	·633	·630	·624	·616	·609	·23
·26	·627	·635	·636	·631	·622	·613	·607	·603	·598	·598	·601	·607	·616	·628	·633	·630	·624	·616	·609	·26
·30	..	..	·636	·634	·633	·628	·624	·618	·612	·609	·610	·613	·619	·625	·628	·628	·624	·619	·613	·30
·33	..	..	·637	·637	·636	·633	·627	·621	·615	·613	·613	·615	·619	·625	·628	·628	·624	·621	·616	·33
·40	..	..	·642	·643	·640	·637	·631	·625	·616	·613	·613	·615	·618	·624	·630	·630	·628	·624	·619	·40
·46	..	..	..	·651	·648	·643	·636	·627	·619	·615	·612	·612	·615	·624	·633	·637	·636	·630	·621	·46
·53	..	..	..	..	·654	·651	·645	·637	·627	·618	·612	·612	·615	·627	·639	·640	·637	·630	·622	·53
·59	..	..	..	..	.	·652	·648	·642	·636	·628	·624	·622	·625	·633	·642	·640	·636	·630	·624	·59
·66	..	..	..	..	..	..	·654	·651	·646	·642	·640	·639	·642	·643	·645	·643	·639	·633	·627	·66
·72	..	..	..	..	.	..	.	·657	·652	·649	·648	·648	·649	·649	·648	·645	·642	·636	·628	·72
·79	..	..	..	..	.	..	..	·657	·652	·651	·651	·654	·655	·655	·651	·648	·643	·639	·634	·79
·86	..	..	..	..	..	..	..	..	·652	·654	·655	·657	·658	·658	·655	·651	·646	·642	·637	·86
·92	..	..	..	..	..	..	..	..	·655	·657	·658	·658	·661	·661	·660	·655	·651	·645	·640	·92
·99	..	..	..	..	..	..	.	..	.	·658	·661	·653	·666	·667	·666	·663	·655	·648	·640	·99
1·05	..	..	..	..	..	..	.	..	.	..	..	..	·667	·670	·667	·670	·663	·652	·640	1·05
1·12	..	..	..	..	..	..	.	..	.	..	..	..	·664	·669	·667	·669	·661	·651	·640	1·12
1·18	..	..	..	..	..	..	..	..	.	..	..	..	·660	·666	·664	·663	·655	·648	·639	1·18
1·25	..	..	..	..	..	..	..	..	..	..	..	..	·661	·663	·661	·657	·649	·642	·636	1·25
1·32	..	..	..	..	..	..	..	..	..	..	..	..	·664	·663	·660	·654	·648	·642	·634	1·32
1·39	..	..	..	..	..	..	..	..	..	..	..	..	·667	·664	·660	·655	·649	..	..	1·39
1·45	..	..	..	..	..	..	..	..	..	..	..	..	·670	·667	·663	·657	·651	..	..	1·45
1·52	..	..	..	..	..	..	..	..	..	..	..	..	·673	·670	·666	·660	·654	..	..	1·52
1·58	..	..	..	..	..	..	..	..	..	..	..	..	·676	..	..	..	..	..	..	1·58
1·65	..	..	..	..	..	..	..	..	..	..	..	..	·681	..	..	..	..	..	..	1·65

COEFFICIENTS *of Vertical Plank Weirs at right angles to the Channel, when the edge is chamfered at the lower arris, and when the head passing over is in contact with the water at and below the Weir; or when the water immediately below the Weir rises to the crest. The maximum coefficient* ·733 *appears to obtain when the height of the Weir is double the depth passing over the crest.*

Head h in feet.	Heights of weirs, in feet, over the bottom of the channel, and corresponding values of the coefficient of discharge c_d in the formula $v = c_d \times \frac{2}{3}\sqrt{2gh}$.									Head h in feet.
	·66′	·82′	·99′	1·15′	1·32′	1·48′	1·65′	1·81′	1·98′	
·30	·727	...	...	...	...	...	...	...	...	·30
·33	·724	...	...	...	...	...	...	...	...	·33
·36	·721	...	...	...	...	...	...	...	...	·36
·39	·718	...	...	...	...	...	...	...	...	·39
·43	·714	...	...	...	...	...	...	...	...	·43
·46	·709	...	...	...	...	...	...	...	...	·46
·49	·702	·708	·715	·724	...	...	...	...	...	·49
·53	·694	·699	·708	·718	...	...	...	...	...	·53
·56	·687	·693	·700	·712	·729	...	...	...	...	·56
·59	·679	·687	·694	·705	·721	...	...	...	...	·59
·63	·676	·682	·689	·700	·717	...	...	...	...	·63
·66	·672	·678	·684	·696	·714	...	...	...	...	·66
·73	·667	·672	·678	·690	·708	·733	...	...	...	·73
·79	·661	·666	·673	·685	·705	·729	...	...	...	·79
·86	·655	·660	·669	·681	·700	·724	...	...	...	·86
·92	·648	·655	·666	·678	·699	·720	...	...	...	·92
·99	·640	·652	·666	·678	·693	·703	·712	·720	·729	·99
1·05	·631	·645	·657	·669	·681	·691	·702	·711	·720	1·05
1·12	·627	·636	·646	·657	·667	·679	·690	·700	·711	1·12
1·19	·625	·636	·646	·657	·666	·675	·685	·694	·703	1·19
1·25	·625	·636	·646	·657	·666	·675	·682	·690	·696	1·25
1·32	·625	·636	·646	·657	·666	·673	·679	·685	·691	1·32
1·39	...	...	...	...	·666	·672	·678	·682	·684	1·39
1·45	...	...	...	...	·664	·670	·675	·679	·684	1·45
1·52	...	...	...	...	·661	·667	·672	·676	·681	1·52
1·58	...	...	...	...	·658	·663	·669	·672	·675	1·58
1·65	...	...	...	...	·655	·658	·663	·666	·667	1·65

The effect of the form of the crest in increasing the coefficients is distinctly observable in the next table, although the weirs experimented on overhung the water above, between the crest and the bottom of the channel.

The following table gives the result of experiments on chamfered plank weirs, for gauging, extending across a channel at right angles to it, when the back-water below was joined to the head-water at passing over, and when there was no air between :—

TABLE *of Experimental Coefficients for Plank Weirs leaning up-stream, when the crest has the down-stream arris rounded to a quadrant; and when the crest is cylindrical and projecting up-stream in the form of a knob.*

Head h in feet.	Plank weir leaning up-stream one-third to one; the lower arris of crest rounded off to a quadrant of a circle with a radius the full thickness of the plank.		Plank weir leaning upwards one-third to one, the crest rounded and projecting in front beyond the plank, so as to be thicker than it.	
	Water free from curve of crest ·13 foot thick.	Water in contact with curve of crest ·17 foot thick.	Water in contact with curve of crest ·3 foot thick.	Water in contact with curve of crest ·33 foot thick.
·16	·589	·651	...	...
·20	·589	·672	...	...
·23	·594	·697	...	...
·26	·612	·697	...	...
·30	·633	·721	...	·670
·33	·642	·747	·604	·686
·36	·649	·766	·625	·700
·39	·655	·768	·648	·714
·43	·661	·795	·669	·727
·46	·667	·802	·687	·741
·49	·675	...	·702	·753
·53	·679	...	·715	·765
·56	·685	...	·729	·775
·59	...	...	·741	·786
·63	...	...	·753	·795
·66	...	...	·762	·802
·69	...	...	...	·808
·72	...	...	...	·813

	feet	feet	feet	feet	feet	feet	feet	feet
Height of weir over the bottom of the channel below	·66	·83	1·00	1·16	1·32	1·48	1·65	2·00
Heads passing over the weir in each case, when absorbed at the crest into the back-water . . .	·23	·31	·38	·45	·51	·59	·66	·92

which shows that the head was drowned (*noyée*) when the depth of the lower channel below the crest of the weir was less than $2\frac{1}{2}$ times the head passing over, taking a general average.

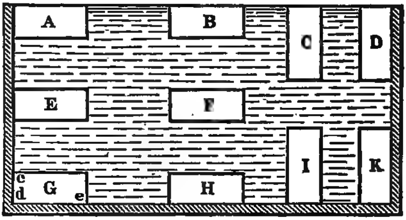
It is necessary here to protest against the notation adopted by Boileau, Morin and others, of giving only two-thirds of the coefficient of discharge, c_d, for notches and weirs, instead of the full and true value. The correct formula for the discharge from a notch or weir is $D = \frac{2}{3} l h \sqrt{2 g h}$. Now they assume a coefficient due to an incorrect formula $D = l h \sqrt{2 g h}$, which reduces c_d to $\frac{2}{3} c_d$ to give the same final results. This leads also to an unnecessary distinction between the coefficients of orifices at the surface, or notches, and orifices sunk to some depth, which, practically, have the same, or nearly the same, general value. Mr. Hughes, at p. 328 of his useful treatise on Water-works, first edition, falls into the same error, for the theoretical discharge per minute over a weir one foot long is $321 h^{\frac{3}{2}}$, and not $481 h^{\frac{3}{2}}$, as he sanctions. In the edition of 1872, however, p. 376, he gives both, with a common factor m, giving Mr. Blackwell as an authority for the former. The factor m must be the coefficient of discharge and cannot, in the same case, have two different values. *The coefficients for a notch and an orifice are substantially the same if correct formulæ for the theoretical discharges be adopted.*

SECTION IV.

VARIATIONS IN THE COEFFICIENTS FROM THE POSITION OF THE ORIFICE.—GENERAL AND PARTIAL CONTRACTION.—VELOCITY OF APPROACH.—CENTRAL AND MEAN VELOCITIES.—PRACTICAL FORMULÆ FOR THE DISCHARGE OVER WEIRS AND NOTCHES.

A glance at TABLE I. will show that the coefficients increase as the orifices approach the surface, to a certain depth dependent on the ratio of the sides, and that this increase increases with the ratio of the length to the depth: some experimenters have found the increase to continue uninterrupted for all orifices up to the surface, but this seems to hold only for depths taken at or near the orifice when it is square or nearly so: it has also been found that the coefficient increases as the orifice approaches to the sides or bottom of a vessel: as the contraction becomes imperfect the coefficient increases. These facts probably arise from the velocity of approach being more direct and concentrated under the respective circumstances. The lateral orifices A, B, C, D, E, F, G, H, I, and K, Fig. 11, have coefficients differing more or less from each other. The coefficient for A is found to be larger than either of those for B, C, E, or D; that for G or K larger than that for H or I; that for H larger than that for I; and that for F, where the contraction is general, least of all. The contraction of the fluid on entering the orifice F removed from the bottom and sides is complete; it is termed, therefore, "*general contraction;*" that at the orifices A, E, G, H, I, K, and D, is interfered with by the sides; it is therefore incomplete, and termed "*partial contraction.*" The increase in the coefficients for the same-sized orifices at the same mean depths may be assumed as proportionate to the length of the perimeter at which the contraction is partial, or from which the lateral flow is shut off; for example, the increase for the orifice G is to that for H as $c\,d + d\,e : d\,e$; and in the same manner the increase for G is to that for E as $c\,d + d\,e : c\,d$. If n be put for the ratio of the contracted portion $c\,d\,e$ to the entire perimeter, and, as before, c_d for the coefficient of general contraction, then the coefficient of partial contraction is equal to

Fig. 11



(35.) $$c_d + \cdot 09\, n = c_d + \cdot 1\, n \text{ nearly,}$$

for rectangular orifices. The value of the second term $\cdot 09\, n$ is derived from various experiments. If ·617 be taken for the mean value of c_d, the expression may be changed into the form $(1 + \cdot 146\, n)\, c_d$. When $n = \frac{1}{4}$, this becomes $1{\cdot}036\, c_d$; when $n = \frac{1}{2}$, it becomes $1{\cdot}073$ c; and when $n = \frac{3}{4}$, contraction is prevented for three-fourths of the perimeter, and the coefficient for partial contraction becomes 1·109 c_d. The form which is given in equation (35) is, however, the simplest; but the value of n must not exceed $\frac{3}{4}$. If in this case $c_d = {\cdot}617$, the coefficient for partial contraction becomes $\cdot617 + \cdot09 \times \frac{3}{4} = \cdot617 + \cdot067 = \cdot684$. Bidone's experiments give for the coefficient of partial contraction $(1 + \cdot152\,n)\,c_d$; and Weisbach's $(1 + \cdot132\,n)\,c_d$.

VARIATION IN THE COEFFICIENTS FROM THE EFFECTS OF THE VELOCITY OF APPROACH.

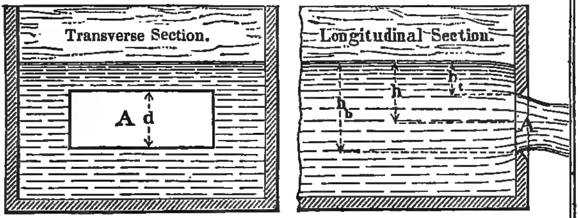
Heretofore it has been generally supposed that the water in the vessel is almost still, its surface level unchanged, and the vessel consequently large compared with the area of the orifice. When the water flows in a channel to the orifice with a perceptible velocity, the contracted vein and the discharge are both found to be increased, other circumstances being the same. If the area of the vessel or channel in front exceed thirty times that of the orifice, the discharge will not be perceptibly increased by the induced velocity in the conduit; but for lesser areas of the approaching channel, corrections due to the velocity of approach become necessary. It is clear that this velocity may arise from either a surface inclination in the channel, an increase of head, or a small channel of approach supplied in some way.

Equation (6) gives the discharge from a rectangular orifice A, Fig. 12, of the length l, with a head measured from still water

$$\text{D} = \frac{2}{3} c_d\, l \sqrt{2\,g} \times \{h_b^{\frac{3}{2}} - h_t^{\frac{3}{2}}\},$$

in which h_b and h_t are measured to the surface at some distance back from the orifice, as shown in the section. The water here, however, must move along the channel towards the orifice with considerable velocity. If A be the area of the orifice, and C the area of the channel, it may be supposed, with tolerable accuracy, that this

Fig. 12



velocity is equal to $\frac{A}{C}v_o$, in which v_o represents the mean velocity in the orifice. If the velocity of approach be represented by v_a, then

(36.) $$v_a = \frac{A}{C} \times v_o,$$

and consequently the theoretical height due to it—the same as when there is no contraction at the *vena-contracta*—is

(37.) $$\left\{ \begin{array}{l} h_a = \frac{A^2}{C^2} \times \frac{v_o^2}{2g} = \cdot 0155 \, \frac{A^2 v_o^2}{C^2} \Big\}; \\ \text{and taking the head due to contraction,} \\ \text{\&c., into account} \\ h_a = \frac{A^2}{C^2} \times \frac{v_o}{2g\,c_d^2} = \cdot 0155 \, \frac{A^2 v_o^2}{c_d^2\,C^2} \Big\}; \end{array} \right.$$

in feet measures.* The height h_a may be considered as an increase of head, converting h_b into $h_b + h_a$, and h_t into $h_t + h_a$. The discharge therefore now becomes

---

* When the approaching velocity passes through the orifice without contraction, it is evident that the head h_a required to produce that

$$(38).\quad D = \tfrac{2}{3} c_d l \sqrt{2g} \left\{ (h_b + h_a)^{\frac{3}{2}} - (h_t + h_a)^{\frac{3}{2}} \right\};$$

which, for notches or weirs, is reduced to

$$(39.)\quad D = \tfrac{3}{2} c_d l \sqrt{2g} \left\{ (h_b + h_a)^{\frac{3}{2}} - h_a^{\frac{3}{2}} \right\},^*$$

as h_t then vanishes. As D is also equal to $A \times v_o$, equation (37) may be changed into

$$(40.)\left\{\begin{array}{l} h_a = \dfrac{D^2}{C^2} \times \dfrac{1}{2g} = \left.\dfrac{\cdot0155\, D^2}{C^2}\right\}, \\ \text{and taking the head due to contraction,} \\ \text{\&c., into account,} \\ h_a = \dfrac{D^2}{C^2} \times \dfrac{1}{2g\, c_d^2} = \left.\dfrac{\cdot0155\, D^2}{c_d^2\, C^2}\right\} \end{array}\right.$$

in feet measures.

If this value for h_a be substituted in equations (38) and (39), the resulting equations will be of a high order and do not admit of a direct solution; and in

---

velocity in the orifice, *with contraction outside at the vena-contracta*, must be $h_a = \frac{A^2}{C^2} \times \frac{v_o^2}{2g \times c_d^2}$ instead of $h_a = \frac{A^2}{C^2} \times \frac{v_o^2}{2g}$. In like manner $h_a = \frac{v_a^2}{c_d^2 \times 2g} = \cdot04\, v_a^2$ in feet measures when v_a is the velocity of approach and $c_d = \cdot617$.

* The formula for the discharge over weirs, taking into account the velocity of approach, $D = 2\cdot95\, c_d\, l \sqrt{h + \cdot115\, v_a^2}$, given by D'Aubuisson, "Traité Hydraulique," seconde édition, pp. 78 et 95, and adopted by some English writers and engineers, is incorrect in principle. In feet measures it becomes $D = 5\cdot35\, c_d\, l\, h \times \sqrt{h + \cdot03494\, v_a^2}$, which *form*,—with alterations in the numerals and measures, was used for calculating discharges of sewers during the METROPOLITAN MAIN DRAINAGE discussion.

(38) and (39), as they stand, h_a involves implicitly the value of D, which is what is sought for. By finding at first an approximate value for the velocity of approach, the height h_a due to it can easily be found, equation (37); this height, substituted in equation (38) or (39), will give a closer value of D, from which again a more correct value of h_a can be determined; and by repeating the operation the values of D and h_a can be had to any degree of accuracy. In general the values found at the second operation will be sufficiently correct for all practical purposes.

It has been already observed that, for *orifices*, it is advisable to find the discharge from a formula in which only one head, that at the centre, is made use of; and though TABLE IV., as shall be shown, enables us to calculate the discharge with facility from either formula, it will be of use to reduce equation (38) to a form in which only the head (h) at the centre is used. The error in so doing can never exceed six per cent., even at small depths, equation (31), and this is more than balanced by the observed increase in the coefficients for smaller heads.

The formula for the discharge from an orifice, h, being the head at the centre, is

$$\text{D} = c_d\sqrt{2\,g\,h} \times \text{A};$$

and when the additional head h_a, due to the velocity of approach, is considered,

$$\text{D} = c_d\sqrt{2\,g\,(h + h_a)} \times \text{A},$$

which may be changed into

$$(41.)^* \qquad \text{D} = \text{A}\sqrt{2\,g\,h} \times c_d\left\{1 + \frac{h_a}{h}\right\}^{\frac{1}{2}}.$$

---

* See equation (41 A), Section VII. *infra.*

equation (39), for notches, may be also changed to the form

$$(42.)\quad \text{D} = \tfrac{2}{3}\text{A}\sqrt{2\,g\,h_b} \times c_d \left\{ \left(1+\frac{h_a}{h_b}\right)^{\frac{3}{2}} - \left(\frac{h_a}{h_b}\right)^{\frac{3}{2}} \right\};$$

this is similar in every way to the equation

$$(43.)\quad \text{D} = \tfrac{2}{3}\text{A}\sqrt{2\,g\,d} \times c_d \left\{ \left(1+\frac{h_t}{d}\right)^{\frac{3}{2}} - \left(\frac{h_t}{d}\right)^{\frac{3}{2}} \right\},$$

for the discharge from a rectangular orifice whose depth is d, with the head h_t, at the upper edge. TABLE III. contains the values of $\left\{ 1+\frac{h_a}{h} \right\}^{\frac{1}{2}}$ in equation (41), and TABLE IV. the values of $\left(1+\frac{h_a}{h_b}\right)^{\frac{3}{2}} - \left(\frac{h_a}{h_b}\right)^{\frac{3}{2}}$ in equation (42), or the similar expression in (43), $\frac{h_a}{h_b}$ or $\frac{h_t}{d}$ being put equal to n; and it may be perceived that the effect of the velocity of approach is such as to increase the coefficient from c_d to $c_d \left\{ 1+\frac{h_a}{h} \right\}^{\frac{1}{2}}$ for orifices sunk some distance below the surface, in which h is the depth of the centre of the orifice; and into

$$c_d \left\{ \left(1+\frac{h_a}{h_b}\right)^{\frac{3}{2}} - \left(\frac{h_a}{h_b}\right)^{\frac{3}{2}} \right\}$$

for weirs when h_a is the height due to the velocity of approach, and h_b the head on the weir. A few examples, showing the application of the formulæ (41), (42), and (43), and the application of TABLES I., II., III., and IV. to them, will be of use. Suppose, for the present, the velocity of approach v_a to be given, and no extra head required to maintain it through the orifice: in other words, when

$h_a = \dfrac{v_a^2}{2g \times \cdot 956^2} = \cdot 017\, v_a^2$ in feet measures nearly.

EXAMPLE I. *A rectangular orifice,* 12 *inches wide by* 4 *inches deep, has its centre placed* 4 *feet below the surface, and the water approaches the head with a velocity of* 28 *inches per second; what is the discharge?*

For an orifice of the given proportions, and sunk to a depth nearly four times its length, find from TABLE I.

$$c_d = \frac{\cdot 616 + \cdot 627}{2} = \cdot 621 \text{ nearly.}$$

As the coefficient of velocity, equation (2), for water flowing in a channel is about ·956, find from column No. 3, TABLE II., the height $h_a = 1\frac{1}{8} = 1\cdot 125$ inch nearly, corresponding to the velocity 28 inches. Equation (41),

$$\text{D} = \text{A}\sqrt{2\,g\,h} \times c_d \left\{ 1 + \frac{h_a}{h} \right\}^{\frac{1}{2}},$$

now becomes

$$\text{D} = 12 \times 4\sqrt{2gh} \times \cdot 621 \left\{ 1 + \frac{1\cdot 125}{48} \right\}^{\frac{1}{2}}.$$

Also find $\sqrt{2gh} = 192\cdot 6$ inches, when $h = 48$ inches, in TABLE II.; therefore

$$\text{D} = 12 \times 4 \times 192\cdot 6 \times \cdot 621 \left\{ 1 + \frac{1\cdot 125}{48} \right\}^{\frac{1}{2}}$$

$= 9244\cdot 8 \times \cdot 621\ \{1 + \cdot 0234\}^{\frac{1}{2}} = 9244\cdot 8 \times \cdot 621 \times 1\cdot 0116$, (as $\{1\cdot 0234\}^{\frac{1}{2}} = 1\cdot 0116$ from TABLE III.) $= 9244\cdot 8 \times \cdot 628$ nearly $= 5805\cdot 7$ cubic inches $= 3\cdot 36$ cubic feet per second. *Or thus*: The value of $\cdot 621 \times (1\cdot 0234)$ being found equal ·628, $\text{D} = \text{A} \times \cdot 628\sqrt{2\,g \times 48}$. Now for the coefficient ·628, and $h = 48$ inches, TABLE II.

gives us $\cdot628\ \sqrt{2\,g \times 48} = 120\cdot96$ inches; hence D $=$ $12 \times 4 \times 120\cdot96 = 5806\cdot08$ cubic inches $= 3\cdot36$ cubic feet, the same as before, the difference of $\cdot38$ in the cubic inches being of no practical value.

If h_a be found from the formula $h_a = \frac{v_a^2}{2\,g\,c_d^2}$, then is $h_a = 2\cdot6$ inches, and the discharge becomes D$=3\cdot40$ cubic feet nearly.

If the centre of the orifice were within 1 foot of the surface, the effect of the velocity of approach would be much greater; for then

$c_d \times \left\{ 1 + \frac{h_a}{h} \right\}^{\frac{1}{2}} =$ (from TABLE I.) $\cdot623$

$\left\{ 1 + \frac{1\cdot125}{12} \right\}^{\frac{1}{2}} =$ (from TABLE III.) $\cdot623 \times 1\cdot047 =$ $\cdot652$ instead of $\cdot628$. In this case the discharge is D $=$ $12 \times 4 \times \cdot652\ \sqrt{2\,g \times 12} = 12 \times 4 \times \cdot652 \times 96\cdot3$ (from TABLE II.) $= 12 \times 4 \times 62\cdot8 = 3014\cdot4$ cubic inches $=$ $1\cdot744$ cubic feet per second. Or find the value of $\cdot652\ \sqrt{2\,g\,h}$ directly from TABLE II. thus:

The value of $\cdot628\ \sqrt{2\,g \times 12} = 60\cdot48 \quad \cdot628$
The value of $\cdot666\ \sqrt{2\,g \times 12} = 64\cdot14 \quad \cdot652$
$38 \quad : \quad 3\cdot66 :: 24 : 2\cdot31.$

Hence $\cdot652\ \sqrt{2\,g\,h} = 60\cdot48 + 2\cdot31 = 62\cdot79$, and the discharge $= 12 \times 4 \times 62\cdot79 \times 3013\cdot92$ cubic inches $=$ $1\cdot744$ cubic feet per second, the same as before.

If h_a be taken equal to $\frac{v_a^2}{2\,g\,c_d^2} = 2\cdot6$ inches, then the resulting value of D $= 1\cdot833$ cubic feet nearly.

EXAMPLE II. *A rectangular notch,* 7 *feet long, has a head of* 8 *inches measured at about* 4 *feet above the* *crest, and the water approaches the over-fall with a velocity of* $16\frac{1}{4}$ *inches per second; what is the discharge?*

For a still head assume $c_d = \cdot 628$ in this case, and then from equation (42)

$$\text{D} = \frac{2}{3}\text{A}\sqrt{2\,g\,h_b} \times c_d \left\{ \left(1 + \frac{h_a}{h_b}\right)^{\frac{3}{2}} - \left(\frac{h_a}{h_b}\right)^{\frac{3}{2}} \right\}.$$

As in the last example, find from TABLE II. (h_a) the height due to the velocity of approach ($16\frac{1}{4}$ inches) to be $\frac{3}{8} = 3\cdot 375$ inch, assuming the coefficient of velocity to be ·956. Therefore, $h_a = \cdot 375$, $h_b = 8$, $c_d = \cdot 628$, and $\text{A} = 7 \times 12 \times 8$; or for measures in feet $\frac{h_a}{h_b} = \cdot 047$, $h_b = \frac{2}{3}$, and $\text{A} = 7 \times \frac{2}{3}$; hence

$$\text{D} = \frac{2}{3} \times 7 \times \frac{2}{3}\sqrt{2\,g \times \frac{2}{3}} \times \cdot 628 \left\{ (1\cdot 047)^{\frac{3}{2}} - (\cdot 047)^{\frac{3}{2}} \right\}.$$

The value of $(1\cdot 047)^{\frac{3}{2}} - (\cdot 047)^{\frac{3}{2}}$ will be found from TABLE IV. equal to 1·0612; the value of $\sqrt{2\,g \times \frac{2}{3}}$ will be found from TABLE II. equal to 6·552, viz. by dividing the velocity 78·630, to be found opposite 8 inches, by 12; hence

$$\begin{aligned} \text{D} &= \frac{2}{3} \times 7 \times \frac{2}{3} \times 6\cdot 552 \times \cdot 628 \times 1\cdot 0612 \\ &= \frac{2}{3} \times 7 \times 4\cdot 368 \times \cdot 628 \times 1\cdot 0612 \\ &= \frac{2}{3} \times 7 \times 4\cdot 368 \times \cdot 666 \text{ nearly.} \\ &= \frac{2}{3} \times 7 \times 2\cdot 909 = 7 \times 1\cdot 939 \end{aligned}$$

$= 13\cdot 573$ cubic feet per second $= 814\cdot 38$ cubic feet per minute. *Or thus:* From TABLE VI., when the coefficient is ·628, the discharge from a weir 1 foot long, with a head of 8 inches, is found to be 109·731 cubic feet per minute. The discharge for a weir 7 feet long, when $\frac{h_a}{h} = {}$·047 is therefore 109·731 × 7 × 1·0612 = 815·12 cubic feet per minute. The difference between this value and that before found, 814·38 cubic feet is immaterial, and has arisen from not continuing all the products to a sufficient number of places of decimals.

If $h_a = \frac{v_a^2}{2\,g\,c_d^2} = {}$·87 inch, then D = 14·5 cubic feet per second, or 870 cubic feet per minute nearly.

In equations (36) and (37), the relations between the channel, orifice, velocity of approach, and velocity in the orifice, are pointed out, viz.,

$$v_a = \frac{A}{C} \times v_o, \text{ and } h_a = \frac{A^2}{C^2} \times \frac{v_o^2}{2\,g} = \frac{D^2}{2\,g\,C^2,},$$

$$\text{in which } h_a = \frac{v^2}{2\,g}$$

(neglecting, for the present, the coefficient of velocity in passing through the orifice). As v_o is the actual velocity in the orifice, $\frac{v_o}{c_d}$ must be the theoretical velocity due to the head $h + h_a$, and therefore

$$h + h_a = \frac{v_o}{c_d^2 \times 2\,g}, \text{ and } h = \frac{v_o^2}{c_d^2 \times 2\,g} - \frac{v_a^2}{2\,g};$$

hence

$$\frac{h_a}{h} = \frac{1}{\frac{v_o^2}{v_a^2 \times c_d^2} - 1} = \frac{c_d^2\,v_a^2}{v_o^2 - c_d^2\,v_a^2} = \frac{c_d^2\,A^2}{C^2 - c_d^2\,A^2}, \text{ for } \frac{v_o^2}{v_a^2} = \frac{C^2}{A^2}.$$

And therefore

$$(44.)\qquad \frac{h_a}{h} = \frac{c_d^2\,\text{A}^2}{\text{C}^2 - c_d^2\cdot\text{A}^2} = \frac{c_d^2}{m^2 - c_d^2}$$

substituting this value in equations (41) and (42), there results

$$(45.)\left\{\begin{array}{l} \text{D} = \text{A}\sqrt{2\,g\,h} \times c_d\left\{1 + \dfrac{c_d^2}{m_2 - c_d^2}\right\}^{\frac{1}{2}} \text{ or,} \\ \text{D} = \text{A}\sqrt{2\,g\,h} \times \dfrac{c_d}{\left(1 - \dfrac{c_d^2}{m^2}\right)^{\frac{1}{2}}}, \end{array}\right.$$

in which $m = \frac{\text{C}}{\text{A}}$, for the discharge from an orifice at some depth; and for the discharge from a weir,

$$(46.)\quad \text{D} = \tfrac{2}{3}\sqrt{2\,g\,h_b} \times c_d\left\{\left(1 + \frac{c_d^2}{m^2 - c_d^2}\right)^{\frac{3}{2}} - \left(\frac{c_d^2}{m^2 - c_d^2}\right)^{\frac{3}{2}}\right\}.$$

The last two equations give the discharge when the ratio of the channel to the orifice $\frac{\text{C}}{\text{A}} = m$ is known, when $h_a = \frac{v_a^2}{2\,g}$, and when at the same time the whole quantity of water passing through the orifice, *that due to the velocity of approach as well as that due to the pressure, is supposed to suffer a contraction whose coefficient is* c_d.

When $h_a = \frac{v_a^2}{2\,g \times c_d^2}$, that is when the velocity of approach, v_a, passes through the orifice without contraction, we shall get

$$(44a.)\qquad \frac{h_a}{h} = \frac{v_a^2}{v_o^2 - v_a^2} = \frac{\text{A}}{\text{C}^2 - \text{A}^2} = \frac{1}{m^2 - 1};$$

consequently, in this case, equation (45) becomes

$$(45a.)\quad \text{D} = \text{A}\sqrt{2\,g\,h} \times c_\text{d} \times \left\{ 1 + \frac{1}{m^2 - 1} \right\}^{\frac{1}{2}} = \text{A}\sqrt{2\,g\,h} \times c_\text{d} \left\{ \frac{m^2}{m^2 - 1} \right\}^{\frac{1}{2}};$$

and equation (46) in like manner changes into

$$(46a.)\quad \text{D} = \tfrac{2}{3}\,\text{A}\sqrt{2\,g\,h_\text{b}} \times c_\text{d} \times \left\{ \left(1 + \frac{1}{m^2 - 1}\right)^{\frac{3}{2}} - \left(\frac{1}{m^2 - 1}\right)^{\frac{3}{2}} \right\}.$$

The last multipliers of these two equations, $\left(1 + \frac{1}{m^2 - 1}\right)^{\frac{1}{2}}$ and $\left(1 + \frac{1}{m^2 - 1}\right)^{\frac{3}{2}} - \left(\frac{1}{m^2 - 1}\right)^{\frac{3}{2}}$, are the same as the like multipliers in (45) and (46), when c_d, within the brackets $= 1$; consequently their values are at once found from the coefficient unity, 1, in the last page of TABLE V., for the respective values of $m = \frac{\text{C}}{\text{A}}$; and also for those of $\frac{h_\text{a}}{h_\text{b}} = \frac{1}{m^2 - 1}$. When $c_\text{d} = 1$, equations (45) and (45a) may be changed into the particular case

$$\text{D} = \text{A} \left\{ \frac{2\,g\,h}{1 - \left(\frac{1}{m}\right)^2} \right\}^{\frac{1}{2}} = m\,\text{A} \left\{ \frac{2\,g\,h}{m^2 - 1} \right\}^{\frac{1}{2}}$$

which is the equation of DANIEL BERNOULLI.

When $\text{A} = \text{C}$, or the orifice is equal to the channel, then $\frac{1}{m^2 - 1}$ becomes infinite, and hence h must be zero. Indeed, this assumption cannot be made consistently, for any given depth of water; and the ratio m can never become so small as unity. A full discussion of the theoretical question would be out of place here. It is only necessary to observe, that the two last columns in TABLE V. give the multipliers of c in equations (45a) and (46a) to find the coefficients suited to $\frac{C}{A} = m$, which in practice should seldom or never be less than 2.

If $n = \frac{c_d^2}{m^2 - c_d^2}$, the values of $\left\{ 1 + \frac{c_d^2}{m^2 - c_d^2} \right\}^{\frac{1}{2}}$, and of $\left\{ 1 + \frac{c_d^2}{m^2 - c_d^2} \right\}^{\frac{3}{2}} - \left\{ \frac{c_d^2}{m^2 - c_d^2} \right\}^{\frac{3}{2}}$, respectively, can be easily had from TABLES III. and IV. TABLE V. has, however, been calculated for different ratios of the channel to the orifice, and for different values of the coefficient of discharge. This table gives at once the values of

$$c_d \left\{ 1 + \frac{c_d^2}{m^2 - c_d^2} \right\}^{\frac{1}{2}} \text{ and } c_d \left\{ \left(1 + \frac{c_d}{m^2 - c_d^2} \right)^{\frac{3}{2}} - \left(\frac{c_d^2}{m^2 - c_d^2} \right)^{\frac{3}{2}} \right\}$$

as new coefficients, and the corresponding value of

$$\frac{h_a}{h} \text{ and } \frac{h_a}{h_b} = \frac{c_d^2}{m^2 - c_d^2}$$

in equations (44) and (46).

It is equally applicable, therefore, to equations (41) and (42) as to equations (45) and (46). For instance, here at once is found the value of $·628 \times \{ (1·047)^{\frac{3}{2}} - (·047)^{\frac{3}{2}} \}$ in EXAMPLE II., p. 95, equal to ·666, as $\frac{h_a}{h_b} =$ ·047, and the next value to it for the coefficient ·628, in the table, is ·046, opposite to which is found ·666, the new coefficient sought. The sectional area of the channel in this case, as appears from the first column, must be about three times that of the weir or notch.

When $\frac{h_a}{h_b} = \frac{1}{m^2 - 1} = \frac{A^2}{C^2 - A^2}$, then in Example II. $\frac{h_a}{h_b} = \cdot 11$ and $\left(1 + \frac{h_a}{h_b}\right)^{\frac{3}{2}} - \left(\frac{h_a}{h_b}\right)^{\frac{3}{2}} = 1{\cdot}133$, Table IV. (or Table V. for the coefficient 1). Hence in this case $\cdot 628 \times 1{\cdot}133 = \cdot 712$ the new coefficient suited to the velocity of approach. Here of course $h_a = \frac{v_a^2}{2\,g\,c_d^2}$ (see Note, p. 89).

Table V. is calculated from coefficients c_d, in still water, which vary from ·550 to 1. Those from ·606 to ·650, and the mean value ·628 are most suited for application in practice. When the channel is equal to the orifice, the supply should equal the discharge, and for open channels, with the mean coefficient ·628, we find, accordingly, from the table, the new coefficient 1·002 for weirs; or 1 very nearly as it should be. It is also found in the same case, viz., when $A = C$, and $c_d = \cdot 628$, that for short tubes, Fig. 13, the resulting new coefficient becomes ·807. This, as will afterwards be seen, agrees very closely with the experimental results. When the coefficients in still water are less than ·628, or more correctly ·62725, the orifice, according to this formula, cannot equal the channel unless other resistances take place, such as from friction in tubes longer than one and a half or two diameters, or in wide-crested weirs. For

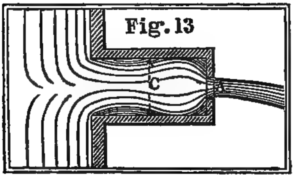
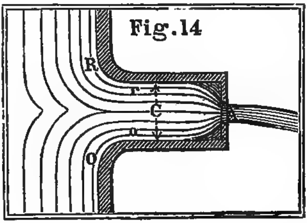


Fig. 13

greater coefficients the junction of the short tube with the vessel must be rounded, Fig. 14, on one or more sides; and in weirs or notches the approaches must slope from the crest and ends to the bottom and sides, and the overfall be sudden. The converging form of the approaches must, however, increase the velocity of approach; and therefore v_a is greater than $\frac{A}{C} \times v_o$ when C is measured between *r o* and R O, Fig. 14, to find the discharge, or new coefficient of an orifice placed at *r o*.

Fig.14



As the coefficients in TABLE V. are best suited for orifices at the end of short cylindrical or prismatic tubes at right angles to the sides or bottom of a cistern, a correction is required when the junction is rounded off as at R O *r o*, Fig. 14. When the channel is equal to the orifice, the new coefficient in equation (45) becomes

$$c_d \left\{ 1 + \frac{c_d^2}{1 - c_d^2} \right\}^{\frac{1}{2}} = c_d \times \left\{ \frac{1}{1 - c_d^2} \right\}^{\frac{1}{2}}.$$

The velocity in the short tube Fig. 14 is to that in the short tube Fig. 13 as 1 to $c_d \left\{ \frac{1}{1 - c_d^2} \right\}^{\frac{1}{2}}$ nearly, or for the mean value $c_d = \cdot 628$, as 1 to ·807. Now, as $\frac{C}{A}$ is assumed equal to $\frac{v_o}{v_a}$ in the cylindrical or prismatic tube, Fig. 13, $\frac{\cdot 807\,C}{A} = \frac{v_o}{v_a}$ in the tube Fig. 14 with the rounded junction, for v_a becomes $\frac{v_a}{\cdot 807}$; hence, in order to find the discharge from orifices at the end of the short tube, Fig. 14, it is only necessary to multiply the numbers representing the ratio $\frac{\text{C}}{\text{A}}$ in the first column, TABLE V., by ·807, or more generally by $c_d \left\{\frac{1}{1 - c_d^2}\right\}^{\frac{1}{2}}$, and find the coefficient opposite to the product. Thus if $c_d = \cdot 628$, then, when $\frac{\text{C}}{\text{A}} = 1$, $c_d \times \left\{\frac{1}{1 - c_d^2}\right\}^{\frac{1}{2}} = \cdot 807$ in the table. If, again, $\frac{\text{C}}{\text{A}} = 3$, then $3 \times \cdot 807, = 2\cdot 421$, the value of $\frac{v_o}{v_a}$ for the tube Fig. 14, and opposite this value of $\frac{\text{C}}{\text{A}}$, taken in column 1, there is found ·651 for the new coefficient. For the cylindrical or prismatic tube, Fig. 13, the new coefficient would be only ·642. When the head h_a is however equal to $\frac{v_a^2}{2\,g\,c_d^2}$ the results must be modified accordingly (see Note, p. 89).* When h is measured from still water in a cistern, Figs. 13 and 14, and v_a the velocity of approach at C, in a short tube, inserted at the sides, or bottom, then we must take $h - h_a$ for the head, h.

---

* Professor Rankine gives the value of the coefficient of discharge, or contraction, for varying values of A and C at a diaphragm *in* a pipe by the empirical formula

$$c_d = \frac{\cdot 618}{\left(1 - \cdot 618 \times \frac{\text{A}^2}{\text{C}^2}\right)^{\frac{1}{2}}}.$$

When $\frac{\text{A}}{\text{C}} = 0$, $c_d = \cdot 618$; and when $\frac{\text{A}}{\text{C}} = 1$, $c_d = 1$; as it should be very nearly for an orifice in a *thin* plate, to which only, and to an orifice A at the end of a short tube, Fig. 14, the formula is suited (see SECTION X).

DIFFERENT EFFECTS OF CENTRAL AND MEAN VELOCITIES IN A SHORT TUBE AND CHANNEL.

There is, however, another element to be taken into consideration, and which it is necessary to refer to more particularly hereafter. It is this, that the central velocity, directly facing the orifice, is also the maximum velocity in a short tube, and not the mean velocity. The ratio of these velocities is 1 : ·835 nearly; hence, in the example, p. 102, where $\frac{c}{A} = 3$, we get $3 \times {\cdot}835 = 2{\cdot}505$ for the value of $\frac{c}{A}$ in column 1, Table V., opposite to which is found ·649, the coefficient for an orifice of one-third of the section of the tube when cylindrical or prismatic, Fig. 13; and $3 \times {\cdot}835 \times {\cdot}807 = 2{\cdot}02$ nearly, opposite to which we shall get ·661 for the coefficient when the orifice is at the end of the short tube, Fig. 14, with a rounded junction. Therefore, $\frac{c}{A} \times {\cdot}835$ equal to the new value of $\frac{c}{A}$ for finding the discharge from orifices at the end of cylindrical or prismatic tubes, and $\frac{c}{A} \times {\cdot}835 \times {\cdot}807 = \frac{c}{A} \times {\cdot}67$ nearly for the new value of $\frac{c}{A}$ when finding the discharge from orifices at the end of a short tube with a rounded junction.

The ratio of the mean velocity in a tube to that facing the orifice cannot be less than ·835 to 1, and varies up to 1 to 1; the first ratio obtaining when the orifice is pretty small compared with the section of the tube, and the other when they are equal. If the curve D C, whose abscissæ (A *b*) represent the ratio of the orifice to the section of the tube, and whose ordinates (*b c*) represent the ratio of the mean velocity in the tube to that facing the orifice, be supposed to be a parabola, then the following values are found:—

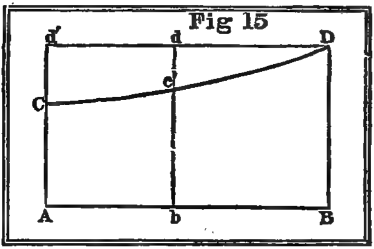


Fig 15

Ratio of the orifice to the channel, or values of $\frac{A}{C} = \frac{A\,b}{A\,B}$	Values of *d c*	Ratio of the mean velocity of approach in a tube or channel to that directly opposite the orifice, or values of *b c*.
·0	·165	·835
·1	·163	·837
·2	·158	·842
·3	·150	·850
·4	·139	·861
·5	·124	·876
·6	·106	·894
·7	·084	·916
·8	·059	·941
·9	·031	·969
1·0	·000	1·000

These values of *b c* are to be multiplied by the corresponding ratio $\frac{C}{A}$ in order to find a new value, opposite to which will be found, in the table, the coefficient for orifices at the ends of short prismatic or cylindrical tubes; and this new value again multiplied by ·807, or more generally by $c_d \left\{\frac{1}{1 - c_d^2}\right\}^{\frac{1}{2}}$*, will give another new value of $\frac{C}{A}$, opposite to which, in the table, will be found the coefficient for orifices at the ends of short tubes with rounded junction.

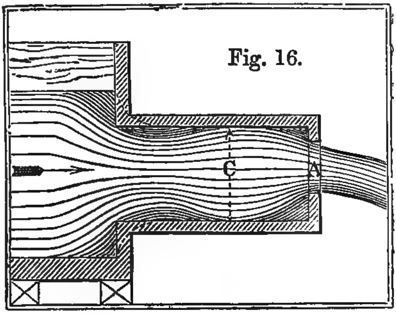


Fig. 16.

Example III.—*What shall be the discharge from an orifice* A, *Fig.* 16, 2 *feet long by* 1 *foot deep, when the value of* $\frac{C}{A}$ *is* 3, *and the depth of the centre of* A 1 *foot* 6 *inches below the surface?*

The theoretical discharge is $D = 2 \times 1 \times \frac{117·945}{12}$ (Table II.) $= 2 \times 9·829 = 19·658$ cubic feet per second. From the table on last page the coefficient for the mean velocity, facing the orifice, is about ·86; hence $\frac{C}{A} \times ·86 = 3 \times ·86 = 2·58$. If the coefficient be taken from Table I., it is found (opposite to 2, the ratio of the length of the orifice to its depth) to be ·617; and, for this coefficient, opposite to 2·58, in Table V., or the next number to it, the required coefficient ·636 is found; hence the discharge is $·636 \times 19·658 = 12·502$ cubic feet per second. If the coefficient in still water be taken at ·628, then we shall obtain the new coefficient ·647, and the discharge would be ·647 × 19·658 = 12·719 cubic feet. If the junction of the tube with the cistern be rounded, as shown by the dotted lines, then multiply 2·58 by ·807, which gives 2·08 for the new value of $\frac{\text{c}}{\text{A}}$, opposite which, in TABLE V., when the first coefficient is ·628, the new coefficient ·659 is found; and the discharge in this case would be ·659 × 19·658 = 12·955 cubic feet per second.

---

* See pp. 97 and 98, with reference to the modifications of equations (45) and (46) into (45*a*) and (46*a*) suited to $h_a = \frac{v_a^2}{2 g c_d^2}$.

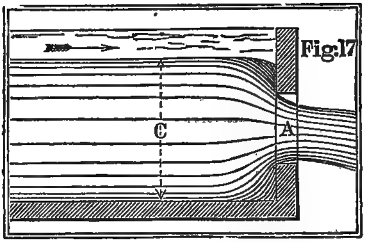


Fig. 17

It is not necessary to take out the coefficient of mean velocity facing the orifice to more than two places of decimals. For gauge notches in thin plates placed in streams and millraces, Fig. 17, the mean coefficient ·628, for still water, may be assumed; thence the new coefficient suited to the ratio $\frac{\text{c}}{\text{A}}$ may be found, as in the first portion of EXAMPLE III.

When h_a is taken equal to $\frac{v_a^2}{2\,g\,c_d^2}$ then from TABLE V. with a coefficient of 1, and 2·58 for the ratio of the channel to the orifice, the value of $\left\{1 + \frac{1}{m^2 - 1}\right\}^{\frac{1}{2}}$ in equation (45*a*) is 1·085. Hence the discharge is 19·658 × ·617 × 1·085 = 19·658 × ·6694 = 13·16 cubic feet per minute in this case.

EXAMPLE IV. *What shall be the discharge through the aperture* A, *equal* 2 *feet by* 1 *foot, when the channel is to the orifice as* 3·375 *to* 1, *and the depth of the* *centre is* 1·25 *foot below the surface, taken at about* 3 *feet above the orifice?*

Here the coefficient of the approaching velocity is ·85 nearly, whence the new value of $\frac{\text{C}}{\text{A}}$ is 3·375 × ·85 = 2·87; and as c_d = ·628, from TABLE V. the new coefficient is ·644. Hence

$\text{D} = 2 \times 1 \times \frac{107{\cdot}669}{12} \times {\cdot}644$ (TABLE II.) $= 2 \times 8{\cdot}972 \times {\cdot}644 = 17{\cdot}944 \times {\cdot}644 = 11{\cdot}556$ cubic feet per second. Weisbach finds the discharge, by an empirical formula, to be 11·31 cubic feet. If the coefficient be sought in TABLE I., it is ·617 nearly, from which, in TABLE V., the new coefficient is found to be ·632: hence 17·944 × ·632 = 11·341 cubic feet per second. If the coefficient ·6225 were used, the new coefficient equals ·638, and the discharge 11·468 cubic feet. *Or thus:* The ratio of the head at the upper edge to the depth of the orifice is $\frac{9}{12} = {\cdot}75$, and from TABLE IV. we find $(1{\cdot}75)^{\frac{3}{2}} - ({\cdot}75)^{\frac{3}{2}} = 1{\cdot}6655$. Assuming the coefficient to be ·644, find from TABLE VI. the discharge per minute over a weir 12 inches deep and 1 foot long which is $\frac{208{\cdot}650 + 205{\cdot}119}{2} = 206{\cdot}884$ cubic feet nearly; and as the length of the orifice is 2 feet, then $\frac{2 \times 206{\cdot}884 \times 1{\cdot}6655}{60} = 11{\cdot}482$ cubic feet per second, which is the correct theoretical discharge for the coefficient ·644, and less than the approximate result, 11·556 cubic feet above found, by only a very small difference. The velocity of approach in this example must be derived from the surface inclination of the stream.

When $h_a = \frac{v_a^2}{2\,g\,c_d^2}$, then with a coefficient 1 and 2·87 for the value of $\frac{\text{C}}{\text{A}}$ the last page of TABLE V. gives 1·067, by interpolation, for the value of $\left\{1 + \frac{1}{m^2 - 1}\right\}^{\frac{1}{2}}$ in equation (45*a*) see pp. 112 and 113. Hence the discharge is 17·944 × ·628 × 1·067 = 17·944 × ·670 = 12·02 cubic feet per second.

FOR NOTCHES, or PONCELET WEIRS, the approaching velocity is a maximum at or near the surface. If the central velocity at the surface facing the notch be 1, the mean velocity from side to side will be ·914. Assume therefore the variation of the central to the mean velocity to be from 1 to ·914; and hence the ratio of the mean velocity at the surface of the channel to that facing the notch or weir cannot be less than ·914 to 1, and varies up to 1 to 1; the first ratio obtaining when the notch or weir occupies a very small portion of the side or width of the channel, and the other when the weir extends for the whole width. Following the same mode of calculation as at p. 104, Fig. 15, the following results will be obtained :—

Ratio of the width of the notch to the width of the channel.	Values of *d e*, Fig. 15.	Values of *b c*, Fig. 15.
·0	·086	·914
·1	·085	·915
·2	·083	·917
·3	·078	·922
·4	·072	·928
·5	·064	·936
·6	·055	·945
·7	·044	·956
·8	·031	·969
·9	·016	·984
1·0	·000	1·000

These values of $b\ c$ are to be used as before, in order to find the value of $\frac{C}{A}$, opposite to which in the tables, and under the heading for weirs, will be found the new coefficient.

EXAMPLE V. *The length of a weir is* 10 *feet; the width of the approaching channel is* 20 *feet; the head, measured about* 6 *feet above the weir, is* 9 *inches; and the depth of the channel* 3 *feet: what is the discharge?*

Assuming the circumstances of the overfall to be such that the coefficient of discharge for heads, measured from still water in a deep weir basin or reservoir, is ·617, then from TABLE VI. the discharge is 128·642 × 10 = 1286·42 cubic feet per minute; but from the smallness of the channel the water approaches the weir with some velocity, and $\frac{C}{A} = \frac{20 \times 3}{10 \times \frac{3}{4}} = 8$. Also the width of the channel is equal to twice the width of the weir, and hence (small table, p. 108,) 8 × ·936 = 7·488 for the new value of $\frac{C}{A}$. From TABLE V. is now found the new coefficient $\frac{\cdot 622 + \cdot 624}{2} = \cdot 623$, and hence the discharge is $\frac{1286 \cdot 42 \times \cdot 623}{\cdot 617} = 1298 \cdot 93$ cubic feet per minute. *Or thus:* As the theoretical discharge, TABLE VI., is 2084·96 cubic feet, then 2084·96 × ·623 = 1298·93, the same as before. In this example, however, the mean velocity approaching the overfall bears to the mean velocity in the channel a greater ratio than 1 : ·936, as, though the head is pretty large in proportion to the depth of the channel, the ratio of the sections $\frac{A}{C} = \frac{1}{8}$ is small. It is therefore more correct to find the multiplier from the small table, p. 104. By doing so the new value of $\frac{C}{A}$ is $8 \times \cdot 838 = 6\cdot 704$. From this and the coefficient ·617 we shall find, as before from TABLE V., the new coefficient to be ·627; hence $2084\cdot 96 \times \cdot 627 = 1307\cdot 27$ cubic feet per minute for the discharge.

The foregoing solution takes for granted that the velocity of approach is subject to contraction before arriving at the overfall, or in passing through it; now, as this reduces the mean velocity of approach from 1 to ·784, TABLE V., when the coefficient for heads in still water is ·617, it is necessary to multiply the value of $\frac{C}{A} = 6\cdot 704$, last found, by ·784, and then $6\cdot 704 \times \cdot 784 = 5\cdot 26$ is the value $\frac{C}{A}$ due to this correction, from which the corresponding coefficient in TABLE V. is found to be ·629, and hence the corrected discharge is $2084\cdot 96 \times \cdot 629 = 1311\cdot 44$ cubic feet.

By using equation (46*a*) in the preceding example, the value of $\left(1 + \frac{1}{m^2 - 1}\right)^{\frac{3}{2}} - \left(\frac{1}{m^2 - 1}\right)^{\frac{3}{2}}$ for $\frac{C}{A} = m = 7\cdot 488$ is, from the last columns of TABLE V. for a coefficient unity, 1·025. Hence the discharge is $2084\cdot 96 \times \cdot 617 \times 1\cdot 025 = 2085 \times \cdot 632 = 1318$ cubic feet per minute in round numbers. If $\frac{C}{A}$ were taken equal to 6·704, then equation (46*a*) would become $2085 \times \cdot 617 \times 1\cdot 031 = 2085 \times \cdot 636 = 1326$ cubic feet nearly.

It is to be borne in mind that the value of the ratio $\frac{C}{A}$ *in* TABLE V. *is simply an approximate value for the* *ratio of the velocity in the channel facing the orifice to the velocity in the orifice itself, and that large differences of value do not always affect the coefficient of discharge materially.* The corrections applied in the foregoing examples were for the purpose of finding this ratio of velocity more correctly than the simple expression $\frac{\text{C}}{\text{A}}$ gives it. The ratio of h_a to h_b also may sometimes vary very considerably without materially affecting the value of c_d; for instance, if $c_d = \cdot628$ for still water, the change of $\frac{h_a}{h}$ from 0 to ·046, and of $\frac{\text{C}}{\text{A}}$ from infinity to 3 causes a variation of, only, from ·628 to ·642 for orifices, and to ·666 for notches, which, practically, does not exceed six per cent. The following auxiliary table

AUXILIARY TABLE, TO BE USED WITH TABLE V. FOR MORE NEARLY FINDING THE COEFFICIENT OF DISCHARGE NEARLY SUITED TO EQUATIONS (45*a*) AND (46*a*).

Ratio of the orifice, to the channel, or $\frac{\text{A}}{\text{C}}$	Multipliers due to the difference of the central and mean velocity only.	Multipliers for finding the new values of $\frac{\text{C}}{\text{A}}$ in Table V., when the water approaches and passes through the orifice, without contraction or loss of velocity.						
		Coefficient ·639	Coefficient ·628	Coefficient ·617	Coefficient ·606	Coefficient ·595	Coefficient ·584	Coefficient ·573
·0	·835	·69	·67	·65	·64	·62	·60	·58
·1	·837	·70	·68	·66	·64	·62	·60	·59
·2	·842	·70	·68	·66	·64	·62	·61	·59
·3	·850	·71	·69	·67	·65	·63	·61	·59
·4	·861	·72	·69	·68	·66	·64	·62	·60
·5	·876	·73	·71	·69	·67	·65	·63	·61
·6	·894	·74	·72	·70	·68	·66	·64	·62
·7	·916	·76	·74	·72	·70	·68	·66	·64
·8	·941	·78	·76	·74	·72	·70	·68	·66
·9	·969	·81	·78	·76	·74	·72	·70	·68
1·0	1·000	·831	·807	·784	·762	·740	·719	·699

finds the correction, and thence the new coefficient, with facility. Thus, if the channel be five times the size of the orifice, and a loss in the approaching velocity takes place equal to that in a short cylindrical tube, then $5 \times \cdot842 = 4\cdot210$ is the new value of $\frac{\text{C}}{\text{A}}$, opposite to which, in TABLE V., will be found the coefficient sought. If the coefficient for still water be ·606, it is found to be ·612 for orifices and ·623 for weirs. But when the water approaches without loss of velocity, from the auxiliary table, ·64 is found for the multiplier instead of ·842, and consequently the new value of $\frac{\text{C}}{\text{A}}$ becomes $5 \times \cdot64 = 3\cdot2$, from which ·617 is found to be the new coefficient for orifices and ·636 for weirs. The auxiliary table is calculated by multiplying the numbers in the second column (see third column, table, p. 104) by the value of $c \times \left\{\frac{1}{1 - c_{\text{d}}^2}\right\}^{\frac{1}{2}}$, which will be found from TABLE V., for the different values of c_{d} in the table, viz.

·639, ·628, ·617, ·606, ·595, ·584, and ·573,
·831, ·807, ·784, ·762, ·740, ·719, and ·699,

to be respectively, as given in the top and bottom lines of figures.

When $\frac{c_{\text{d}}^2}{m^2 - c_{\text{d}}^2}$ in equations (45) and (46) is equal to $\frac{1}{m^2 - 1}$ in equations (45*a*) and (46*a*), then must $c_{\text{d}} = 1$, and $c_{\text{d}} \left\{1 + \frac{c_{\text{d}}^2}{m^2 - c_{\text{d}}^2}\right\}^{\frac{1}{2}}$ in equation (45) becomes equal to $\left\{1 + \frac{1}{m^2 - 1}\right\}^{\frac{1}{2}}$ in equation (45*a*); and

$c_d\left\{\left(1+\frac{c_d^2}{m^2-c_d^2}\right)^{\frac{3}{2}}-\left(\frac{c_d^2}{m^2-c_d^2}\right)^{\frac{3}{2}}\right\}$ in equation (46) also becomes equal to $\left\{\left(1+\frac{1}{m^2-1}\right)^{\frac{3}{2}}-\left(\frac{1}{m^2-1}\right)^{\frac{3}{2}}\right\}$ in equation (46*a*); and therefore the coefficient found from the last three columns of TABLE V. for $c_d = 1$ will give the multiplier for c_d, outside the brackets, in (45*a*) and (46*a*), to find the new coefficients. Thus in the last example $m = 5$, and hence TABLE V. for $c_d = 1$, is given $\left\{1+\frac{1}{m^2-1}\right\}^{\frac{1}{2}} = 1{\cdot}021$ and $\left\{\left(1+\frac{1}{m^2-1}\right)^{\frac{3}{2}} - \left(\frac{1}{m^2-1}\right)^{\frac{3}{2}}\right\} = 1{\cdot}055$. Hence $1{\cdot}021 \times {\cdot}606 = {\cdot}619$ nearly; and $1{\cdot}055 \times {\cdot}606 = {\cdot}639$ nearly, the new coefficients found from the other method being ·617 and ·636, the difference by both methods being of no great practical importance.

It is necessary to observe, that in equations (45), (46), (45*a*), and (46*a*), the head due to the velocity of supply or approach, h_a, *must be extra to the head, h, in the formula and no part of it:* and that—as is indicated by the equations—m can never be so small as unity, for then $\frac{1}{m^2-1}$ would be infinite. These equations are, therefore, only strictly applicable to orifices in the short tubes, Fig. 15 and Fig. 16; when the head h_a due to the velocity of approach is included in the head h measured from still water in a large cistern.

The initial value of the coefficient of discharge, c_d itself, varies considerably with the position and form of the orifice; for a mean value of ·707 it changes, in equation (45), according to the relation of C and A into

$\dfrac{\cdot 707}{\left(1 - \cdot 5\frac{A^2}{c^2}\right)^{\frac{1}{2}}}$; and for a value of ·618 for an orifice, central in a thin plate, Professor Rankine's empirical formula, note p. 102, is in practice applicable.

PRACTICAL FORMULÆ FOR THE DISCHARGE OVER WEIRS.

In order to reduce the preceding formulæ for weirs and notches to some of the forms in common use, with definite combined numerical coefficients, by substituting 8·025 for $\sqrt{2\ g}$, equation (39), becomes for feet measures, as $\frac{2}{3} \times 8{\cdot}025 = 5{\cdot}35$,

$$\text{(A.)}\quad D_a = 5{\cdot}35\ c_d\ l\ \left\{(h_b + h_a)^{\frac{3}{2}} - h^{\frac{3}{2}}\right\}.$$

For inch measures, as $\sqrt{2\ g} = 27{\cdot}8$, the discharge, taken also in cubic feet, becomes

$$\text{(B.)}\quad D = \cdot 01072\ c_d\ l\ \left\{(h_b - h_a)^{\frac{3}{2}} - h_a^{\frac{3}{2}}\right\}.$$

When the length l is taken in feet and the depth in inches, it is

$$\text{(C.)}\quad D = \cdot 1287\ c_d\ l\ \left\{(h_b - h_a)^{\frac{3}{2}} - h_a^{\frac{3}{2}}\right\}.$$

The last three equations being for seconds of time, *when the time is taken at one minute*, for all measures in feet the discharge in cubic feet is

$$\text{(D.)}\quad D = 321\ c_d\ l\ \left\{(h_b + h_a)^{\frac{3}{2}} - h_a^{\frac{3}{2}}\right\}.$$

This when c_d is taken at ·614 becomes

$$\text{(D}_1\text{.)}\quad D_a = 197\ l\ \left\{(h'_b + h_a)^{\frac{3}{2}} - h_a^{\frac{3}{2}}\right\}.$$

For a coefficient of ·617

$$\text{(D}_2\text{.)}\quad D_a = 198\ l\ \left\{(h_b + h_a)^{\frac{3}{2}} - h_a^{\frac{3}{2}}\right\}$$

For a coefficient of ·623

$$(\mathrm{D}_3.)\quad \mathrm{D_a} = 200\, l\, \{(h_b + h_a)^{\frac{3}{2}} - h_a^{\frac{3}{2}}\}.$$

For a coefficient of ·628

$$(\mathrm{D}_4.)\quad \mathrm{D_a} = 201{\cdot}6\, l\, \{(h_b + h_a)^{\frac{3}{2}} - h_a^{\frac{3}{2}}\}.$$

For a coefficient of ·648

$$(\mathrm{D}_5.)\quad \mathrm{D_a} = 208\, l\, \{(h_b + h_a)^{\frac{3}{2}} - h_a^{\frac{3}{2}}\}.$$

For a coefficient of ·667

$$(\mathrm{D}_6.)\quad \mathrm{D_a} = 214\, l\, \{(h_b + h_c)^{\frac{3}{2}} - h_a^{\frac{3}{2}}\}.$$

For a coefficient of ·712

$$(\mathrm{D}_7.)\quad \mathrm{D_a} = 228{\cdot}6\, l\, \{(h_b + h_a)^{\frac{3}{2}} - h_a^{\frac{3}{2}}\}.$$

And for a coefficient of ·810

$$(\mathrm{D}_8.)\quad \mathrm{D_a} = 260\, l\, \{(h_b + h_a)^{\frac{3}{2}} - h_a^{\frac{3}{2}}\}.$$

For inch measures the discharge in cubic feet is

$$(\mathrm{E}.)\quad \mathrm{D_a} = {\cdot}6433\, c_d\, l\, \{(h_b + h_a)^{\frac{3}{2}} - h'^{\frac{3}{2}}_a\}.$$

And for lengths (l) in feet and depths (h_b) in inches the discharge also in cubic feet becomes

$$(\mathrm{F}.)\quad \mathrm{D_a} = 7{\cdot}72\, c_d\, l\, \{h_b + h_a)^{\frac{3}{2}} - h_a^{\frac{3}{2}}\}.$$

The latter equation, when the coefficient of discharge, c_d, is taken at ·614 becomes

$$(\mathrm{F}_1.)\begin{cases} \mathrm{D_a} = 4{\cdot}74\, l\, \{h_b + h_a)^{\frac{3}{2}} - h_a^{\frac{3}{2}}\}, \text{ and} \\ \mathrm{D} = 4{\cdot}74\, l\, h_b^{\frac{3}{2}}, \text{ when the velocity of approach vanishes.} \end{cases}$$

For a coefficient of ·617

$$(\mathrm{F}_2.)\begin{cases} \mathrm{D_a} = 4{\cdot}76\, l\, \{(h_b + h_a)^{\frac{3}{2}} - h_a^{\frac{3}{2}}\}, \text{ and} \\ \mathrm{D} = 4{\cdot}76\, l\, h_b^{\frac{3}{2}} \text{ when the velocity of approach vanishes.} \end{cases}$$

For a coefficient of ·623

$$(\mathrm{F}_3.)\begin{cases}\mathrm{D} = 4{\cdot}81\ l\ \{(h_{\mathrm{b}} + h_{\mathrm{a}})^{\frac{3}{2}} - h_{\mathrm{a}}^{\frac{3}{2}}\}, \text{ and} \\ \mathrm{D} = 4{\cdot}81\ l\ h_{\mathrm{b}}^{\frac{3}{2}} \text{ with no perceptible approach.}\end{cases}$$

For a coefficient of ·628

$$(\mathrm{F}_4.)\begin{cases}\mathrm{D}_{\mathrm{a}} = 4{\cdot}85\ l\ \{(h_{\mathrm{b}} + h_{\mathrm{a}})^{\frac{3}{2}} - h_{\mathrm{a}}^{\frac{3}{2}}\}, \text{ and} \\ \mathrm{D} = 4{\cdot}85\ l\ h_{\mathrm{b}}^{\frac{3}{2}} \text{ with no perceptible approach.}\end{cases}$$

For a coefficient of ·648

$$(\mathrm{F}_5.)\begin{cases}\mathrm{D}_{\mathrm{a}} = 5\ l\ \{(h_{\mathrm{b}} + h_{\mathrm{a}})^{\frac{3}{2}} - h_{\mathrm{a}}^{\frac{3}{2}}\}, \text{ and} \\ \mathrm{D} = 5\ l\ h_{\mathrm{b}}^{\frac{3}{2}} \text{ with no perceptible approach.}\end{cases}$$

For a coefficient of $\frac{2}{3}$ or ·667

$$(\mathrm{F}_6.)\begin{cases}\mathrm{D}_{\mathrm{a}} = 5{\cdot}14\ l\ \{(h_{\mathrm{b}} + h_{\mathrm{a}})^{\frac{3}{2}} - h_{\mathrm{a}}^{\frac{3}{2}}\}, \text{ and} \\ \mathrm{D} = 5{\cdot}14\ l\ h_{\mathrm{b}}^{\frac{3}{2}} \text{ with no perceptible approach.}\end{cases}$$

For a coefficient of ·712

$$(\mathrm{F}_7.)\begin{cases}\mathrm{D}_{\mathrm{a}} = 5{\cdot}5\ l\ \{(h_{\mathrm{b}} + h_{\mathrm{a}})^{\frac{3}{2}} - h_{\mathrm{a}}^{\frac{3}{2}}\}, \text{ and} \\ \mathrm{D} = 5{\cdot}5\ l\ h_{\mathrm{b}}^{\frac{3}{2}} \text{ with no perceptible approach.}\end{cases}$$

And finally for a coefficient of ·81

$$(\mathrm{F}_8.)\begin{cases}\mathrm{D}_{\mathrm{a}} = 6{\cdot}3\ l\ \{(h_{\mathrm{b}} + h_{\mathrm{a}})^{\frac{3}{2}} - h_{\mathrm{a}}^{\frac{3}{2}}\}, \text{ and} \\ \mathrm{D} = 6{\cdot}3\ l\ h_{\mathrm{b}}^{\frac{3}{2}} \text{ when the velocity of approach} \\ \quad \text{vanishes.}\end{cases}$$

The theoretical value of h_{a} in each of the foregoing equations is in terms of the velocity of approach v_{a}

$$h_{\mathrm{a}} = \frac{v_{\mathrm{a}}^2}{2\,g},$$

in which $2\,g$ must be taken equal to 64·403 for heads in feet, and equal to 772·84 for heads in inches. But it is evident that in order to produce the velocity per second v_{a} passing through the notch with a nearly still-water basin above it, that h must be increased from its theoretical value $\frac{v_a^2}{2\,g}$ to $\frac{v_a^2}{c_d^2\,2\,g}$, in which expression c_d is the coefficient of discharge due to the particular notch, or weir, and its attendant circumstances; whence

$$h_a = \frac{v_a^2}{c_d^2\,2\,g} = \frac{\text{Theoretical head}}{c_d^2}.$$

Now, unquestionably the most general coefficient both for notches and submerged orifices, in thin plates, for gauging whether triangular, rectangular, or circular, is ·617, when the orifice or notch is small compared with the approaching channel; whence for measures in feet

$$h_a = \cdot 0408\ v_a^2, \text{ and } v_a = 4\cdot 95\ \sqrt{h}\ .$$

For measures in inches,

$$h_a = \cdot 0034\ v_a^2, \text{ and } v_a = 17\cdot 2\ \sqrt{h_a}.$$

And for measures in which v_a is expressed in feet per second, and h_a in inches

$$h_a = \cdot 49\ v_a^2, \text{ and } v_a = 1\cdot 43\ \sqrt{h_a}\ ;$$

which shows that *half the square of the approaching velocity in feet is equal to the head* h_a *in inches; very nearly.* By substituting these values of h_a, found in terms of the approaching velocity, according to the standards used in the equations from (A) to (F) inclusive, and also in equation (F_2), we shall be enabled to find the proper discharge from a notch in a *thin plate.* The values of h_a, as given above, can be found at once in inches from the observed values of v_a, to be also taken in inches, for coefficients varying from ·584 to ·974, by means of TABLE II. Thus, with a coefficient of ·617, we shall find, for an approaching velocity of 36 inches per second, that h_a becomes $4\frac{2}{5} = 4\cdot 4$ inches nearly, while for a coefficient of ·666, it is only $3\frac{3}{4}$ = 3·8 inches; and for a coefficient of 1, the theoretical head is but $1\frac{3}{4}$ = 1·7 inch nearly.

For the very nature of the case the approaching velocity must continue nearly unimpaired through the notch with but a very slight reduction arising from the viscidity of the water when it enters the aperture, and separates from the lateral fluid. But in order to give this unimpaired velocity by means of an extra head h_a, it is evident that h_a must be increased above the theoretical value by the amount due to the coefficient of discharge; or, as before stated, h_a must be increased from $\frac{v_a^2}{2g}$ to $\frac{v_a^2}{c_d^2 2g}$. This value of h_a is, perhaps, something too large, owing to the reduction of v_a at the moment it enters the notch and is acted upon by the overfall, drawing it away, as it were, from the lateral water above the crest.

The numerical results of the respective formulæ from (A) to (F_8), inclusive, can be obtained by modifying the form as in equation (42) into

$$\left\{\begin{array}{l} \mathrm{D}_a = \mathrm{D} \times \left\{\left(1+\frac{h_a}{h_b}\right)^{\frac{3}{2}} - \left(\frac{h_a}{h_b}\right)^{\frac{3}{2}}\right\} \text{ or,} \\ \mathrm{D}_a = c_d \times \frac{2}{3}\, l\, h_b \sqrt{2\, g\, h_b} \times \left\{\left(1+\frac{h_a}{h_b}\right)^{\frac{3}{2}} - \left(\frac{h_a}{h_b}\right)^{\frac{3}{2}}\right\} \end{array}\right.$$

in which D is the discharge found, when there is no velocity of approach, by the common form $\mathrm{D} = 5{\cdot}35 \times c_d\, l\, h^{\frac{3}{2}}$, for which separate values are given in equations from (F_1) to (F_8) inclusive; and numerical values in TABLE VI.; and $\left\{\left(1+\frac{h_a}{h_b}\right)^{\frac{3}{2}} - \left(\frac{h_a}{h_b}\right)^{\frac{3}{2}}\right\}$ a multiplier suited to the velocity of approach, the values of which can be found from TABLE IV. Suppose, for example, $D = 158{\cdot}1$ cubic feet per minute, $h_b = 10$ inches, and $h_a = 4$ inches, which is that due to an approaching velocity of 3 feet per second with a coefficient of ·648; then the multiplier becomes $(1 + {\cdot}4)^{\frac{3}{2}} - {\cdot}4^{\frac{3}{2}} = 1{\cdot}4035$, TABLE IV. Hence the discharge due to an approaching velocity of 3 feet is $158{\cdot}1 \times 1{\cdot}4035 = 221{\cdot}9$ cubic feet, or an increase of about 40 per cent. Also, if the common formula were used, it is plain that the coefficient ·648 should be increased to $\cdot 648 \times 1{\cdot}4035$, or to ·909 nearly, which approximates within 10 per cent. of the theoretical value. Nothing can show more clearly THE NECESSITY FOR VARYING THE COEFFICIENTS WHEN THE ORDINARY FORMULÆ ARE USED, EVEN FOR A NOTCH IN A THIN PLATE: for other notches the coefficients, even for still water above the crest, vary considerably.

The form of the equation used by D'Aubuisson and several other writers is

$$D_a = C\, l \sqrt{h_b^3 + c\, v_a^2\, h_b^2}$$

in which C and c are numerical coefficients, and v_a the velocity of approach. This form is incorrect in principle, although the values of C and c can be so taken as to give resulting values for D_a approximately correct. For feet measures, and time in seconds, Professor Downing makes, after D'Aubuisson, p. 37 of his translation,

$$D_a = c_d \times 5{\cdot}35\, l \sqrt{h_b^3 + {\cdot}03494\, v_a^2\, h_b^2}.$$

Doctor Robinson* gives for like measures and time, values varying from

---

* Proceedings Royal Irish Academy, vol. iv. p. 212. $\cdot 1395\, v_a^2$ is nine times the theoretical head, and too much.

$$D_a = 3{\cdot}55\, l \sqrt{h_b + {\cdot}1395\, v_a^2\, h_b^2}, \text{ to}$$
$$D_a = 3{\cdot}2\, l \sqrt{h_b^3 + {\cdot}1395\, v_a^2\, h_b^2}.$$

Mr. Taylor finds (for the Government Referees, see Report on the Main Drainage of the Metropolis, 13th July, 1858, p. 32) the discharge in cubic feet, per minute, when the depth is taken in inches, and the length in feet to be,

$$D_a = 5{\cdot}5\, l \sqrt{h_b^3 + {\cdot}8\, v_a^2\, h_b^2};$$

and the Messrs. Hawksley, Bidder, and Bazalgette assume, (p. 33 *ibid,*) for like measures,

$$D_a = 5\, l \sqrt{h_b^3 + {\cdot}1875\, v_a^2\, h_b^2},$$

which they consider is in "*excess.*" The following table, copied and extended from the report just referred to, shows the results of the last two formulæ, and of our equations (F_5) and (F_7), in which the depth,

Formulæ.	Mean velocities approaching the notch in feet per second, and discharges in cubic feet per minute.						
	0	·5	1	1·5	2	2·5	3
$D_a = 5\sqrt{h_b^3 + {\cdot}1875\, v_a^2\, h_b^2}$ . .	158·1	158·5	159·5	161·4	164	167	171
Equation (L) when the head, h_a, due to the velocity of approach is taken at only its theoretical value . . .	158·1	159·2	162·1	166·8	173	180	189
Equation (F_5) when the head, h_a, due to the velocity of approach is increased for the coefficient of velocity ·648 . .	158·1	160	167	177	190	205	222
Equation (F_7) when the head, h_a, due to the velocity of approach is taken at only its theoretical value . .	173·9	175·1	178·3	183·5	190·1	198·3	207·5
Equation (F_7) when the head due to the velocity of approach is increased for the coefficient of velocity ·712	173·9	176	182	192	204	218	234
$D_a = 5{\cdot}5\sqrt{h_b^3 + {\cdot}8\, v_a^2\, h_b^2}$ . .	173·9	175·7	180·8	188·9	199·8	213	228

h_b, must be taken equal to 10 inches, and the length, l, equal to 1 foot.

In equations (L) and (N) we can get, TABLE II., the values of the head, h_a, due to velocity of approach v_a, as follows:

v_a = ·5, ·1, 1·5, ·2, 2·5, 3·0 ; in feet per second.
h_a = ·047, ·186, ·419, ·745, 1·16, 1·68 ; theoretical head in inches.

Then

h_a = ·111, ·447, ·997, 1·77, 2·76, 4· ; for a coefficient of ·648,

and

h_a = ·093, ·366, ·833, 1·47, 2·29, 3·31 ; for a coefficient of ·712.

Whence as $h_b = 10$ inches, we shall have in equation (42),

$\frac{h_a}{h_b}$ = ·011, ·045, ·1, ·18, ·28, ·4 ; for a coefficient of 648,

and

$\frac{h_a}{h_b}$ = ·009, ·037, ·083, ·15, ·23, ·33 ; for a coefficient of 712 ;

and hence, by means of TABLE IV. $\left(1 + \frac{h_a}{h_b}\right)^{\frac{3}{2}} - \left(\frac{h_a}{h_a}\right)^{\frac{3}{2}}$ becomes of the following respective values suited to the above velocities,

1·015, 1·059, 1·122, 1·205, 1·3, 1·403 ; for a coefficient of ·648,

and

1·013, 1·049, 1·104, 1·175, 1·254, 1·344 ; for a coefficient of ·712.

These latter values multiplied, in order, by the initial values of the discharges, 158·1 and 173·9, in the above table, give the discharges in the third and fifth lines corresponding; due to the respective velocities of approach.

The accordance between the results in the last two lines of the table is remarkable. TABLE V. shows that if the coefficient be ·667 when the water above the crest is still, it will be increased to ·712 when the ap-

TABLE *showing the actual discharges per foot in length over a weir gauge 4 feet long in a sewer; and the discharges by different formulæ brought forward during the Metropolitan Sewage Discussion. In these formulæ* D *is* THE DISCHARGE IN CUBIC FEET PER MINUTE, h THE HEAD IN INCHES, *and* v_a THE VELOCITY OF APPROACH IN FEET PER SECOND. *See p. 22, Letter to the Right Hon. Lord John Manners, M.P., from the Government Referees, dated 16th August, 1858. The manner of obtaining the experimental values is not stated in the Letter. The experimental discharges give values for the coefficient of discharge,* c_d, *from ·695 to ·737 nearly.*

Descriptions and Formulæ.	Heads, initial velocities, and corresponding discharges in cubic feet per minute for each foot in length of weir.															
HEADS OF WATER ON WEIR.	1 inch, ·083 feet.		2 inches, ·167 feet.		3 inches, ·25 feet.		4 inches, ·333 feet.		5 inches, ·417 feet.		6 inches, ·5 feet		7 inches, ·583 feet.		8 inches, ·667 feet.	
Initial velocities of stream in feet per second . .	·033	·71	·092	·98	·16	·73	·18	1·02	·32	1·27	·43	2·[illegible]	·53	1·81	·44	2·14
Quantities calculated by the formula $D=5{\cdot}5\sqrt{h^3+{\cdot}8\,h^2\,v_a^2}$	5·5	6·5	15·5	18·3	28·6	30·5	44·1	48·3	61·9	68·8	81·4	115·5	103·5	119·4	125·6	150·2
Quantities calculated by the formula $D=1{\cdot}06\left\{(3\,h+v_a^2)^{\frac{3}{2}}-v_a^3\right\}$	5·5	6·5	15·6	18·5	28·7	30·7	44·2	48·8	62 2	69·6	82·1	114·0	104·0	120·5	125·0	151·6
Quantities calculated by the formula $D=4{\cdot}8\sqrt{h^3+{\cdot}1875\,h^2\,v_a^2}$	4·8	5·0	13·5	14·1	24·9	25·3	38·4	39·3	53·7	54·9	70·9	78·7	89·1	92·72	108 9	110·6
Actual quantities measured	5·7	6·7	15·8	18·3	28·0	29·9	43·8	48·2	61·1	67·6	82·7	113·0	105·0	118·0	125·0	152·8
Values of the coefficients, x, in the formula $D=x\sqrt{h^3+{\cdot}8\,h^2\,v_a^2}$ as obtained from the actual quantities discharged . . .	5·7	5·66	5·58	5·50	5·37	5·38	5·46	5·48	5·43	5·41	5·56	5·38	5·59	5·45	5·47	5·61

proaching channel is about 1·83 times the section of the water in the notch, and this only when $\frac{h_a}{h}$ is taken as in equation (44). If the arrises of the two-inch thick waste board be rounded, the coefficient must also be considerable, although uncertain; but as the equation $D_a = 5{\cdot}5 \sqrt{h_b^3 + {\cdot}8\, v_a^2 h_b^2}$ appears to have been framed by Mr. Taylor, to express special experiments made for Mr. Simpson, in which the quantities varied from 5 to 152 cubic feet per minute, and for heads on a four-foot weir varying from 1 inch to 8 inches,* it must be concluded that the coefficient for heads measured from still water above the crest in those experiments suited to the form of the weir used, and to its attendant circumstances, is ·712.

Equations (39) and those from (A) to (F_3) may be easily changed into forms in which only the depth h_b, the velocity of approach, and the coefficient of velocity (in this case equal to that of discharge) c_d, are introduced. It is, however, only necessary here to reduce the general form (A) p. 114, for feet measures, which becomes, after substituting for h_a its value $\frac{v_a^2}{c_d^2 \times 2g}$, and making some reductions,

$$(A_1. \begin{cases} D_a = 5{\cdot}35 c_d \times \left(\frac{1}{c_d^2 \times 64{\cdot}4}\right)^{\frac{3}{2}} l\{(c_d^2 \times 64{\cdot}4 h_b + v_a^2)^{\frac{3}{2}} - v_a^3\} \\ D_a = \frac{{\cdot}01034}{c_d^2} l\,\{(64{\cdot}4\, c_d^2\, h_b + v_a^2)^{\frac{3}{2}} - v_a^3\}; \end{cases}$$

and for time in minutes the discharge is

---

* Vide p. 22, Letter dated 16th August, 1858, from the Government Referees to the Right Hon. Lord John Manners, on the subject of the Metropolitan Main Drainage.

$$(A_2.)\quad D_a = \frac{\cdot621}{c_d^2}\, l\,\{(64\cdot4\, c_d^2\, h_b + v_a^2)^{\frac{3}{2}} - v_a^3\}\,;$$

in which v_a still continues the velocity in feet per second, as determined from observation. These formulæ may be again reduced to many others. If h_b be taken in inches (A_2) becomes

$$(A_3.)\qquad D_a = \frac{\cdot621}{c_d^2}\, l\,\{\,(5\cdot37\, c_d^2\, h_b + v_a^2)^{\frac{3}{2}} - v_a^3\}.$$

Mr. Pole, in a letter to Mr. Simpson and Captain Galton, already referred to, gives the special value,

$$D_a = 1\cdot06\, l\,\{\,(3\, h_b + v_a^2)^{\frac{3}{2}} - v_a^3\},$$

which corresponds very closely with the experiments made for Mr. Simpson. If $c_d = \cdot712$, which also closely corresponds with those experiments, our equation (A_3) becomes for them

$$(A_4.)\qquad D_a = 1\cdot225\, l\,\{\,(2\cdot72\, h_b + v_a^2)^{\frac{3}{2}} - v_a^3\}\,;$$

but the amount of the discharge must always depend on the coefficient c_d, equation (A_3) suited to the special circumstances of the case under consideration.

The form of equation for the discharge proposed by Mr. Boyden* includes the effects of the end contractions: it is

$$D = C\,\{\,l - b\, n\, h_b\}\, h_b^{\frac{3}{2}}$$

in which $C = \frac{2}{3}\, c_d \sqrt{2\, g}$, n the number of end contractions, l the length of the weir, h_b the head measured from the surface of the water above the curvature of approach, and b a coefficient due to the nature of the end contractions. The mean numerical expression for this formula, derived by Francis from his experiments, is for feet measures, per second,

---

* Francis's Lowell Hydraulic Experiments, p. 74.

$$\text{D} = 3{\cdot}33\,(l - {\cdot}1\,n\,h_{\text{b}})\,h_{\text{b}}^{\frac{3}{2}},\ *$$

but the value of C varied from 3·303 to 3·3617. These results give corresponding values of $c_{\text{d}} = {\cdot}617$ to $\cdot 628$, and when $\text{C} = 3{\cdot}33$, $c_{\text{d}} = {\cdot}623$. The experimental results compared with this formula have been referred to at p. 71.

Francis's Lowell experiments on a wooden dam 10 feet long, level and 3 feet wide at the crest, with a head slope of 3½ to 1 in a channel 10 feet wide, give, for heads between 6 and 20 inches, a mean coefficient of ·663 or ·565. This for feet measures would give for the discharge per second

$$\text{D} = 3{\cdot}02\,h^{\frac{3}{2}}.$$

For greater depths, on this width of crest, the discharge would probably rise as high as $3{\cdot}1\,h^{\frac{}{2}}$ or $3{\cdot}3\,h^{\frac{3}{2}}$. The section of the dam was the same as that erected by the Essex Company across the Merrimack River, at Lawrence, Massachusetts. See, also, TABLE OF COEFFICIENTS, p. 68.

In equation (13), p. 42, there is given a general expression for the value of D through a triangular notch. Professor Thomson, of the University of Glasgow, in a paper read at the British Association at Leeds in 1858, says:—

"The ordinary rectangular notches, accurately experimented on as they have been, at great cost and with high scientific skill, in various countries, with the view of determining the necessary formulas and coefficients for their application in practice, are for many purposes suitable and convenient. They are, however, but ill adapted for the measurement of very

---

* Francis's Lowell Hydraulic Experiments, p. 119.

variable quantities of water, such as commonly occur to the engineer to be gauged in rivers and streams. If the rectangular notch is to be made wide enough to allow the water to pass in flood times, it must be so wide that for long periods, in moderately dry weather, the water flows so shallow over its crest, that its indications cannot be relied on. To remove, in some degree, this objection, gauges for rivers or streams are sometimes formed, in the best engineering practice, with a small rectangular notch cut down below the general level of the crest of a large rectangular notch. If now, instead of one depression being made for dry weather, we use a crest wide enough for use in floods, we conceive of a large number of depressions extending so as to give the crest the appearance of a set of steps of stairs, and if we conceive the number of such steps to become infinitely great, we are led at once to the conception of the triangular instead of the rectangular notch. The principle of the triangular notch being thus arrived at, it becomes evident there is no necessity for having one side of the notch vertical, and the other slanting; but that, as may in many cases prove more convenient, both sides may be made slanting, and their slopes may be alike. It is then to be observed, that by the use of the triangular notch, with proper formulas and coefficients derivable by due union of theory and experiments, quantities of running water from the smallest to the largest may be accurately gauged by their flow through the same notch. The reason of this is obvious, from considering that in the triangular notch, when the quantity flowing is very small, the flow is confined to a small space admitting of accurate measurement; and that the space for the flow of water increases as the quantity to be measured increases, but still continues such as to admit of accurate measurement.

"Further, the ordinary rectangular notch, when applied for the gauging of rivers, is subject to a serious objection from the difficulty or impossibility of properly taking into account the influence of the bottom of the river on the flow of the water to the notch. If it were practicable to dam up the river so deep that the water would flow through the notch as if coming from a reservoir of still water, the difficulty would not arise. This, however, can seldom be done in practice, and although the bottom of the river may be so far below the crest as to produce but little effect on the flow of the water when the quantity flowing is small, yet when the quantity becomes great, the velocity of approach comes to have a very material influence on the flow of the water, but an influence which is usually difficult, if not impracticable, to ascertain with satisfactory accuracy. In the notches now proposed of a triangular form, the influence of the bottom may be rendered definite, and such as to affect alike (or at least by some law that may be readily determined by experiment) the flow of the water when very small, or when very great, in the same notch. The method by which I propose that this may be effected consists in carrying out a floor, starting exactly from the vertex of the notch, and extending both up-stream and laterally, so as to form a bottom to the channel of approach, which will both be smooth and will serve as the lower bounding surface of a passage of approach unchanging in form while increasing in magnitude, at the places at least which are adjacent to the vertex of the notch. The floor may be either perfectly level, or may consist of two planes, whose intersection would start from the vertex of the notch, and would pass upstream perpendicularly to the direction of the weir board; the two planes slanting upwards from their intersection more gently than the sides of the notch. The level floor, although theoretically not quite so perfect as the floor of two planes, would probably for most practical purposes prove the more convenient arrangement.

"With reference to the use of the floor it may be said, in short, that by a due arrangement of the notch and the floor, a discharge orifice and channel of approach may be produced, of which (the upper surface of the water being considered as the top of the channel and orifice) the form will be unchanged or but little changed, with variations of the quantity flowing; very much less certainly than is the case with rectangular notches.

"Whatever may be the result in this respect, the main object must be to obtain, for a moderate number of triangular notches of different forms, and both with and without floors at the passage of approach, the necessary coefficients for the various forms of notches and approaches selected, and for various depths in any one of them, so as to allow of water being gauged for practical purposes, when in future convenient, by means of similarly formed notches and approaches. The utility of the proposed system of gauging it is to be particularly observed, will not depend upon a perfectly close agreement of the theory described with the experiments, because a table of experimental coefficients for various depths, or an empirical formula slightly modified from the theoretical one, will serve all purposes.

"To one evident simplification in the proposed system of gauging, as compared with that by rectangular notches, I would here advert, namely, that in the proposed system the quantity flowing comes to be a function of only one variable—namely, the measured head of water—while in the rectangular notches it is a function of at least two variables, namely, the head of water, and the horizontal width of the notch; and is commonly also a function of a third variable very difficult to be taken into account, namely, the depth from the crest of the notch down to the bottom of the channel of approach, which depth must vary in its influence with all the varying ratios between it and the other two quantities of which the flow is a function.

"The proposed system of gauging also gives facilities for taking another element into account which often arises in practice — namely, the influence of back water on the flow of the water in the gauge, when, as frequently occurs in rivers, it is found impracticable to dam the river up sufficiently to give it a clear overfall free from the back or tail water. For any given ratio of the height of the tail water above the vertex of the notch to the height of head water above the vertex of the notch, I would anticipate that the quantities flowing would still be approximately at least, proportional to the $\frac{5}{2}$ power of the head, as before; and a set of coefficients would have to be determined experimentally for different ratios of the height of the head water to the height of the tail water above the vertex of the notch.

"I have got some preliminary experiments made on a right-angled notch in a vertical plane surface, the sides of the notch making angles of 45° with the horizon, and the flow being from a deep and wide pool of quiet water, and the water thus approaching the notch uninfluenced by any floor or bottom. The principal set of experiments as yet made were on quantities of water varying from about 2 to 10 cubic feet per minute; and the depths or heads of the water varied from 2 inches to 4 inches in the right-angled notch. From these experiments I derive the formula

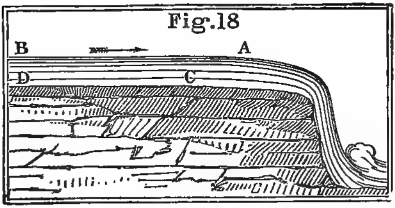
$$Q = 0{\cdot}317\, H^{\frac{5}{2}}$$

where Q is the quantity of water in cubic feet per minute, and H the head as measured vertically in inches from the still water level of the pool down to the vertex of the notch. This formula is submitted at present temporarily as being accurate enough for use for ordinary practical purposes for the measurement of water by notches similar to the one experimented on, and for quantities of water limited to nearly the same range as those in the experiments; but as being, of course, subject to amendment by more perfect experiments extending through a wider range of quantities of water."

In the first edition of this book we gave the general form of the equation for the discharge through triangular notches, and also showed *the general application of the coefficients* ·617 *to* ·628 *for all forms of* *orifices and notches in thin plates.* ·617, as shown in note p. 42, gives a result identical with the practical results of Professor Thomson's experiments. The great advantage of the triangular notch for gauging is, that the sections for all depths flowing over are similar triangles, and therefore the coefficient probably remains constant, or nearly so, not only for one but for all species of triangles, when the depth at the apex is not very small indeed in proportion to the width flowing over at the surface.

The disadvantage of the proposed triangular form of depression, if permanent in a dam, would be that the angular point should be at a lower level than the top of a horizontal crest to maintain the same level, above of the water, during floods; and therefore the power of the water and head would be reduced at the period when most required for mill-power or navigation purposes; that is, during dry weather. For drainage purposes the winter level or that during floods, must evidently be kept down, unless when the banks are steep, and along rapids; but these remarks do not apply to dams erected across millraces or streams where the banks are, generally, considerably above floods; they only refer to occasions for permanent gauging, to find the relations of evaporation, absorption, and discharge for large catchment areas. For notch gauging, to determine the useful effect of water engines, rectangular forms in thin plates have the coefficients already well determined, and the calculations are easy. Rivers and large quantities of flowing water in the absence of a weir, are best gauged by selecting some portion where the section and velocity are nearly uniform and determining each of these from the soundings, and the observed velocities between them, from bank to bank. The jagged, loose and irregular crests on most mill-weirs are unfit to gauge from.

Fig. 18



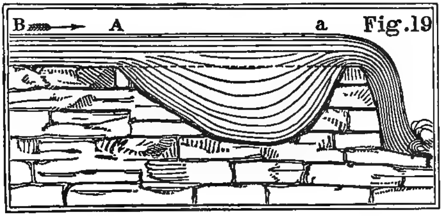
In weirs at right angles to channels with parallel sides, the sectional area can never equal that of the channel unless it be measured at or above the point A, where the sinking of the overfall commences; and unless also the bed C D and surface A B have the same inclination. In all open channels, as millraces, streams, rivers, the supply is derived from the surface inclination of A B, and this inclination regulates itself to the discharging power of the overfall. When the overfall and channel have the same width, and the latter is considerable, then, as shall appear hereafter, $91 \sqrt{h s}$ is the mean velocity in the channel, where h is the depth in feet and s the rate of inclination of the surface A B. Also $\frac{2}{3} \sqrt{2 g h}$ is the theoretical velocity of discharge at the overfall, of equal depth with the channel, and, when both velocities are equal,

$$\frac{2}{3} \sqrt{2 g h} = 5{\cdot}35 \sqrt{h} = 91 \sqrt{h s};$$

from which is found

$$s = \frac{1}{289} = {\cdot}00346,$$

the inclination of B A when the supply is equal to the theoretical discharge at the overfall. If the coefficient at the overfall were ·628, or, which is nearly the same thing, if a large and deep weir basin intervene between the weir and channel, Fig. 19, A a would be level, the



velocity of approach would be destroyed, and then

$$5{\cdot}35 \times {\cdot}628 \sqrt{h} = 3{\cdot}36 \sqrt{h} = 91 \sqrt{h\,s};$$

and thence the inclination of B A is

$$s = \frac{1}{734} = {\cdot}00136$$

very nearly. When discussing the surface inclination of rivers, it will be seen that the conditions here assumed and the resulting surface inclinations would produce velocities that would destroy the regimen and involve a considerable loss of head. If the quantity discharged under both circumstances be the same, and h be the depth in the first case, Fig. 18, then the head in the latter case, Fig. 19, is equal $\left(\frac{5{\cdot}35}{3{\cdot}36}\right)^{\frac{2}{3}} h = 1{\cdot}36\, h$ very nearly, from which and the surface inclination the extent of the backwater may be found with sufficient accuracy. When, in Fig. 19, the inclination of A B exceeds $\frac{1}{734}$, the head at a must exceed the depth of the river above A. Further on, SECTION X., some remarks will be found on the backwater curve.

SECTION V.

SUBMERGED ORIFICES AND WEIRS.—CONTRACTED RIVER CHANNELS.

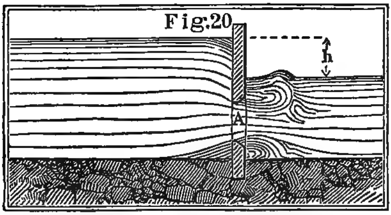


Fig.20

The available pressure at any point in the depth of the orifice A, Fig. 20, is equal to the difference of the pressures on each side. This difference is equal to the pressure due to the height h, between the water surfaces on each side of the orifice; in this case, the velocity is

$$v = c_d \sqrt{2\,g\,h}; \tag{47.}$$

and the discharge

$$D = l\,d\,c_d \sqrt{2\,g\,h}; \tag{48.}$$

in which, as before, l is the length, and d the depth of the rectangular orifice A.

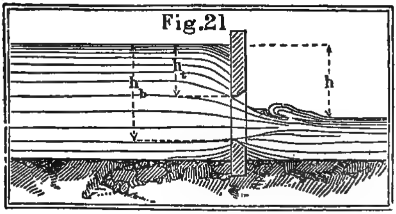


Fig.21

When the orifice is partly submerged, as in Fig. 21, $h_b - h = d_2$ may be put for the submerged depth, and $h - h_t = d_1$, the remaining portion of the depth; whence $d_1 + d_2 = d$ is the entire depth. The discharge through the submerged depth

d_2 is $c_d\, l\, d_2 \times \sqrt{2\, g\, h}$, and the discharge through the upper portion d_1 is

$$\tfrac{2}{3}\, c_d\, l \sqrt{2\, g}\, \{h^{\frac{3}{2}} - h_t^{\frac{3}{2}}\};$$

whence the whole discharge—assuming the coefficient of discharge c_d is the same for the upper and lower depths—is

$$(49.) \quad D = c_d\, l \sqrt{2\, g} \left\{ d_2 \sqrt{h} + \tfrac{2}{3}\, (h^{\frac{3}{2}} - h_t^{\frac{3}{2}}) \right\}$$

We may, however, equation (31), assume that

$$\tfrac{2}{3}\, c_d\, l \sqrt{2\, g}\ (h^{\frac{3}{2}} - h_t^{\frac{3}{2}}) = c_d\, d\, l \sqrt{2\, g \left(h - \frac{d_1}{2}\right)}$$

very nearly, and hence

$$(50.) \quad D = c_d\, l\, d_2 \sqrt{2\, g\, h} + c_d\, l\, d \sqrt{2\, g \left(h_t - \frac{d_1}{2}\right)}.$$

As $h_t + \dfrac{d_1}{2} = h - \dfrac{d_1}{2}$ this equation may be changed into

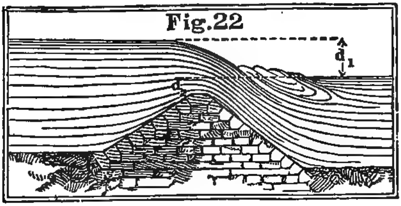
$$(51.) \quad D = c_d\, l\, d_2 \sqrt{2\, g\, h} + c_d\, l\, d_1 \sqrt{2\, g \left(h_t + \frac{d_1}{2}\right)}.$$

In either of these forms the values of

$$c_d \sqrt{2\, g\, h},\ c_d \sqrt{2 g \left(h - \frac{d_1}{2}\right)}, \text{and } c_d \sqrt{2\, g \left(h_t + \frac{d_1}{2}\right)}$$

can be had from TABLE II., and the value of the discharge D thence easily found. When $h - h_a = h_b - h$, the mean value of c_d may be taken at about ·585; that is when the backwater rises to the centre of the orifice.

When the water approaches the orifice with a determinate velocity, the height h_a due to that velocity can be found from TABLE II., and the discharge is then found by substituting $h+h_a$ and h_t+h_a for h and h_t in the above equations.



In the submerged weir, Fig. 22, h becomes equal to d_1, and $h_t=0$; the discharge, equation (49), then becomes

$$(52.)\begin{cases}\mathrm{D}=c_d\, l\, d_2\sqrt{2\,g\,d_1}+\frac{2}{3}\,c_d\, l d_1\sqrt{2\,g\,d_1}, \text{ or}\\ \mathrm{D}=c_d\, l\sqrt{2\,g\,d_1}\left\{d_2+\frac{2}{3}d_1\right\}.\end{cases}$$

When the water approaches with a velocity due to the height h_a, then h becomes $h+h_a$, $h_t=h_a$, and equation (49) becomes

$$(53.)\ \mathrm{D}=c_d l\sqrt{2g}\left\{d_2\sqrt{d_1+h_a}+\frac{2}{3}\,(d_1+h_a)^{\frac{3}{2}}-h_a^{\frac{3}{2}}\right\}.$$

IN THE IMPROVEMENT OF THE NAVIGATION OF RIVERS, it is sometimes necessary to construct weirs so as to raise the upper waters by a given depth, d_1. The discharge D is in such cases previously known, or easily determined, then from the values of d_1, and D, and equation (52), the value of the rise over the crest,

$$(54.)\qquad d_2=\frac{\mathrm{D}}{c_d\, l\sqrt{2\,g\,d_1}}-\frac{2}{3}d_1;$$

or, by taking the velocity of approach into account,

$$(55.)\quad d_2=\frac{\mathrm{D}}{c_d\, l\sqrt{2\,g\,(d_1+h_a)}}-\frac{2}{3}\,\frac{(d_1+h_a)^{\frac{3}{2}}-h_a^{\frac{3}{2}}}{\sqrt{d_1+h_a}}.$$

This value of d_2 must be the depth of the top of the weir below the original surface of the water, in order that this surface should be raised by a given depth, d_1. When h_a is small compared with d_2, $\frac{2}{3}(d_1+h_a)$ may be put for $\frac{2}{3} \times \frac{(d_1 + h_a)^{\frac{3}{2}} - (h)^{\frac{3}{2}}}{\sqrt{d_1+h_a}}$ in equation (55).

Example VI.—*A river whose width at the surface is* 70 *feet, whose hydraulic mean depth is* 4·4 *feet, and whose cross sectional area is* 325 *feet, has a surface inclination of* 1 *foot per mile; to what depth below, or height above the surface must a weir at right angles to the channel be raised, so that the depth of water immediately above it shall be increased by* 3½ *feet?*

When the hydraulic mean depth is 4·4 feet, and the fall per mile 1 foot, from Table VIII. the mean velocity of the river is 29·98 or 30 inches very nearly per second. The discharge is, therefore, 325 × 2½ = 812·5 cubic feet per second, or 48750 cubic feet per minute. Hence, $\frac{48750}{70} = 696{\cdot}4$ cubic feet, must pass over each foot in length of the weir per minute. Assuming the coefficient $c_d = {\cdot}628$ in the first instance, from Table VI. the head passing over a weir corresponding to this discharge is 27·4 inches; but as the head is to be increased by 3½ feet, or 42 inches, it is clear that the weir must be *perfect;* that is, have a clear overfall, and rise 42 − 27·4 = 14·6 inches over the original water surface. In order that the weir may be submerged, or *imperfect,* the head could not be increased by more than 27·4 inches. *Therefore, assume the* example, *that the increase shall be only* 18 *instead of* 42 *inches;* the weir then becomes submerged, and, from equation (54),

$$d_2 = \frac{696{\cdot}4}{{\cdot}628\sqrt{18'' \times 2g}} - \tfrac{2}{3} \times 18'' \text{ (as } l = 1 \text{ foot).}$$

The value of the first part of this expression is found from Tarle VI. or Table II. equal to

$$\frac{696{\cdot}4}{\frac{12}{18} \times \frac{3}{2} \times 370{\cdot}341} = \frac{696{\cdot}4}{370{\cdot}341} = 1{\cdot}88 \text{ feet} = 22{\cdot}56 \text{ in.};$$

hence $22{\cdot}56 - \frac{36}{3} = 10{\cdot}56$ inches is the value of d_2; that is, the submerged weir must be built within 10·56 inches of the surface to raise the head 18 inches above the former level. If, however, the velocity of approach be taken into account, it is equal to $\frac{812{\cdot}5}{430} = 2$ feet per second very nearly; and the height, or value of h_a, due to this velocity, taken from Table II., is $\frac{3}{4} = {\cdot}75$ inch nearly; therefore, from equation (55),

$$d_2 = \frac{696{\cdot}4}{{\cdot}628\sqrt{2g \times 18{\cdot}75}} - \tfrac{2}{3} \times \frac{(18{\cdot}75)^{\frac{3}{2}} - ({\cdot}75)^{\frac{3}{2}}}{\sqrt{18{\cdot}75}}.$$

The value of $\frac{696{\cdot}4}{{\cdot}628\sqrt{2g \times 18{\cdot}75}} =$ (from Table VI.) is

$$\frac{696{\cdot}4}{\frac{3}{2} \times \frac{12}{18{\cdot}75} \times 393{\cdot}75} = \frac{696{\cdot}4}{378{\cdot}8^*} = 1{\cdot}84 \text{ feet} = 22{\cdot}08 \text{ in.};$$

also $\frac{2}{3} \times \frac{(18{\cdot}75)^{\frac{3}{2}} - ({\cdot}75)^{\frac{3}{2}}}{\sqrt{18{\cdot}75}} = \frac{2}{3} \times 18{\cdot}75 - \frac{2}{3} \times \frac{({\cdot}75)^{\frac{3}{2}}}{\sqrt{18{\cdot}75}}$

$$= 12{\cdot}5 - \tfrac{2}{3} \times \frac{{\cdot}65}{4{\cdot}33} = 12{\cdot}5 - {\cdot}1 = 12{\cdot}4.$$

Hence $d_2 = 22{\cdot}08 - 12{\cdot}4 = 9{\cdot}68$ inches, or about 1 inch less than the value previously found from equation (54). The mean coefficient of discharge was here assumed to be ·628. Experiments on submerged weirs show that the value of c_d varies from ·5 up to ·8, but as this coefficient would reduce the value of d_2, or the depth of the top of the weir below the surface, it is safer (where a given depth above a weir must be obtained) to use the lesser and ordinary coefficients of perfect weirs, with a clear overfall, for finding the crest levels of submerged weirs, when it is necessary to construct them. If the coefficient ·8 were used in the previous calculation, then would have been found $d_2 = \frac{\cdot 628 \times 22\cdot 08}{\cdot 8} - 12\cdot 4 = 17\cdot 33 - 12\cdot 4 = 4\cdot 93$ in., or not much more than half the previous value; and this would only increase the whole height of the weir by $9\cdot 68 - 4\cdot 93 = 4\cdot 75$ inches.

---

* This is found from Table II. more readily.

As $\text{D} = \frac{2}{3} c_d l \sqrt{2 g} \{(d_1 + h_a)^{\frac{3}{2}} - h_a^{\frac{3}{2}}\}$ for a perfect weir with a free overfall, it is clear that when D is greater than $\frac{2}{3} c_d l \sqrt{2 g} \{(d_1 + h_a)^{\frac{3}{2}} - h_a^{\frac{3}{2}}\}$, the weir is imperfect or submerged. For backwater curve see SECTION X.

In the following table of coefficients from Lesbros* d_2 is measured from that point below the weir where its value is a minimum. On examining equation (52), it will be seen that the equation $\text{D} = c_d l (d_1 + d_2) \sqrt{2 g d_1}$ adopted by Lesbros is incorrect, and can only be safely used within the limits of his experiments.

---

* *Vide* p. 84, deuxième édition, Hydraulique, par Arthur Morin. Paris, 1858.

COEFFICIENTS FOR SUBMERGED NOTCHES.

Values of $\frac{d_1}{d_1 + d_2}$	Values of the coefficient c_d in the formula $D = c_d\, l\, (d_1 + d_2) \times \sqrt{2\, g\, d_1}$.	Values of $\frac{d_1}{d_1 + d_2}$	Values of the coefficient c_d in the formula $D = c_d\, l\, (d_1 + d_2) \times \sqrt{2\, g\, d_1}$.
·001	·227	·060	·519
·002	·295	·080	·517
·003	·363	·100	·516
——	——	·150	·512
·004	·430	·200	·507
·005	·496	·250	·502
·006	·556	·300	·497
·007	·597	·350	·492
·008	·605	·400	·487
·009	·600	·450	·480
·010	·596	——	——
·015	·580	·500	·474
·020	·570	·550	·466
·025	·557	·600	·459
·030	·546	·700	·444
·035	·537	·800	·427
·040	·531	·900	·409
·045	·526	1·000	·390
·050	·522	,,	,,

The experimental values are those shown between the horizontal lines, the others above the upper ones, and below the lower ones, were deduced from calculation by Lesbros.

The true value of the discharge is expressed by the equation $D = c_d\, l \left\{ \frac{2}{3} d_1 + d_2 \right\} \times \sqrt{2\, g\, d_1}$, and the values of c_d in the above table are, therefore, too small, applied to the correct formula. When $d_1 = d_2$ the table gives $c_d = \cdot 474$. Now for weirs in which the sheet passing over is "drowned," the general value of the coefficient is about ·67; this would give the coefficient for the lower portion d_2, in the true formula, equal to ·503, and a mean coefficient c_d in the correct formula (52) equal to ·569 nearly. When $d_2 = 200\, d_1$, the apparent limits of the experiments on the other side, then the mean value of $c_d = \cdot 496$ nearly in equation (52). These results would show that the coefficient due to the submerged depth d_2, in the first and last experiments, is equal to about ·5 nearly, (but varies to ·6 nearly in some of the middle experiments,) or thereabouts, and, therefore, equation (52) for submerged weirs, as the coefficient for the upper part d_1 is ·67, would become

(52A.) $D = l \times \{\cdot 445\, d_1 + \cdot 5\, d_2\} \times \sqrt{2\, g\, d_1}$;

which for feet measures would become again

(52B.) $D = l \times \sqrt{d_1} \times \{3\cdot 56\, d_1 + 4\, d_2\}$,

for the discharge in cubic feet per second over a submerged weir, Fig. 22.

CONTRACTED RIVER CHANNELS.

When the banks of a river, whose bed has a uniform inclination, approach each other, and contract the width of the channel in any way, as in Fig. 23, the water will rise in the channel above the contracted portion A, until the increased velocity of discharge compensates for the reduced cross section. If, as before, d_1 be put for the increase of depth immediately above the contracted width, and d_2 for the previous depth of the channel, the quantity of water passing through the lower depth, d_2, is equal to $c_d\, l\, d_2 \sqrt{2\, g\, d_1}$, in which l

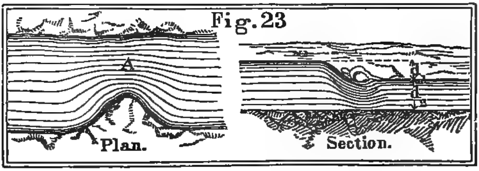


Fig. 23

is the width of the contracted channel at A, and the quantity of water overflowing through d_1 equal to $\frac{2}{3} \times c_d\, l\, d_1 \sqrt{2\, g\, d_1}$; and hence the whole discharge through A is

$$(56.) \qquad \mathrm{D} = c_d\, l \sqrt{2\, g\, d_1} \left(d_2 + \frac{2}{3} d_1\right).$$

When the object is to find the width l of the contracted channel, so that the depth of water in the upper stretch shall be increased by a given depth d_1, then shall

$$(57.) \qquad l = \frac{\mathrm{D}}{c_d \sqrt{2\, g\, d_1} \left(d_2 + \frac{2}{3} d_1\right)}$$

When the velocity of approach is considerable, or when the height h_a due to it becomes a large portion of d_1, its effect must not be neglected. In this case, as before, the discharge through the depth d_2 is equal to $c_d\, l\, d_2 \times \sqrt{2\, g\, (d_1 + h_a)}$; and the discharge through the depth d_1 equal to $\frac{2}{3}\, c_d\, l \sqrt{2\, g}\, \{(d_1 + h_a)^{\frac{3}{2}} - h_a^{\frac{3}{2}}\}$; and hence the whole discharge is

$$(58.) \quad \mathrm{D} = c_d\, l \sqrt{2\, g}\, \left\{d_2\, (d_1 + h_a)^{\frac{1}{2}} + \frac{2}{3} \left[(d_1 + h_a)^{\frac{3}{2}} - h_a^{\frac{3}{2}}\right]\right\};$$

from which may be found

$$(59.) \quad l = \frac{\mathrm{D}}{c_d \sqrt{2\, g}\, \left\{d_2\, (d_1 + h_a)^{\frac{1}{2}} + \frac{2}{3} \left[(d_1 + h_a)^{\frac{3}{2}} - h_a^{\frac{3}{2}}\right]\right\}}.$$

If the projecting spur or jetty at A be itself submerged, these formulæ must be extended; the manner of doing so, however, presents no difficulty, as it is only necessary to find the discharges of the different sections according to the preceding formulæ, and then add them together; but the resulting formula so found is too complicated to be of much practical value.

HEADS ARISING FROM PIERS AND BACKWATER ABOVE BRIDGES.

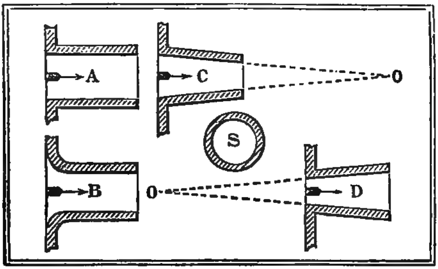
Equations (56), (57), (58), and (59), are applicable to cases of contraction of river channels caused by the construction of bridge-piers and abutments, when the width l is put for the sum of the openings between them. The value of the coefficient c_d will depend on the peculiar circumstances of each case; it was shown that it rises from ·5 to ·7 in some cases of submerged weirs, and for cases of contracted channels it rises sometimes as high as ·8, particularly when they are analogous to those for the discharge through mouthpieces and short tubes. When the heads of the piers are square to the channel, the coefficient may be taken at about ·6; when the angles of the cut-waters or sterlings are obtuse, it may be taken at about ·7; and when curved and acute, at ·8. With this coefficient, a head of $2\frac{5}{8}$ inches will give a velocity of very nearly 36 inches, or 3 feet per second; but as a certain amount of loss takes place from the velocity of the tail-water being in general less than that through the arch, also from obstructions in the passage, and from square-headed and very short piers, the coefficient may be so small in some cases as ·5, which would require a head of $6\frac{3}{4}$ inches to obtain the same velocity. This head is to the former as 54 to 21. The selection of the proper coefficient suited to any particular case is, therefore, a matter of the first importance in determining the effect of obstructions in river channels: this subject shall be referred to again, but it is necessary to observe here, that the form of the approaches, the length of the piers compared with the distance between them, or span, and the length and form of the obstruction compared with the width of the channel, must be duly considered before the coefficient suited to the particular case can be fixed upon. Indeed, the coefficients will always approximate towards those, given in the next section, for mouth-pieces, shoots, and short tubes similarly circumstanced. For some further remarks on contracted channels, see SECTION X.

SECTION VI.

SHORT TUBES, MOUTH-PIECES, AND APPROACHES.—ALTERATION IN THE COEFFICIENTS FROM FRICTION BY INCREASING THE LENGTH.—COEFFICIENTS OF DISCHARGE FOR SIMPLE AND COMPOUND SHORT TUBES.—SHOOTS.

The only orifices heretofore referred to were those in thin plates or planks, with a few incidental exceptions. It has been shown, page 36, Fig. 4, that a rounding off, next the water, of the mouth-piece increases the coefficient; and when the curving assumes the form of the *vena-contracta*, the coefficient increases to ·986, or nearly unity for the outer orifice. The discharge from a short cylindrical tube A, Fig. 24, whose length is from one and a half to three times the diameter, is found to be very nearly an arithmetical mean between the theoretical discharge and the discharge through a circular orifice in a thin plate of the same diameter as the tube, or ·814 nearly. If, however, the inner arris be rounded, or chamfered off in any way, the coefficient will increase until, in the tube B, Fig. 24, with a properly-rounded junction, it becomes unity very nearly.

Fig. 24.



In the conical short tubes C and D the coefficients are found to vary according to some function of the converging or diverging angles O, O, and according as the lesser or greater diameter is taken to calculate from. When the length of the tube exceeds twice the diameter, the friction of the water against the sides may be taken into account.

The following table, calculated, for a coefficient of friction ·00699, due to a discharging *velocity of about eighteen inches per second,* see SECTION VIII., shows the resistance arising from friction in pipes of different lengths in relation to the diameter, and will be found of considerable practical value. It will be perceived that the calculations are made for three different orifices of entry. First, when the arrises are rounded, as in B, Fig. 24, with a coefficient of ·986; secondly, COEFFICIENTS FOR SHORT AND LONG TUBES.

Velocities about 1·5 foot per second.

Number of diameters in the length of the pipe.	Corresponding coefficients of discharge, showing the effects of friction.			Number of diameters in the length of the pipe.	Corresponding coefficients of discharge, showing the effects of friction.		
2 diameters	·986	·814	·715	650 diameters	·228	·225	·223
5 ,,	·936	·779	·690	700 ,,	·220	·217	·215
10 ,,	·884	·747	·668	750 ,,	·213	·211	·209
15 ,,	·840	·720	·649	800 ,,	·206	·205	·203
20 ,,	·801	·695	·630	850 ,,	·201	·199	·197
25 ,,	·767	·673	·615	900 ,,	·195	·193	·192
30 ,,	·737	·653	·598	950 ,,	·190	·189	·187
35 ,,	·711	·634	·584	1000 ,,	·186	·184	·183
40 ,,	·693	·617	·570	1100 ,,	·177	·176	·175
45 ,,	·665	·601	·558	1200 ,,	·170	·169	·168
50 ,,	·646	·586	·546	1400 ,,	·158	·157	·156
100 ,,	·513	·480	·458	1600 ,,	·148	·147	·146
150 ,,	·439	·418	·403	1800 ,,	·139	·139	·138
200 ,,	·389	·375	·364	2000 ,,	·132	·132	·131
250 ,,	·354	·345	·334	2200 ,,	·126	·126	·125
300 ,,	·327	·318	·311	2400 ,,	·120	·120	·120
350 ,,	·304	·297	·292	2600 ,,	·116	·116	·116
400 ,,	·287	·280	·276	2800 ,,	·112	·112	·112
450 ,,	·271	·266	·262	3000 ,,	·108	·108	·108
500 ,,	·258	·254	·250	3200 ,,	·105	·105	·104
550 ,,	·247	·243	·240	3400 ,,	·102	·102	·101
600 ,,	·237	·234	·231	3600 ,,	·099	·099	·099

See p. 199.

when the arrises are square, as in A, with a coefficient of ·815; and, thirdly, when the pipe projects into the vessel, when the coefficient of entry becomes reduced to ·715. The velocity is

$$v = c_{\mathrm{d}} \sqrt{2 g h},$$

h being measured to the centre; lower end of the tube.

It is seen from this table, that the effect of adding to the length of the pipe is greatest next the orifice of entry. The effect of a few diameters added to the length in long pipes is, practically, immaterial; but in short pipes it is considerable.

As for orifices in thin plates, so also for short tubes, the coefficients are found to vary according to the depth of the centre below the surface of the water, and to increase as the depths and diameter of the tube decrease. Poleni first remarked that the discharge through a short tube was greater than that through a simple orifice, of the same diameter, in the proportion of 133 to 100, or as ·617 to ·821.

CYLINDRICAL SHORT TUBES, A, FIG. 24.

The experiments of Bossût, as reduced by Prony, give the following coefficients, at the corresponding depths, for a cylindrical tube A, Fig. 24, 1 inch in diameter and 2 inches long. The depths are given in Paris feet in the original, but the coefficients remain the same, practically, for depths in English feet.

COEFFICIENTS FOR SHORT TUBES, FROM BOSSÛT.

Heads in feet.	Coefficients.	Heads in feet.	Coefficients.	Heads in feet.	Coefficients.
1	·818	6	·806	11	·805
2	·807	7	·806	12	·804
3	·807	8	·805	13	·804
4	·807	9	·805	14	·804
5	·806	10	·805	15	·803

Venturi's experiments give a coefficient ·823 for a short tube A, 1½ inch in diameter and 4½ inches long, at a depth of 2 feet 8½ inches, the coefficient through an orifice in a thin plate of the same diameter and at the same depth being ·622. The author has calculated these coefficients, from the original experiments. The measures were in Paris feet and inches, from which the calculations were directly made; and as the difference *in the coefficient* for small changes of depth or dimensions is immaterial or vanishes, as may be seen by the foregoing small table, and as 1 Paris inch or foot is equal to 1·0658 English inches or feet, the former measures exceed the latter by only about $\frac{1}{15}$th. It may therefore be assumed that *the coefficient* for any orifice, at any depth, is the same, whether the dimensions be in Paris or English feet, or inches. This remark will be found generally useful in the consideration of the older continental experiments, and will prevent unnecessary reductions from one standard to another where the coefficients only have to be considered.

The mean value derived from the experiments of Michelotti, at depths from 3 to 20 feet, and with short tubes A from $\frac{1}{2}$ inch to 3 inches in width, is $c_d = \cdot 814$. Buff's experiments* give the following results for a tube $\frac{3}{10}$ of an inch wide and $\frac{5}{10}$ of an inch long nearly.

BUFF'S COEFFICIENTS FOR SMALL SHORT TUBES.

Head in inches.	Coefficient.	Head in inches.	Coefficient.	Head in inches.	Coefficient.
1¼	·855	6	·840	23	·829
2½	·861	14	·840	32	·826

The increase for smaller tubes and for lesser depths appears by comparing these results with the foregoing, and from the results in themselves, generally. Weisbach's experiments give a mean value for $c_d = \cdot 815$, and for depths of from 9 to 24 inches the coefficients ·843, ·832, ·821, ·810 respectively, for tubes $\frac{4}{10}$, $\frac{8}{10}$, $\frac{12}{10}$ and $\frac{16}{10}$ of an inch wide, the length of each tube being three times the diameter. D'Aubuisson and Castel's experiments with a tube ·61 inch diameter and 1·57 inch long, give ·829 for the coefficient at a depth of 10 feet. When a pipe projects into a cistern and has a sharp edge, the coefficient falls so low as ·715.

---

* Annalen der Physik und Chemie von Poggendorff, 1839, Band 46, p. 243.

The coefficients in the following two short tables were calculated by the author from Rennie's experiments with glass orifices and tubes, TABLE 7, p. 435, Philosophical Transactions for 1831. The form of the orifices, or length of the shorter tubes is not stated, but it is probable from the result, that the arrises of the ends were in some way rounded off; it is stated they were "enlarged." Indeed, the discharges from the short tube or orifice of ¼ inch diameter exceed the theoretical ones in the proportion of 1·261 to 1, and 1·346 to 1. These results could not have been derived from a simple cylindrical tube, but might have arisen from the arrises being more or less rounded at both ends, and the orifice partaking of the nature of a compound tube, which may be constructed, as shall hereafter be shown, so as to increase the theoretical discharge from 1 up to 1·553. The resulting coefficients for the ½ and ¾ inch tubes, approach very closely to those obtained by other

COEFFICIENTS FOR SHORT TUBES, THE ENDS ENLARGED.

Head in feet.	¼ inch diameter.	½ inch diameter.	¾ inch diameter.	1 inch diameter.
1	1·231	·831	·766	·912
2	1·261	·839	·820	·920
3	1·346	·838	·821	·880
4	1·261	·831	·829	·991

experimenters, but those for the inch tube are too high, unless the arris at the ends was also rounded. The coefficients derived from the experiments with a cylindrical glass tube 1 foot long, as here given, are very variable; like the others they are, however, valuable, as exhibiting the uncertainty attending "experiments of this nature," and the necessity for minutely observing and recording every circumstance which tends to alter and modify them. Indeed, for small tubes, a very slight difference in the measurement of the diameter must alter the result a good deal, particularly when it is recollected that measurements are seldom taken more closely than the sixteenth of an inch, unless in special cases. As the author, however, states, p. 433 of the work referred to, that the "diameters of the tubes at their extremities were carefully enlarged to prevent wire edges from diminishing the sections;" this circumstance alone must have modified the discharges, and would account for most of the differences.

COEFFICIENTS DERIVED FROM EXPERIMENTS WITH A GLASS TUBE ONE FOOT LONG.

Heads in feet.	¼ inch diameter.	½ inch diameter.	¾ inch diameter.	1 inch diameter.
1	·892	·703	·691	·760
2	·914	·734	·718	·749
3	·831	·723	·709	·777
4	·914	·725	·677	·815

The coefficient for rectangular short tubes differs in no way materially from those given for cylindrical ones, and may be taken on an average at ·814 or ·815.

SHORT TUBES WITH A ROUNDED MOUTH-PIECE, B, FIG. 24.

When the junction of a short tube with a vessel takes the form of the contracted vein, Figs. 3 and 4, pp. 35 and 36, the mean value of the coefficient $c_d = \cdot 956$, and the actual discharge is found to be from 93 to 99 per cent. of the theoretical discharge. Weisbach, for a tube $1\frac{1}{2}$ inch long and $\frac{9}{10}$ inch diameter, rounded at the junction, found at 1 foot deep $c_d = \cdot 958$, at 5 feet deep $c_d = \cdot 969$, and at 10 feet deep $c_d = \cdot 975$. These experiments show an increase in the coefficients, in this particular case, for an increase of depth. Any other form of junction than that of the contracted vein, will reduce the discharge, and the coefficients will vary from ·715 to ·814, and to ·986, according to the change in the junction from the cylindrical, projecting into the vessel, to the square and properly curved forms. The coefficients derived from Venturi's experiments will be given hereafter.

SHORT CONICAL CONVERGENT TUBES, C, FIG. 24.

The experiments of D'Aubuisson and Castel lead to the following coefficients of discharge and velocity* from a conically convergent tube C at a depth of 10 feet. The original angles and coefficients are here interpolated so as to render the table more convenient to refer to, for practical purposes, than the original. The diameter of the tube at the smaller or discharging orifice in the experiments was ·61 inches, and the

---

* Traité d'Hydraulique, Paris, p. 60.

COEFFICIENTS FOR CONICAL CONVERGENT TUBES.

Converging angle o.	Coefficient of discharge.	Coefficient of velocity.	Converging angle o.	Coefficient of discharge.	Coefficient of velocity.
1°	·858	·858	14°	·943	·964
2°	·873	·873	16°	·937	·970
3°	·908	·908	18°	·931	·971
4°	·910	·909	20°	·922	·971
5°	·920	·916	22°	·917	·973
6°	·925	·923	26°	·904	·975
8°	·931	·933	30°	·895	·976
10°	·937	·950	40°	·869	·980
12°	·942	·955	50°	·844	·985

length of the axis 1·57 inch; that is, the length was 2·6 times the smaller diameter of the tube. The coefficient became ·829 for the cylindrical tube, *i. e.* when the angle at o was nothing. The angle of convergence o determines, from the proportions, the length of the inner and longer diameter of the tube. The coefficients of discharge increase up to ·943 for an angle of $13\frac{1}{2}$ or 14 degrees, after which they again decrease; but the coefficients of velocity increase as the angle of convergence, o, increases from ·829, when the angle is zero up to ·985 for an angle of 50 degrees.

When D is the discharge and A the area of the section, as before shown, $D = c_d A \sqrt{2 g h}$; but as, in conically convergent or divergent tubes, the inner and outer areas (or, as they may be called, the receiving and discharging sections) vary, it is clear that, the discharge being the same, and also the theoretical velocity $\sqrt{2 g h}$, the coefficient c_d must vary inversely with the sectional area A, and that $c_d \times A$ must be constant. For the coefficients tabulated, the sectional area to be used is that at the smaller or outside end of a convergent tube C, Fig. 24.

For a short tube C, whose length is ·92 inch, lesser diameter 1·21 inch, and greater diameter 1·5 inch, the author has found, from Venturi's experiments, that $c_d = {·}607$ if the larger diameter be used in the calculation, and $c_d = {·}934$ when the lesser diameter is made use of, the discharge taking place under a pressure of 2 feet 8½ inches.

The earlier experiments of Poleni, when reduced, furnish us with the following coefficients: A tube 7·67 inches long, 2·167 inches diameter at each end, gave $c_d = {·}854$; the like tube with the inner or receiving orifice increased to 2¾ inches, $c_d = {·}903$; increased to 3·5 inches, $c_d = {·}898$; increased to 5 inches, $c_d = {·}888$; and increased to 9·83 inches, $c_d = {·}864$. The depth or head was 21·33 inches, the discharging orifice 2·167 inches diameter, and the length 7·67 inches, in each case.

In the conically divergent tube D, Fig. 24, the coefficient *of discharge* is larger than for the same tube C, convergent, when the water fills both tubes, and the smaller sections, or those at the same distances from the centres O O, are made use of in the calculations. A tube whose angle of convergence, O, is 5, nearly, with a head of from 1 to 10 feet, whose axial length is 3½ inches, smaller diameter 1 inch, and larger diameter 1·3 inch, gives, when placed as at C, ·921 for the coefficient; but when placed as at D, the coefficient increases to ·948. In the first case the smaller area, used in both calculations, being the receiving, and in the other the discharging, orifice.

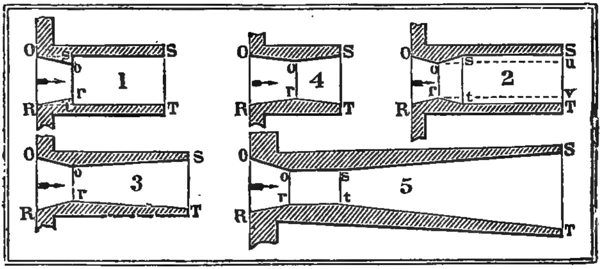
The coefficient *of velocity* is, however, larger for the tube C than for the tube D, and the discharging jet of water has a greater amplitude in falling. The effects of conically diverging tubes will, however, be better perceived from the experiments on compound short tubes.

EFFECTS OF COMPOUND ADJUTAGES AND ADMISSION OF AIR INTO SHORT TUBES.

If the tube A Fig. 24, be pierced all round with small holes at the distance of about half its diameter from the reservoir, the discharge will be immediately reduced in the proportion of ·814 to ·617. Venturi found the reduction for a tube 1½ inch diameter and 4½ inches long, at a depth of 2 feet 10½ inches, as 41 to 31, or as ·823 to 6·22. As long as one hole remained open, the discharge continued at the same reduced rate; but when the last hole was stopped, the discharge again increased to the original quantity. If a small hole be pierced in a tube 4 diameters long, at the distance of 1½ or 2 diameters at nearest to the junction, the discharge will remain unaffected. This shows that the contraction in the cylindrical tube extends only a short distance from the junction, probably 1¼ or 1½ diameter, including the whole curvature of the contraction.

The contraction at the entrance into a tube from a reservoir accounts for the coefficients for a short tube A, Fig. 24, and the short tubes, diagrams 1 and 2, Fig. 25, being each the same decimal nearly, when O R : $o\,r$: : 1 : ·8, or when $o\,r$ is not less than O R × ·79, and is at the distance of nearly $\frac{\text{O R}}{2}$ from O R. The form of the junction O *o r* R remaining as described, the following coefficients will enable us to judge of the discharging powers of differently formed short mouthpieces. They have been deduced and calculated, principally, from Venturi's experiments.*

Fig. 25.



These coefficients show very clearly that any calculations from the mere head of water and size of the orifice, without taking into consideration the form of the discharging tube and its connection with the reservoir, are very uncertain; and that the discharge can only be correctly obtained when all the circumstances of the case, including the form of the discharging orifice and its approaches have been duly considered.

When a tube similar to diagram 5, Fig. 25, has the junction O *o r* R rounded, as in Fig. 4, page 36, the outer extremity *s t* S T, such that *s t* = *o r*, *s* S = 9 *s t*, and the diameter S T = 1·8 times the diameter *s t*, with a *short* central cylindrical piece *o r s t* between, the

---

* See Nicholson's translation of Venturi's Experimental Inquiries, published in the Tracts on Hydraulics, London, 1836. The coefficients in the table, next page, were all calculated for the first time by the Author.

TABLE OF COEFFICIENTS FOR SHORT TUBES AND MOUTH-PIECES.

Description of orifice, mouth-piece, or short tube.	Coefficients for the diameter O R.	Coefficients for the diameter *o r*.
1. An orifice 1½ inch diameter in a thin plate	·622	·974
2. A cylindrical tube 1½ inch diameter and 4½ inches long, A, Fig. 24	·823	·823
3. A short tube with a sharp end projecting into the cistern	·715	·715
4. A cylindrical tube, B, Fig. 24, having the junction rounded, as in Fig. 4, page 36	·611	·956
5. A short conical convergent mouth-piece, C, Fig. 24, of the proportions of O *o r* R, Fig. 25	·607	·934
6. The like tube divergent, with the smaller diameter at the junction with the reservoir; length 3½ inches, lesser diameter 1 inch, and greater diameter 1·3 inch	·561	·948
7. The tube, O *o u v r* R, diagram 2, Fig. 25, when O R=1½ inch, *o r*=1·21 inch, *u v*=1·21 inch, and *o u*=*r v*=2 inches, the cylindrical portion being shown by dotted lines	·600	·923
8. The same tube when *o u*=11 inches	·567	·873
,, The same tube when *o u*=23 inches	·531	·817
9. The tube, O *o s* S T *t r* R, diagram 2, Fig. 25, in which O R=*s t*=S T=1½ inch, from O to *s* 1¾ inch, and *s* S=3 inches, gives the same coefficient as the cylindrical tube, result No. 2 (see No. 19), viz.	·823	1·266
10. The tube, diagram 1. Fig. 25, O R=1½ inch	·804	1·237
11. The same tube, having the spaces *o s* O and *r t* R between the mouth-piece O *o r* R and the cylindrical tube O S T R open to the influx of the water.	·785	1·209
12. The double conical tube, O *o* S T *r* R, diagram 3, Fig. 25, when O R=S T=1½ inch, *o r*=1·21 inch, O *o*=·92 inch, and *o* S=4·1 inches	·928	1·428
13. The like tube when, as in diagram 4, Fig. 25, O *o r* R=*o* S T *r*, and O *o* S=1·84 inch	·823	1·266
14. The like tube when, S T=1·46 inch, and *o* S=2·17 inches	·823	1·266
15. The like tube when S T=3 inches, and *o* S=9¾ inches	·911	1·400
16. The like tube when *o* S=6½ inches, and S T enlarged to 1·92 inch	1·020	1·569
17. The like tube when S T=2¼ inches, and *o* S=12⅓ inches	1·215	1·855
18. A tube, diagram 5, Fig. 25, when *o s*=*r t*=3 inches, *o r*=*s t*=1·21 inch, and the tube *o* S T *r* the same as described in No. 12, viz. S T=1½ inch, and *s* S=4·1 inches	·895	1·377
19. The tube, diagram 2, Fig. 25, when S T is enlarged to 1·97 inch, and *s* S to 7 inches, the other dimensions remaining as in No. 9	·945	1·454
20. When the junction of *o s r t* with *s* S T *t*, diagram 2, Fig. 25, is improved, the other parts remaining as described in No. 9	·850	1·309
21. Another experiment gives	·847	1·303

coefficient of discharge, corresponding to the diameter *o r* = *r s* will increase to 1·493 or 1·555; that is, the discharge is $\frac{1\cdot493}{\cdot622} = 2\cdot4$, or $\frac{1\cdot555}{\cdot622} = 2\cdot5$ times as much as through an orifice (whose diameter is *o r*) in a thin plate; and $\frac{1\cdot555}{\cdot822} = 1\cdot9$ times as much as through a short cylindrical tube A, Fig. 24, whose diameter is also *o r*. Venturi was of opinion that this discharge continued even when the central cylindrical portion *o r s t* was of considerable length; but this was a mistake, as the maximum discharge is obtained when it is reduced so that O *o r* R and S *s t* T shall join, as in diagram 3, Fig. 25. It is seen from No. 16 of the foregoing coefficients that $\frac{1\cdot569}{\cdot622} = 2\cdot52$ and $\frac{1\cdot569}{\cdot822} = 1\cdot91$ are, perhaps, nearer to the maximum results obtainable by comparing the discharge from a compound tube O *o* S T *r* R, diagram 3, Fig. 25, with those through an orifice in a thin plate, and through a short cylindrical tube. When the form of the tube becomes curvilineal throughout, as in Fig. 26, S T = 1·8 *o r* and *o* S = 9 *o r*, the coefficient suited to the diameter *o r* will be 1·57 nearly, and the discharge will be $\frac{1\cdot57}{\cdot622} = 2\cdot52$ times as much as through an orifice *o r* in a thin plate.

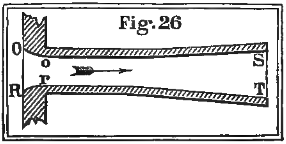


Fig. 26

The whole of the preceding coefficients have been determined from circumstances in which the coefficient for an orifice in a thin plate was ·622, and for a short cylindrical tube ·822 or ·823. When the circumstances of head and approaches in the reservoir are such as to increase or decrease those primary coefficients, the other coefficients for compound adjutages will have to be increased or decreased proportionately.

After examining the foregoing results, it appears sufficiently clear that the utmost effect produced by the formation of the compound mouth-piece O *o* S T *r* R, with the exception of No. 17, is simply a restoration of the loss effected by contraction in passing through the orifice O R in a thin plate, *and that the coefficient* 2·5 *applied to the contracted section at o r is simply equal to the theoretical discharge, or the coefficient unity, applied to the primary orifice* O R; for, as orifice O R : orifice *o r* : : 1 : ·64, very nearly, when O *o r* R takes the form of the *vena-contracta,* and the coefficient of discharge for an orifice *o r* in a thin plate is ·622, then the ratio of the theoretical discharge through the orifice O R, is to the actual discharge through an orifice *o r*, so is 1 to ·622 × ·64, so is 1 : ·39808 : : 1 : ·4 very nearly; and as ·4 × 2·5 = 1, it is clear that the form of the tube O *o* S T *r* R, when it produces the foregoing effect, simply restores the loss caused by contraction in the *vena-contracta.* Venturi's sixteenth experiment, from which the coefficients in No. 17 of the Table are derived, gives the coefficient 1·215 for the orifice O R. This indicates that a greater discharge than the theoretical through the receiving orifice may be obtained. It is, however, observable that Venturi, in his seventh proposition, does not rely on this result, and Eytelwein's experiments do not give a larger coefficient than 2·5 applied to the contracted orifice *o r*, which, as above shown, is equal to the theoretical discharge through O R.

SHOOTS.

When the sides and under edge of an orifice or notch increase in thickness, so as to be converted into a shoot or small channel, open at the top, the coefficients reduce very considerably, and to some extent beyond what the increased resistance from friction, particularly for small depths, appears to indicate. Poncelet and Lesbros* found for orifices 8″ × 8″, that the addition of a horizontal shoot 21 inches long reduced the coefficient from ·604 to ·601, with a head of about 4 feet; but for a head of 4½ inches the coefficient fell from ·572 to ·483. For notches 8″ wide, with the addition of a horizontal shoot 9′ 10″ long, the coefficient fell from ·582 to ·479 for a head of 8″; and from ·622 to ·340 for a head of 1″. Castel also found for a notch 8″ wide, with the addition of a shoot 8″ long, inclined 4° 18′, the mean coefficient for heads from 2″ to 4½″, to be ·527 nearly. The effects arising from friction alone will be perceived from the short table at the beginning of this section, p. 146.

The orifice of entry into a shoot and its position with reference to the sides and bottom modify the discharge, the head remaining constant. Lesbros † has given the coefficients suited to different positions of shoots both within and without a cistern, and from notches and submerged orifices; but, however valuable these are in some respects, they are of little practical use to the engineer. The general principles which are involved in the modification of these coefficients have, however, been already pointed out when discussing the effects of the position of the orifice, and the addition of short tubes, on the discharge. Equation (74B), p. 196, is here applicable.

---

* Traité d'Hydraulique, pp. 46 et 94.

† *Vide* Morin's Hydraulique, deuxième édition, pp. 29 et 40.

SECTION VII.

LATERAL CONTACT OF THE WATER AND TUBE.—ATMOSPHERIC PRESSURE.—HEAD MEASURED TO THE DISCHARGING ORIFICE.—COEFFICIENT OF RESISTANCE.—FORMULA FOR THE DISCHARGE FROM A SHORT TUBE.—DIAPHRAGMS.—OBLIQUE JUNCTIONS.—FORMULA FOR THE TIME OF THE SURFACE SINKING A GIVEN DEPTH.—LOCK CHAMBERS.—SLUICES.—TIDAL SLUICES.

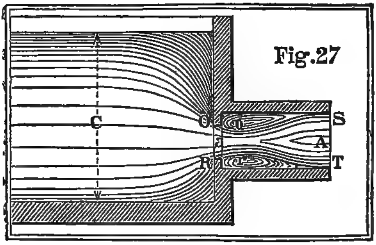


Fig.27

The contracted vein *o r* is about ·8 times the diameter O R; but it is found, notwithstanding, that water, in passing through a short tube of not less than 1½ diameter in length, fills the whole of the discharging orifice S T. This is partly effected by the outflowing column of water carrying forward and exhausting the air between it and the tube, and by the external air then pressing on the column so as to enlarge its diameter and fill the whole tube. When once the water approaches closely to the tube, or is caused to approach, it is attracted and adheres with some force to it. The water between the tube and the *vena-contracta* is, however, rather in a state of eddy than of forward motion, as appears from the experiments, with the tube, diagram 2, Fig. 25, giving the same discharge as the simple cylindrical tube. If the entrance be contracted by a diaphragm, as at O R, Fig. 27, the water will also generally fill the tube, if it be only sufficiently long. Short cylindrical tubes do not fill when the discharge takes place in an exhausted receiver; but even diverging tubes, D, Fig. 24, will be filled, under atmospheric pressure, when the angle of divergence, O, does not exceed 7 or 8 degrees, and the length be not very great nor very short.

When a tube is fitted to the bottom or side of a vessel, it is found that the discharge is that due to the head measured from the surface of the water to the lower or discharging extremity of the tube. It must, however, be sufficiently long, and not too long, to get filled throughout. Guiglielmini first referred this effect to atmospheric pressure, but the first simple explanation is that given by Dr. Mathew Young, in the Transactions of the Royal Irish Academy, vol. vii., p. 56. Venturi, also, in his fourth proposition, gives a demonstration.

The values of the coefficients for short cylindrical tubes, which are given p. 156, have been derived from experiments. Coefficients which agree pretty closely with them, and which are derived from the coefficients for the discharge through an orifice in a thin plate, may, however, be calculated as follows: Let C be the area of the approaching section, Fig. 27, A the area of the discharging short tube, and a the area of the orifice O R which admits the water from the vessel into the tube: also put, as before, h for the head measured from the surface of the water to the centre of the tube, and diaphragm O R; v for the actual velocity of discharge at ST; v_a for the velocity of approach in the section C towards the diaphragm O R; and c_c for the coefficient of contraction in passing from O R to $o\ r$; then $C \times v_a = A \times v$, the contracted section $o\ r = c_c \times a$, and consequently the velocity at the contracted section is equal to $\frac{A\,v}{a\,c_c} = \frac{C\,v_a}{a\,c_c}$. Now a theoretical head equal to

$$\frac{v^2 - v_a^2}{2\,g} = \frac{v^2\left(1 - \frac{A^2}{C^2}\right)}{2\,g}$$

is necessary to change the velocity v_a into v by the action of gravity; but as the water at the contracted section $o\ r$, moving with a velocity $\frac{A\,v}{a\,c_c}$, strikes against the water between it and T S, moving, from the nature of the case, with a slower velocity,* a certain loss of effect takes place from the impact. If this be, supposed, sudden, then writers on mechanics have shown *that a loss of head, equal to that due to the difference of the velocities,* $\frac{A\,v}{a\,c_c} - v$, *before and after the impact must take place.* This loss of head is therefore equal to

$$\frac{\left(\frac{A}{a\,c_c} - 1\right)^2 v^2}{2\,g},$$

---

* *Vide* Sir Robert Kane's translation of Rühlman's book on Horizontal Water Wheels, p. 49.

whence the whole head,

$$(60.)\qquad h = \frac{\left(1 - \frac{\text{A}^2}{\text{C}^2}\right)v^2 + \left(\frac{\text{A}}{a\,c_c} - 1\right)^2 v^2}{2\,g},$$

from which the velocity from a short tube, is found to be

$$(61.)\quad v = \sqrt{2gh}\left\{\frac{1}{1 - \frac{\text{A}^2}{\text{C}^2} + \left(\frac{\text{A}}{a\,c_c} - 1\right)^2}\right\}^{\frac{1}{2}}.$$

Now, as $\sqrt{2\,g\,h}$ would be the velocity of discharge were there no resistances, or loss sustained, it is evident that $\left\{\frac{1}{1 - \frac{\text{A}^2}{\text{C}^2} + \left(\frac{\text{A}}{a\,c_c} - 1\right)^2}\right\}^{\frac{1}{2}}$ becomes as it were a coefficient of velocity. When the diameter of the diaphragm O R becomes equal to the diameter S T of the tube, $\text{A} = a$, and as the coefficient of velocity becomes equal to the coefficient of discharge when there is no contraction, in such case this coefficient, which we call *c o f*, is expressed by the formula

$$(62.)\qquad c\,o\,f. = \left\{\frac{1}{1 - \frac{\text{A}^2}{\text{C}^2} + \left(\frac{1}{c_c} - 1\right)^2}\right\}^{\frac{1}{2}},*$$

---

* When the diaphragm is placed in a tube of uniform bore, then $\text{C} = \text{A}$, and

$$c\,o\,f. = \frac{1}{\frac{\text{A}}{a\,c_c} - 1} = \frac{c_c}{\frac{\text{A}}{a} - c_c},$$

and the loss of head, in passing the diaphragm, becomes

$$h = \left(\frac{\text{A}}{a\,c_c} - 1\right)^2 \times \frac{v^2}{2\,g}.$$

It is evident from the equations that $\frac{\text{A}}{a}$ and c_c depend mutually on and when the approaching section C is very large compared with the area A,

$$(63.)\quad c\,o\,f. = \left\{\frac{1}{1+\left(\frac{1}{c_c}-1\right)^2}\right\}^{\frac{1}{2}}.$$

If c_c = ·64, the last equation gives *c o f.* = ·872; if c_c = ·601, *c o f.* = ·833; if c_c = ·617, *c o f.* = ·847; and if c_c = ·621, *c o f.* = ·856. These results are in excess of those derived from experiment with cylindrical short tubes, perfectly square at the ends and of uniform bore. As some loss, however, takes place in the eddy between *o r* and the tube, and from the friction at the sides, not taken into account in the above calculation, they will account for the differences of not more than from 4 to 6 per cent. between the calculation and experiment. If c_c be assumed for calculation equal ·590, then *c o f.* = ·821; and as this result agrees very closely with the experimental one, c_c should be taken of this value in using the foregoing formulæ, from (60) to (63), for practical purposes. The thickness of the diaphragm itself and the relation of that thickness to the diameter, as well as the form of the orifice *a*, are necessary elements in the consideration of this question.

COEFFICIENT OF RESISTANCE. — LOSS OF MECHANICAL POWER IN THE PASSAGE OF WATER THROUGH THIN PLATES AND PRISMATIC TUBES.

The coefficients of contraction, velocity, and dis-

---

each other, and that they cannot be assumed arbitrarily. See equations (66), (67), (123), (124), and (125), with the corresponding remarks.

charge have been already defined. *The coefficient of resistance is the ratio of the head due to the resistance, to the theoretical head due to the actual or final velocity.* If v be this latter velocity, the theoretical head due to it is $\frac{v^2}{2g}$; and if c_r be the coefficient of resistance, then the head due to the resistance itself is, from our definition, $c_r \times \frac{v^2}{2g}$. Now if c_v be the coefficient of velocity, the theoretical velocity of discharge must be $\frac{v}{c_v}$, and the head due to it is equal $\frac{v^2}{c_v^2 \times 2g}$; but as the theoretical head due to v is $\frac{v^2}{2g}$, then

$$\frac{v^2}{c_v^2 \times 2g} - \frac{v^2}{2g} = \left(\frac{1}{c_v^2} - 1\right)\frac{v^2}{2g}$$

is the head due to the resistance; and, therefore, from the definition, the coefficient of resistance is

$$(64.) \qquad c_r = \frac{1}{c_v^2} - 1;$$

from which the coefficient of velocity is found

$$(65.) \qquad c_v = \left\{\frac{1}{c_r + 1}\right\}^{\frac{1}{2}}.$$

These equations enable us to calculate the coefficient of resistance from the coefficient of velocity, and *vice versâ.* If $c_v = 1$, $c_r = 0$, as it should be. The following short table, calculated from equation (65), will be of use. In short tubes, the coefficient of velocity c_v is equal to the coefficient of discharge c_d.* The coefficient of velocity for an orifice in a thin

---

* See the tables of resistances, discharge, and contraction, pp. 169 and 171.

COEFFICIENTS OF VELOCITY AND RESISTANCE.

Coefficient of velocity.	Coefficient of resistance.	Coefficient of velocity.	Coefficient of resistance.	Coefficient of velocity.	Coefficient of resistance.
·990	·020	·910	·208	·830	·452
·970	·063	·890	·263	·820	·488
·950	·109	·870	·320	·814	·508
·930	·156	·850	·383	·810	·525

plate, or for a mouth-piece, Fig. 4, is ·974; while that for a short prismatic tube, A, Fig. 24, is ·814 nearly. The coefficient of resistance in the former case is ·054, and in the latter ·508; there is, therefore, 9·4 times as great a loss of mechanical power in the passage through short prismatic tubes, as through orifices in thin plates or tubes with a rounded junction, as in Fig. 4, the quantities of water discharged and the discharging orifices being the same.

If the quantities discharged and the heads be the same in both cases, then

$$\frac{v_t^2}{2\,g \times \cdot 814^2} = \frac{v_o^2}{2\,g \times \cdot 974^2} \text{ is equal to the head;}$$

that is, $\frac{v_t^2}{\cdot 663 \times 2\,g} = \frac{v_o^2}{\cdot 949 \times 2\,g}$, or $\cdot 949\, v_t^2 = \cdot 663\, v_o^2$; whence we get $v_t^2 = \cdot 698\, v_o^2$ and $v_o^2 = 1 \cdot 431\, v_t^2$ for the relation of the discharging velocities, v_o, from an orifice, and, v_t, from a short tube. The height due to the resistance is therefore, $\left(\frac{1}{\cdot 814^2} - 1\right)\frac{v_t^2}{2\,g}$ for short prismatic tubes, and $\left(\frac{1}{\cdot 974^2} - 1\right)\frac{1 \cdot 431\, v_t^2}{2\,g}$ for orifices in thin plates. These are to each other as ·508 to ·054 × 1·431, or as 5·08 to ·773, that is to say, *the loss of mechanical power arising from the resistance in passing through short tubes is* 6·57 *times as great as when the water passes through thin plates or mouthpieces, as in* Fig. 4; and the discharging mechanical power in plates, is to that in tubes as 1·431 to 1, or as 1 : ·698, the heads and quantities discharged being the same.

The whole loss of mechanical power in the passage is 5·4 per cent. for the plates, and about 51 per cent. for short tubes. If the loss compared with the whole head be sought, then, when v is the discharging velocity, $\frac{v}{\cdot 814}$ is the theoretical velocity due to the head in short tubes, and its square $\frac{v^2}{\cdot 814^2} = \frac{v^2}{\cdot 663}$ is as the whole head; therefore, the whole head is to the head due to the discharging velocity as $\frac{v^2}{\cdot 663}$ to v^2, or as 1 to ·663; and as ·508 is the coefficient of resistance * for the dischanging velocity, $\cdot 508 \times \cdot 663 = \cdot 337$ is the coefficient of resistance due to the whole head; this is equal to a loss of 34 per cent. nearly, or about one-third. In like manner, $\cdot 974^2 \times \cdot 054 = \cdot 0512$ is found for the coefficient when the discharge takes place through thin plates, or $5\frac{1}{8}$ per cent. of the whole head.

DIAPHRAGMS.

When a diaphragm, O R, Fig. 27, is placed at the entrance of a short tube, it is shown, page 162, that a loss of head equal $\dfrac{\left(\dfrac{A}{a\,c_c}-1\right)^2 v^2}{2\,g}$ takes place when v is the discharging velocity, whence the coefficient of resistance is equal to $\left(\dfrac{A}{a\,c_c}-1\right)^2$,* according to the definition. The coefficient of contraction c_c, as before shown, page 164, should be taken equal to ·590 in the application of formula (63); and, as it must also be taken equal to about ·621 when the area of the tube A is very large compared with the area a of the orifice O R in the diaphragm, it may be assumed that when $\dfrac{a}{A}$ is equal to

---

* Table, p. 166.

0, ·1, ·2, ·3, ·4, ·5, ·6, ·7, ·8, ·9, and 1
successively, the coefficient c_c must be taken equal to
·621, ·618, ·615, ·612, ·609, ·606, ·603, ·600, ·597, ·593, and ·590,

taken in the same order. As the approaching section C may be considered exceedingly large, the value of the coefficient of discharge or velocity, as the tube O R S T is supposed full, in equation (61), becomes

$$(66.)\qquad c_d = \left\{\frac{1}{1+\left(\dfrac{A}{a\,c_c}-1\right)^2}\right\}^{\frac{1}{2}},$$

and the coefficient of resistance

$$(67.)\qquad c_r = \left(\frac{A}{a\,c_c}-1\right)^2;$$

---

* For the sudden alteration in the velocity passing through a diaphragm, we must reject the hypothesis of D'Aubuisson, Traité d'Hydraulique, p. 238, and adopt that of Navier, taking the loss of head to correspond to the square of the difference and not to the difference of the squares of the velocities *in* and *after* passing the orifice. The coefficient of contraction must, however, be varied to suit the ratio of the channels, as it is in this and the following pages.

from which equations and the above values of c_c, corresponding to $\frac{a}{A}$, the following values of the coefficients of discharge and resistance through the tube O R S T, Fig. 27 have been calculated.

COEFFICIENTS OF CONTRACTION, DISCHARGE, AND RESISTANCE FOR DIAPHRAGMS.

Ratio $\frac{a}{A}$	Coefficient c_c.	Coefficient c_d. for the orifice A.	Coefficient c_r.	Ratio $\frac{a}{A}$.	Coefficient c_c.	Coefficient c_d. for the orifice A.	Coefficient c_r.
0·0	·621	·000	infinite.	0·6	·603	·493	3·115
0·1	·618	·066	231·	0·7	·600	·587	1·907
0·2	·615	·139	50·8	0·8	·597	·675	1·198
0·3	·612	·219	19·8	0·9	·593	·753	·762
0·4	·609	·307	9·6	1·0	·590	·821	·483
0·5	·606	·399	5·3	...	...	...	...

In this table c_c is the coefficient of contraction, c_d the coefficient of discharge, suited to the larger section of the pipe A, at S T; and c_r the coefficient of resistance. The discharge is found from equation (61), as C is here very large compared with A, to be

$$\text{(67A.)}\quad D = A\sqrt{2gh}\left\{\frac{1}{1+\left(\frac{A}{a\,c_c}-1\right)^2}\right\}^{\frac{1}{2}}$$

$$= A\sqrt{2gh}\left\{\frac{1}{1+c_r}\right\}^{\frac{1}{2}} = c_d\,A\sqrt{2gh}.$$

The coefficient of resistance c_r is here equal $\left(\frac{A}{a\,c_c}-1\right)^2$, and the coefficient of discharge $c_d = \frac{1}{(1+c_r)^{\frac{1}{2}}}$. *

---

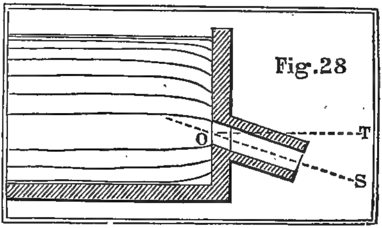
* For the loss sustained by contraction in the bore of a pipe by a diaphragm, see equations (123), (124), and (125). The actual value of

The tube must be so placed, that the water, after passing the diaphragm, shall fill it; for instance, between two cisterns, when the height h must be measured between the water surfaces, or when the tube is sufficiently long to be filled; in this case, however, *the height must be determined from the discharging velocity*, as a portion of the head is required to overcome the friction, which shall be referred to more particularly in the next section.

The table shows that the head due to the resistance is 5·3 times that due to the discharging velocity, when the area of the diaphragm is half the area of the tube; that is, the whole head required is 6·3 times that due to the velocity, and that the coefficient of discharge is reduced to ·399. In order to find the coefficients suited to the smaller area of the orifice in the diaphragm O R, when it is to be used in calculations of the discharge, divide the numbers corresponding to $\frac{a}{\text{A}}$ into those of c_d, opposite to them in the table. Thus, when $\frac{a}{\text{A}} = {\cdot}8$, then the coefficient of discharge suited to the area a, is equal $\frac{\cdot 675}{\cdot 8} = {\cdot}844$, and so of other values of the ratio $\frac{a}{\text{A}}$. The coefficients in the table, page 169, are for the larger orifice A in the formula $\text{D} = \text{A}\, c_d \sqrt{2\, g\, h}$.

c_c in equation (67A) depends on the thickness of the diaphragm as well as on the relation of a and A. The form of the orifice a also affects the value of c_c.

SHORT TUBES OBLIQUE AT THE JUNCTION.



When a tube is attached obliquely, as in Fig. 28, the author has found that if the number of degrees in the angle T O S, formed by the direction of the tube O S, with the perpendicular O T, be represented by ϕ, then $\cdot 814 - \cdot 0016\,\phi$ will give the coefficient of discharge corresponding to the obliquely attached short tube in the Figure. This formula is, however, empirical, but it is simple, and agrees pretty closely with experimental results. As the coefficient of resistance is equal $\frac{1}{c_v^2} - 1$, equation (64), then here $c_r = \frac{1}{(\cdot 814 - \cdot 0016\,\phi)^2} - 1$; from these equations the following table for heads measured to the middle of the outside orifice has been calculated:—

COEFFICIENTS OF DISCHARGE AND RESISTANCE FOR OBLIQUE JUNCTONS.

ϕ in degrees.	Coefficient of discharge.	Coefficient of resistance.	ϕ in degrees.	Coefficient of discharge.	Coefficient of resistance.
0°	·814	·508	35°	·758	·740
5	·806	·539	40	·750	·778
10	·798	·569	45	·742	·816
15	·790	·602	50	·734	·856
20	·782	·635	55	·726	·897
25	·774	·669	60	·718	·940
30	·766	·704	65	·710	·984

The coefficient of resistance for a tube at right angles to the side, is to the like coefficient when it makes an angle of 45 degrees as ·508 to ·816, or as 1 to 1·6 nearly; and the loss of head is greater in the same proportion. If the short tube be more than three or four diameters in length, friction will have to be taken into account. The head *h* is measured to the centre of the outside orifice.

FORMULA FOR FINDING THE TIME THE SURFACE OF WATER IN A CISTERN TAKES TO SINK A GIVEN DEPTH.—DISCHARGE FROM ONE VESSEL OR CHAMBER INTO ANOTHER.—LOCK CHAMBERS.

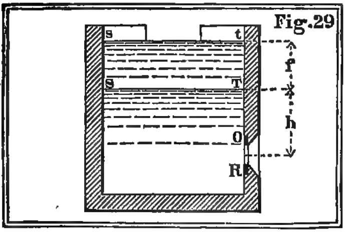


Fig.29

In experiments for finding the value of the coefficients of discharge, one of the best methods is to observe the time the water discharged from the orifice takes to sink the surface in a prismatic cistern a given depth; the ratio of the observed to the theoretical time will then give the coefficient sought. A formula for finding the time is, therefore, of much practical value. In Fig. 29, the theoretical time of falling from *s t* to S T, in seconds, is

$$(68.)\quad t=\frac{A}{4\cdot0125\,a}\{(h+f)^{\frac{1}{2}}-h^{\frac{1}{2}}\}=\cdot249\,\frac{A}{a}\{(h+f)^{\frac{1}{2}}-h^{\frac{1}{2}}\}$$

in which *a* is the area of the orifice O R, and A the area of the prismatic vessel at *s t* or S T: this formula is for measures in feet. For measures in inches, we have

$$(69).\qquad t=\frac{A}{13\cdot9\,a}\{h+f)^{\frac{1}{2}}-h^{\frac{1}{2}}\}.$$

This time is double the time required to discharge the same quantity if the head at the orifice remained constant.

EXAMPLE VII.—*A cylindrical vessel* 5·74 *inches in diameter has an orifice* ·2 *inch in diameter at a depth of* 16 *inches below the surface, measured to the centre; it is found that the water sinks* 4 *inches in* 51 *seconds; what is the coefficient of discharge?*

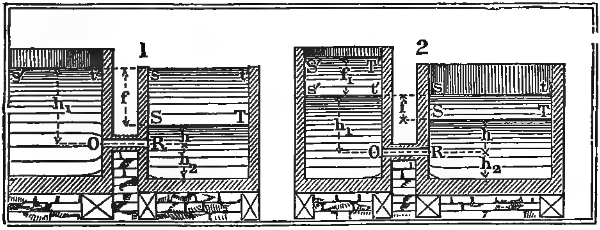
The theoretical time t is found from equation (69), equal

$$\frac{5\cdot74 \times \cdot7854}{13\cdot9 \times \cdot2^2 \times \cdot7854}\{16^{\frac{1}{2}} - 12^{\frac{1}{2}}\} = \frac{32\cdot9476}{\cdot556}\{4 - 3\cdot4641\}$$

$= \frac{17\cdot6566}{\cdot556} \times \cdot5359 = 31\cdot8$ seconds; hence, $\frac{31\cdot8}{51} = \cdot624$ is the coefficient sought. When the orifice O R and the horizontal section of the vessel are similar figures, $\frac{A}{a}$ is equal $\frac{S\ T^2}{O\ R^2}$; and therefore, for circular cisterns and orifices, it is unnecessary to introduce the multiplier ·7854.

Formulæ for the time water in a prismatic vessel takes to fall a given depth, when discharged from an orifice at the side or bottom are given above. The time the surface S T, diagram 1, Fig. 29*a*, takes to rise

Fig. 29*a*.



to *s t*, when supplied through an orifice or tube O R, from an upper large chamber or canal, whose surface *s′ t′* remains always at the same level, is $\frac{2\,\text{A} f^{\frac{1}{2}}}{c_d\, a \sqrt{2g}}$,* and thence the time of rising from R to S for measures in feet is

$$(69\text{A.})\qquad t = \frac{\text{A}}{4{\cdot}0125\, c_d a}\{h_1^{\frac{1}{2}} - f^{\frac{1}{2}}\}$$

and for measures in inches

$$(69\text{B.})\qquad t = \frac{\text{A}}{13{\cdot}9 c_d a}\{h_1^{\frac{1}{2}} - f^{\frac{1}{2}}\},$$

in which A is the area of the horizontal section at S T; a the sectional area of the communicating channel or orifice O R; c_d the coefficient of discharge suited to it, and h_1 and f, as shown in the diagram.

In order to find the time of filling the lower vessel to the level S T, supposing it at first empty, the contents of the portion below O R are equal to $\text{A}h_2$, and the time of filling it equal to

$$(69\text{C.})\qquad \frac{\text{A}h_2}{8{\cdot}025\, c_d\, a h_1^{\frac{1}{2}}}$$

then the time of filling up to any level S T, for measures in feet, is equal to the sum of (69A) and (69C); that is,

$$(69\text{D.})\qquad \text{T} = \frac{\text{A}h_1^{\frac{1}{2}}}{8{\cdot}025\, c_d a}\left\{2 + \frac{h_2}{h_1} - \frac{2 f^{\frac{1}{2}}}{h_1^{\frac{1}{2}}}\right\}$$

$$= \frac{\text{A}\,(2h_1 + h_2 - 2 f^{\frac{1}{2}} h_1^{\frac{1}{2}})}{8{\cdot}025\, c_d\, a h_1^{\frac{1}{2}}},$$

and for measures in inches

---

* The time of rising from S to *s* (diagram 1) is exactly double the time it would take to fill the same depth *below* R, if the pressure *f* remained uniform.

(69E.) $$\mathrm{T} = \frac{\mathrm{A} h_1^{\frac{1}{2}}}{27{\cdot}8\, c_d a}\left\{2 + \frac{h_2}{h_1} - \frac{2 f^{\frac{1}{2}}}{h_1^{\frac{1}{2}}}\right\} = \frac{\mathrm{A}\,(2h + h_2 - 2 f^{\frac{1}{2}} h_1^{\frac{1}{2}})}{27{\cdot}8\, c_d a h_1^{\frac{1}{2}}}.$$

When S T coincides with *s t*

(69F.) $$\mathrm{T} = \frac{\mathrm{A}\,(2h_1 + h_2)}{8{\cdot}025 c_d a h_1^{\frac{1}{2}}},$$

for measures in feet, and

(69G.) $$\mathrm{T} = \frac{\mathrm{A}\,(2h_1 + h_2)}{27{\cdot}8\, c_d a h_1^{\frac{1}{2}}},$$

for measures in inches. These equations are exactly suited to the case of a closed lock-chamber filled from an adjacent canal.

When the upper level S′ T′ is also variable, as in Diagram 2, the time which the water in both vessels takes to come to the same uniform level *s′ t′ s t*, which is known or easily found, is

(69H.) $$t = \frac{2\mathrm{A}\mathrm{A}_1(h_1 + f_1 - h)^{\frac{1}{2}}}{c_d a\,(\mathrm{A} + \mathrm{A}_1)\sqrt{2g}} = \frac{2\mathrm{A}\mathrm{A}_1(f + f_1)^{\frac{1}{2}}}{c_d a(\mathrm{A} + \mathrm{A}_1)\sqrt{2g}};$$

in which $h_1 + f_1 - h = f + f_1$ is the difference of levels at the beginning of the flow; A_1 the horizontal section of the upper chamber; and the other quantities as in Diagram 1. As $\mathrm{A}_1 . f_1 = \mathrm{A}_1 . f$, then

$$f + f_1 = \frac{\mathrm{A}_1 + \mathrm{A}}{\mathrm{A}} f_1 = \frac{\mathrm{A}_1 + \mathrm{A}}{\mathrm{A}_1} f.$$

Now, in order to find the time of falling a given depth f_x below the first level S′ T′, the head above *s′ t′ s t* is equal to $f_1 - f_x$ in the upper vessel, and the depth below it in the lower vessel is equal to $\frac{\mathrm{A}_1\,(f_1 - f_x)}{\mathrm{A}}$; whence the difference of levels in the two vessels at the end of the fall *d*, is

$$f_1 - d + \frac{A_1(f_1 - f_x)}{A} = \frac{A + A_1}{A}(f_1 - f_x).$$

The time of falling through any given depth, f_x is, therefore, from equation (69H),

$$(69I.)\quad t = \frac{2AA_1(f+f_1)^{\frac{1}{2}}}{c_d a(A+A_1)\sqrt{2g}} - \frac{2AA_1\left\{\left(\frac{A_1+A}{A}\right)(f_1 - f_x)\right\}^{\frac{1}{2}}}{c_d a(A+A_1)\sqrt{2g}}$$

$$= \frac{2AA_1}{c_d a(A+A_1)\sqrt{2g}}\left\{(f+f_1)^{\frac{1}{2}} - \left(\frac{(A+A_1)(f_1-f_x)}{A}\right)^{\frac{1}{2}}\right\},$$

When $f_x = f_1$ this is reduced to $t = \dfrac{2AA_1(f+f_1)^{\frac{1}{2}}}{c_d a(A+A_1)\sqrt{2g}}$, and farther when $A = A_1$ this last is again farther reduced to $t = \dfrac{A_1 f_1^{\frac{1}{2}}}{c_d a\sqrt{g}} = \dfrac{1{\cdot}4142\, A_1 f_1}{c_d a\sqrt{2gf_1}}$. in which $\sqrt{2g} = 8{\cdot}025$ for measures in feet, and equal $27{\cdot}8$ for measures in inches. The whole time of filling to a level the lower *empty vessel*, is found by adding the time of filling the portion below R, determined in a manner similar to equations (68), to be

$$(69K.)\quad \frac{2A_1}{c_d a\sqrt{2g}}\left\{(h_1+f_1)^{\frac{1}{2}} - \left(h_1 + f_1 - \frac{h_2 A}{A_1}\right)^{\frac{1}{2}}\right\},$$

to the time of filling above R, given in equation (69H), when h is taken equal to zero. Equations (69H), (69I), and (69K) are applicable to the case of the upper and lower chambers of a double lock, after making the necessary change in the diagrams.

The above equations require further extensions when water flows into the upper vessel while also flowing from it into the lower; such extensions are, however, of little practical value, and we therefore omit them. For sluices in flood-gates with square arrises, c_d may be taken at about ·545, but with rounded arrises, the coefficient will rise much higher. See SECTIONS III. and VI.

SLUICE OPES.—FLOOD AND TIDAL SLUICE OPES.

Equations (41), (42), (43), (48), (49), (50), and (51) give the discharge from sluice opes under different circumstances when fully open, submerged, or partly submerged. Rejecting the velocity of approach, and measuring the depth, h to the centre, or centre of gravity, of the orifice, whose area is a, equation (41), becomes, for the case in Fig. 12, with any form of section, rectangular or circular, entirely open and without back water.

$$(41\text{A.})\begin{cases} \text{D} = c_d\, a \sqrt{2gh} = 8{\cdot}025\, c_d\, a \sqrt{h}, \text{ for one second; or} \\ \text{D} = 481{\cdot}5\, c_d\, a \sqrt{h}, \text{ for one minute; or} \\ \text{D} = 4{\cdot}95\, a \sqrt{h} \text{ when } c_d = {\cdot}617 \text{ in one second; and} \\ \text{D} = 297\, a \sqrt{h} \text{ in one minute.} \end{cases}$$

All in feet measures.

In the case, Fig. 20, in which the sluice is covered, or entirely submerged, then the discharge in one minute, in feet measures, equation (48) becomes also

$$(47\text{A.})\qquad \text{D} = 60\, c_d\, a \sqrt{2\,g\,h} = 297\, a \sqrt{h};$$

in which, however, h is now the difference of level between the surfaces of the upper and lower waters. If c_d be taken ·582 instead of ·617 then

$$(47\text{B.})\qquad \text{D} = 280\, a \sqrt{h}.$$

But if the coefficient run up to ·712 then

$$(47\text{C.})\qquad \text{D} = 343\, a \sqrt{h}.$$

When the sluice-ope is partly submerged, as in

Fig. 21, putting a_1 for the area of the open portion and a_2 for that portion submerged, then for time in minutes, equation (50) or (51) becomes

$$(50 \,\&\, 51\text{A.}) \quad \text{D}=60\left(c_d\, a_2 \sqrt{2\,g\,h} + c_d\, a_1 \sqrt{2\,g\,\frac{h+h_t}{2}}\right)$$

$$= 297\left(a \quad \sqrt{h} + a_1 \sqrt{\frac{h\ +\ h_t,}{2}}\right)$$

for $c_d = {\cdot}617$ as a mean value in both a_2 and a_1. It however generally differs in both. For a coefficient of ·582 it becomes

$$(50 \,\&\, 51\text{B.}) \quad \text{D} = 280\left(a_2 \sqrt{h} + a_1 \sqrt{\frac{h\ +\ h_t}{2}}\right).$$

And for an average coefficient of ·712 in A_1 and A_2 it is

$$(50 \,\&\, 51\text{C.}) \quad \text{D} = 343\left(a_2 \sqrt{h} + a_1 \sqrt{\frac{h\ +\ h_t}{2}}\right).$$

In these formulæ the pressure at the sluice remains unchanged and the acting heads constant.

WHEN THE HEADS VARY. The general differential equation for the discharge, whatever be the law of the rise and fall, f is evidently

$$(70.)\ d\,\text{D}=c_d\,a\sqrt{2\,g} \times d\,t\sqrt{f}=c_d\,a\sqrt{2\,g}\times d\,t\sqrt{h_1-h}.$$

The integration of this equation depends on the relation between f and t. For the ordinary cases of filling prismatic ponds from upper levels the preceding equations from (68) to (69K) may be used as follows to find D. If the upper surface remain at the same level as in 1, Fig. 29a; while the surface of the water below rises from the discharge through the sluice, and if the lower pond be a prism, whose horizontal section is A, then dividing h A, the quantity by the time t as given in equation (69A), the average discharge in average seconds of the whole time t is

(70A.) $$\mathrm{D} = \frac{4{\cdot}0125\, c_d\, a\, h}{h_1^{\frac{1}{2}} - f^{\frac{1}{2}}}.$$ And for the time t;

$$\mathrm{D} = \frac{4{\cdot}0125\, c_d\, h\, t}{h_1^{\frac{1}{2}} - f_1^{\frac{1}{2}}} = \mathrm{A}\, h.$$

which are independent of the area A of the lower vessel. When $h = h_1$ then $f = o$ and this becomes

(70B.) $$\mathrm{D} = 4{\cdot}0125\, c_d\, a\, h_1^{\frac{1}{2}}\, t = \mathrm{A}\, h_1$$

which is exactly *one-half what it would be in the same time if the head h_1 remained constant, $h = 0$, and therefore the lower level not rising* higher than O R.

If the lower level remain unchanged and the upper varies, then, calling the section of the upper prism A_1, the difference of level h_1, and f the fall in the time t, the average discharge in one second is found in a similar manner to be

(70C.) $$\mathrm{D} = \frac{4{\cdot}0125\, c_d\, a\, f}{h_1^{\frac{1}{2}} - (h_1 - f)^{\frac{1}{2}}}.$$ And for the time t;

$$\mathrm{D} = \frac{4{\cdot}0125\, c_d\, a\, f\, t}{h_1^{\frac{1}{2}} - (h_1 - f)^{\frac{1}{2}}} = f\, \mathrm{A}_1,$$

when $f = h_1$ this is reduced to

(70D.) $$\mathrm{D} = 4{\cdot}0125\, c_d\, a\, h_1^{\frac{1}{2}}\, t = h_1\, \mathrm{A}_1$$

the whole discharge in the time of falling through h_1. In fact the values (70B) and (70D) for the relations between the time, discharge, and head are the same, when either level, above or below, is fixed and the water rises or falls in the prismatic pond. When the pond or cistern is of any irregular shape and section the contents can be divided by horizontal sections into any suitable number of parts of equal height, when the whole time of filling or emptying will be the sum of the times calculated for each horizontal lamina, and the greater the number of lamina the more correct the result.

When the levels of both ponds vary by the water passing from one into the other, diagram 2, Fig. 29a. Measuring f and f_1 from the common level and calling f_x the height fallen in any time t, then from equation (69I) by dividing into f_x A and multiplying by t

$$(70\text{E.})\ \text{D} = \frac{c_d\, a\, \text{A}^{\frac{1}{2}} (\text{A} + \text{A}_1) \sqrt{2\, g\, f_x}\; t}{2\text{A}\left\{ \{\text{A}(f + f_1)\}^{\frac{1}{2}} - \{(\text{A} + \text{A}_1)(f_1 - f_x)\}^{\frac{1}{2}} \right\}} = f_x \text{A}_1.$$

When $f_x = f_1$ and the water is one level in both ponds this becomes

$$(70\text{F.}) \qquad \text{D} = \frac{c_d\, a\, (\text{A} + \text{A}_1) \sqrt{2\, g\, f_1}\; t}{2\, \text{A}\, (f + f_1)^{\frac{1}{2}}} = f_1\, \text{A}_1.$$

And when A is infinitely large, h and $f = 0$ this is farther reduced to

$$(70\text{G.}) \qquad \text{D} = \frac{c_d\, a \sqrt{2\, g\, f_1} \times t}{2} = f_1\, \text{A}_1$$

as it should be. In each of the last three equations the factor $\frac{\sqrt{2\,g}}{2} = 4{\cdot}0125$ for feet measures. It may be said of these formulæ that the product f_x A_1, or f_1 A_1, gives the quantity at once, but in many problems f_x, f_1 and t have to be found from each other.

Assuming the form of the ordinary formula

$$\text{D} = c_d\, a \sqrt{2\, g f_1} = 8{\cdot}025\, c_d\, a \sqrt{f_1}$$

in one second, or $481{\cdot}5\ c_d\ a \sqrt{f_1}$ in one minute for a steady head f_1. Then for a variable head as in Fig. 29, and 1 Fig. 29a, the time of discharging a given quantity is doubled: or, which is the same thing, the coefficient c_d becomes now $\cdot 5\ c$ in the first form.

In the case of two equal ponds equation (70F) would become

$$\text{D} = \cdot 707\, c_d\, a \sqrt{2\, g\, f_1} \times t,$$

and the time 1·4142 times that required to discharge a quantity equal to f_1 A with an invariable head f_1. But as the head at the beginning was $f_1 + f = 2\,f_1$ if this latter head were used, the quantity being the same,

$$\text{D} = \frac{\cdot 707\, c_d\, a \sqrt{4\, g\, f_1} \times t}{4} = \cdot 177\, c_d\, a \sqrt{2\, g \cdot 2\, f_1} \times t,$$

or a little over one-sixth of what it would be if the head at the beginning $2\,f_1$, remained invariable for the same time t.

FLOOD AND TIDAL SLUICES.

The opes for these are intended for the drainage of low embanked lands. They are fitted with gates, or doors, generally hung on their upper side and self-acting so as to shut when the outside water rises over the inside level, and to open when it sinks to or below it, so as to pass off the inside water. The effective head is reduced during the time the water takes to

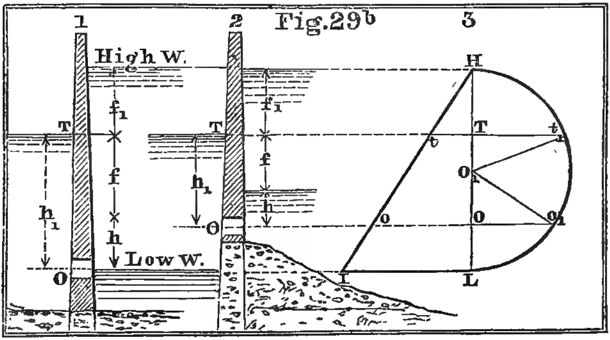


Fig. 29b

rise, between the bottom of the aperture—or low water if over the aperture,—and the inside level of the water on the lands. During the time of fall a like reduction takes place; and, therefore, the discharging power of the sluice is considerably reduced. The relations between the times and the heads risen or fallen through being known, the integration of equation (70) can be effected directly or by approximation, h being any function of t. When the rise or fall of the flood is everywhere proportionate to the time, as in H L I, diagram 3, Fig. 29b; where H L represents the fall and H I the time; then in diagrams 1 and 2, if t be the time of rising through $h + f = h_1$, the time of rising through h is $\frac{h\,t}{h_1}$ and integrating equation 70 accordingly,

$$(70\text{A.}) \qquad \text{D} = \frac{2\,t\,h}{3\,h_1} c_d\, a \sqrt{2\,g\,h};$$

for the discharge in the time of rising through h, which when $h_1 = h$ becomes

$$(70\text{B.}) \quad \text{D} = \tfrac{2}{3}\, c_d\, t\, a \sqrt{2\,g\,h_1} = \cdot 667\, c_d\, t\, a \sqrt{2\,g\,h_1}.$$

The coefficient c_d is therefore reduced one-third when $h_1 = f$, when $h_1 = 2\,f$ or at half flood, and indeed throughout h_1, for either diagram 1 or 2, the coefficient $\frac{2\,c_d}{3}$ remains constant. The gate or door at O is supposed shut in the time of rising and falling through f_1 and fully open in the time of rise or fall through $h + f = h_1$. When not fully open the discharge becomes still further reduced.

TIDES.—If the whole fall H L be that of a tide from high to low water, then on the assumption that when

H $t_1\,o_1$ L is the arc of a semicircle it represents time, T, of rising or falling through H L, the time of rising through O L is represented by the arc L o_1, and the time of rising through T O by the arc $o_1\,t_1$ the direct integration of equation (70), which then becomes by reduction

$$(70\text{C.})\ d\text{D} = c_d\,a\,\sqrt{2\,g} \times d\,t_h \sqrt{h_1 - \frac{(h_1+f_1)}{2}\left(1 - \cos.\ \frac{180°\,t_h}{\text{T}}\right)}$$

in which t_h is the time of rising through h; and $\frac{h_1 + f_1}{2}$, the semirange of the tide, gives the discharge. Putting the angle L $\text{O}_1\,o_1 = \theta$ this may be changed into

$$(70\text{D.})\ d\,\text{D} = c_d a \sqrt{2\,g} \times \frac{\text{T}}{180°} \times d\,\theta \sqrt{h_1 - \frac{h_1+f_1}{2}(1 - \cos\theta)}.$$

Or as it can be otherwise expressed, putting $2\sin.^2\frac{\theta}{2}$ for $1 - \cos\theta$

$$(70\text{E.})\, d\,\text{D} = c_d a \sqrt{2\,g} \times \frac{2\,\text{T}}{\pi} \times d\left(\frac{\theta}{2}\right) \sqrt{h_1 - (h_1+f_1)\sin^2\frac{\theta}{2}}$$

The integration of any of these forms can only be effected approximately. With tides from 20 feet to 6 feet, inside heights, or values of h_1, from 14 feet to 1 foot, and times of rising through those heights from 235 to 53 minutes, Mr. Cotton calculated the coefficients from (70E) and found them to vary from ·748 to ·785. If applied to the common form these give

$$(70\text{F.}) \qquad \text{D} = (\cdot748 \text{ to } \cdot785)\ c_d\ t\ a \sqrt{2\,g\,h_1},$$

for the discharge within the limits of the calculations. These "TIDAL COEFFICIENTS," as I shall call them, are too high for most cases occurring in practice, and require the sluice to be placed at or below low water of a

6h. 13m. tide, as in diagram 1, Fig. 29b. The fall and rise of the tide at the end of the ebb and beginning of the flow would, for several minutes, be practically nothing, and the coefficient would then be unity, which is the limit for a very small value of h at low water in diagram 1. At semirange for a small rise h, and an orifice placed there, the rise would be as the times and the coefficient would be ·667; both giving ·833 for an arithmetical mean, which is evidently too high, as the coefficient unity holds only for a comparatively small height. If, however, in diagram 2, Fig. 29b, the sluice or mouth of the culvert be high up, and near below the semirange of the tidal wave, which is the more common case in practice, then the coefficient would reduce to $\frac{2}{3} = ·667$ for its limit. All this supposes the surface of the inside water or reservoir at T in both diagrams to remain constant, and as it should reduce something in the outflow until the tide rises for some height up h_1 there is still a greater reason for selecting $\frac{2}{3}$ rds. or the minimum coefficient of the range, and to represent the discharge from a tidal sluice fully open by

(70G.) $D = \frac{2}{3}\, c_d\, a \sqrt{2g\, h_1} \times t = 5·35\, c_d\, a \sqrt{h_1} \times t,$

for feet measures and time in seconds; or

(70H.) $D = 321\, c_d\, a \sqrt{h_1} \times t$

for measures in feet and time in minutes.

The value of c_d is best taken from the table calculated from equation (74B), SECTION VIII. If the length of the culvert* or pipe under an embankment at the mouth of which the sluice is placed be 20 diameters or 80 mean radii $c_d = \cdot 731$ then (70G) becomes

---

* The orifice O is, in practice, generally a pipe or culvert of some length built under an embankment.

(70I.) $$\text{D} = 234\cdot 6\, a \sqrt{h_1} \times t.$$

If the length of the culvert be 40 diameters or 160 mean radii then $c_d = \cdot 668$ and the discharge would be

(70K.) $$\text{D} = 214\, a \sqrt{h_1} \times t.$$

And for a coefficient of ·623

(70L.) $$\text{D} = 200\, a \sqrt{h_1} \times t.$$

The time of rise or fall of the tide may be taken at 6h. 13m. in diagram 1, and the time of rising through h_1 be represented by the arc L o_1 t_1, diagram 3, but in diagram 2 the time corresponding to h_1 is the arc o_1 t_1. For a uniform rise these times would be represented by I t, and I o.

Sluice-doors when self-acting should open fully so as to be free above the top of the ope; and not to fall below it until the rising water is at the level of the surface of the inside reservoir; when it should shut if well constructed. They hang in the greater number of executed works at an angle θ, with the vertical which varies with the force of the outflow. The aperture, a, in this case is no longer the cross section of the culvert, but the orifice now may be supposed as made up of a plane the width of the culvert, at right angles to the door, equal to a sin. θ, having two vertical triangular *open cheeks* of the height of the culvert one on each side, between the vertical plane on the sloping door and the top. These triangular cheeks vary in area from zero to their maximum value, which is when the door hangs at an angle of 45°. If the door be set back in the culvert these cheeks are stopped and the outlet becomes a sin. θ. Further formulæ for such contractions would be mere waste, practically considered; and they are therefore not given. A good sluice gate, with its mountings, from the axis of suspension downwards, should have the same specific gravity as the outside water, should act in a cistern so as to be entirely immersed at all times, and the centre of pressure a little below the centre of gravity. But this is no place for questions of construction, or the application of hydrodynamical principles to them. If the object were to calculate, at first, the sectional area of a culvert required for a given discharge it should not be made less, in practice, than double those easily derived from the above formulæ, which would vary with the ratio of the lengths to the hydraulic mean depth of the culvert.

In these sluices, flood or tidal, the time of rising and falling through f_1 is lost in each flood, or in each tide; but as sea water is more dense than fresh water the time lost is a little more. There is also a back leakage through the sluice when shut. When the sluice can be placed at or below low water springs there is an advantage if not overbalanced by the expense; but in general it is sufficient to place it at or below the low water in the tail-race which, itself, must have a surface fall to the low water of neap and spring tides along the shore, if it be of any length. Otherwise, unless artificially constructed and covered over, it would fill in. The range of the tide varies considerably even in the same place from the lowest neaps to the highest springs. The mid-tide is nearly constant and the velocity of ebb and flow indicated by change of level is then a maximum for each tide. For 30 degrees on each side comprising the time of falling through the central half of the whole range the times, diagram 3, are nearly as the changes of level. In the remaining half range, comprising one quarter above and the other quarter below, the relation is more complex, and varies with time, wind, and weather. In Dundalk I have known two high waters within a few hours of each other, the first ebb having commenced and continued for some time until it was stopped by a return flow. Hence, in order to estimate approximately the discharge from a tidal sluice, we must calculate the discharge for each tide and each day, suitable to diagram 1 or diagram 2; noting that as the range varies from springs to neaps so must the head, h_1, when the surface at T of the backwater remains constant. It is necessary to keep this surface at all times from twelve to eighteen inches at least below the adjacent lands, and more if the element of expense permits. This level regulates the depth and size of the sluice or sluices.

Self-acting sluices can be hung on vertical as well as horizontal axes. When at the surface, for weirs across rivers, the centre of pressure for crest sluices is at two-thirds of the depth below the surface. As the water falls below the top so does this centre; but it can never rise higher than half the depth, the position when most deeply immersed. The position of the horizontal axis of such sluices lies therefore below the middle, and is regulated by the circumstances of each case, which are referred to farther on.

SECTION VIII.

FLOW OF WATER IN UNIFORM CHANNELS.—MEAN VELOCITY.—MEAN RADII AND HYDRAULIC MEAN DEPTHS. —BORDER.—TRAIN.—HYDRAULIC INCLINATION.— EFFECTS OF FRICTION.—FORMULÆ FOR CALCULATING THE MEAN VELOCITY.—APPLICATION OF THE FORMULÆ AND TABLES TO THE SOLUTIONS OF THREE USEFUL PROBLEMS.

In rivers the velocity is a maximum along the central line of the surface, or, more correctly, over the deepest part of the channel; and it decreases thence to the sides and bottom: but when backwater arises from any obstruction, either a submerged weir, Fig. 22, or a contracted channel, Fig. 23, the velocity in the channel approaching the obstruction is a maximum at the depth of the backwater, below the surface, and it decreases thence to the surface, sides, and bottom. When water flows in a pipe of any length, the velocity at the centre is greatest, and it decreases thence to the sides or circumference of the pipe. If the pipe be supposed divided into two portions in the direction of its length, the lower portion or channel will be analogous to a small river or stream, in which the velocity is greatest at the central line of the surface, and the upper portion will be simply the lower reversed. A pipe flowing full may, therefore, be looked upon as a double stream, and it will soon appear that the formulæ for the discharge from each kind are all but identical, though a pipe may discharge full at all inclinations, while the inclinations in rivers or streams, having uniform motion, never exceed a few feet per mile.

MEAN VELOCITY.

It is found, by experiment, that the mean velocity is nearly independent of the depth or width of the channel, the central or maximum velocity being the same. From a number of experiments, Du Buât derived empirical formulæ equivalent to

$$v = \frac{\mathrm{v}_b + \mathrm{v}}{2} = \mathrm{v} - \mathrm{v}^{\frac{1}{2}} + \frac{1}{2},\ \mathrm{v}_b = (\mathrm{v}^{\frac{1}{2}} - 1)^2, \text{ and } \mathrm{v} = (\mathrm{v}^{\frac{1}{2}} + 1)^2;$$

in these equations v is the mean velocity, v the maximum surface velocity, and v_b the velocity at the sides, or bottom, expressed in French inches. Tables calculated from these formulæ do not give correct results for measures in English inches, though they are those generally adopted. Disregarding the difference in the measures, which are as 1 to 1·0678, it will be found that, in the generality of channels, the mean velocity is not an arithmetical mean between the velocity at the central surface line and that at the bottom, though nearly so between the mean bottom and mean surface velocities. Dr. Young,* modifying Du Buât's formula, assumes for English inches that $v + v^{\frac{1}{2}} = \mathrm{v}$, and hence $v = \mathrm{v} + \frac{1}{2} - (\mathrm{v} + \frac{1}{4})^{\frac{1}{2}}$. This gives results very nearly the same as the other formula for v, but something less, particularly for small surface velocities. For instance, Du Buât's formula gives ·5 inch for the mean velocity when the central surface velocity is 1 inch, whereas Dr. Young's makes it ·38 inch. For large velocities both formulæ agree very closely, disregarding the difference between the measures, which is only seven per cent. They are best suited to very small channels or pipes, but unless at mean velocities of about 3 feet per second, they are wholly inapplicable to rivers.

---

* Philosophical Transactions, 1808, p. 487.

Prony found, from Du Buât's experiments, that for measures in metres $v = \left(\frac{2{\cdot}37187 + \mathrm{v}}{3{\cdot}15312 + \mathrm{v}}\right)\mathrm{v}$, in which v is also the maximum surface velocity. This, reduced for measures in English feet, becomes

$$(71.) \qquad v = \left(\frac{7{\cdot}783 + \mathrm{v}}{10{\cdot}345 + \mathrm{v}}\right)\mathrm{v};^{*}$$

and for measures in English inches,

$$(71\text{A}.) \qquad v = \left(\frac{93{\cdot}39 + \mathrm{v}}{124{\cdot}14 + \mathrm{v}}\right)\mathrm{v}.$$

For medium velocities $v = {\cdot}81\ \mathrm{v}$. The experiments from which these formulæ were derived were made with small channels. The author has calculated the values of v from that of v, equation (71A), and given the results in columns 3, 6, and 9 in TABLE VII. Ximenes, Funk, and Brünning's experiments in larger channels give the mean velocity at the centre of the depth equal ·914 v, when the central or maximum surface velocity

---

[*] Francis, Lowell Experiments, p. 150, finds this formula to give 15 per cent. less than the result found by weir measurement from the formula $\mathrm{D} = 3{\cdot}33\ (l - {\cdot}1\ n\ h)\ h^{\frac{3}{2}}$, the quantity discharged being about 250 cubic feet per second, and the velocity about 3·2 feet. It appears, however, that Francis uses the mean surface velocity, and not the maximum surface velocity required by the formula : if the latter were used, the difference would be reduced to 6 per cent., or thereabouts, in equation (72).

is v; but as the velocity also decreases in nearly the same ratio at the surface from the centre to the sides of the channel, we shall get the mean velocity in the whole section equal $\cdot 914 \times \cdot 914\ \mathrm{v} = \cdot 835\ \mathrm{v}$; and hence, for large channels,

$$(72.) \qquad v = \cdot 835\ \mathrm{v},$$

in which equation v is the *maximum velocity at the surface.* The author has also calculated the values of *v* from this formula, and given the results in columns 2, 5, and 8 of Table VII. This table will be found to vary considerably from those calculated from Du Buât's formula in French inches, hitherto generally used in this country, and much more applicable for all practical purposes.

MEAN RADIUS.—HYDRAULIC MEAN DEPTH.—BORDER.—COEFFICIENT OF FRICTION.

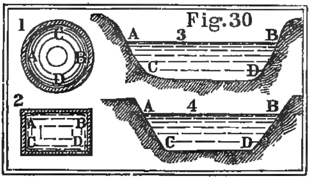


Fig. 30

If, in the diagrams 1 and 2, Fig. 30, exhibiting the sections of cylindrical and rectangular tubes filled with flowing water, the areas be divided respectively by the perimeters A C B D A and A B D C A, the quotients are termed "*the mean radii*" of the tubes, diagrams 1 and 2; and the wetted perimeters in contact with the flowing water are termed "*the borders.*" In the diagrams 3 and 4, the surface A B is not in contact with the channel, and the width of the bed and sides, taken together, A C D B, becomes "*the border.*" "*The mean radius*" is equal to the area A B D C A divided by the length of the border A C D B. "*The hydraulic mean depth*" is the same as "*the mean radius,*" this latter term being perhaps most applicable to pipes flowing full, as in diagrams 1 and 2; and the former to streams and rivers which have a surface line A B, diagrams 3 and 4. Throughout the following equations, the value of the "mean radius," "hydraulic mean depth," or quotient, $\frac{\text{area A B D C A}}{\text{border B D C A}}$,* is designated by the letter r, remarking here that *for cylindrical pipes flowing full, or rivers with semicircular beds, it is always equal to half the radius, or one-fourth of the diameter.*

Du Buât was the first to observe that the head due to the resistance of friction for water flowing in a uniform channel increased directly as the length of the channel l, directly as the border, and inversely† as the

---

* M. Girard has conceived it necessary to introduce the coefficient of correction 1·7 as a multiplier to the border for finding r, to allow for the increased resistance from aquatic plants; so that, according to his reduction,

$$r = \frac{\text{area}}{1{\cdot}7 \text{ border}}.$$

See Rennie's First Report on Hydraulics as a branch of Engineering; Third Report of the British Association, p. 167; also, equation (85), p. 216. The Author has known cases in very irregular channels in which for this sort of correction

$$r = \frac{\text{area}}{4 \text{ border}}.$$

In other words, where the velocity found from the common formula, from the fall per mile, required to be reduced one-half to find the actual mean velocity.

† Pitot had previously, in 1726, remarked that the diminution arising from friction in pipes is, *cæteris paribus*, inversely as the diameters.

area of the cross-flowing section, very nearly; that is, as $\frac{l}{r}$. It also increases as the square of the velocity, nearly; therefore the head due to the resistance must be proportionate to $\frac{v^2\, l}{2\, g\, r}$. If $c_f \times \frac{v^2\, l}{2\, g\, r} = h_f$, then c_f is the coefficient for the head due to the resistance of friction, as h is the head necessary to overcome the friction at the given velocity; c_f is therefore termed "*the coefficient of friction.*" It is found to increase as the velocity decreases.

HYDRAULIC INCLINATION.—TRAIN.

If l be the length of a pipe or channel, and h_f the height due to the resistance of friction of water flowing in it, then $\frac{h_f}{l}$ is *the hydraulic inclination.* In Fig. 31 the tubes A B, C D, of the same length l, and whose discharging extremities B and D are on the same horizontal plane B D, will have the same hydraulic inclination and the same discharge, no matter what the actual inclinations or the depth of the entrances at A and C may be, if they are of the same kind and bore; and as the velocities in A B and C D are the same, the height

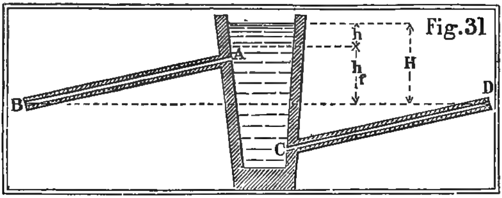


Fig.31

h due to them must be the same when the circumstances of the orifices of entry A and C are alike. The whole head is $H = h + h_f$ (see pp. 161 and 162, &c.) The hydraulic inclination is not therefore the whole head H, divided by the length l of the pipe, as it is sometimes mistaken for, but the height h_f (found by subtracting the height h, due to the entrance at A or C, and the velocity in the pipe, from the whole height) divided by the length l. When the height h is very small compared with the head h_f due to friction, or to the whole height H, as it is in very long tubes with moderate heads; $\frac{H}{l}$ may be substituted for $\frac{h_f}{l}$ without error; but for short pipes up to 1000 diameters in length the latter only should be used in applying Du Buât's and some other formulæ, which only allow for the head due to friction; otherwise the results will be too large, and only fit to be used approximately in order to determine the height h from the velocity of discharge thus found. When the horizontal pipe C D, Fig. 32, is equal in every way to the inclined pipe A B, and the head at A is that due to the velocity in C D, the discharge from the pipe A B will be equal to that from C D; but a peculiar property belongs to the pipe A B in the position in which it is

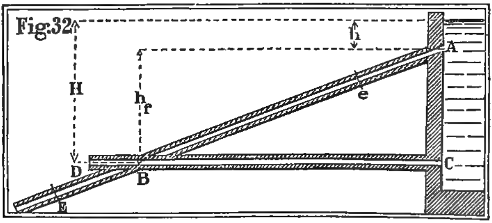


Fig.32

here placed; for *if it be cut short at any point e, or lengthened to any extent, to* E, *the discharge will remain the same and equal to that through the horizontal pipe* C D. The velocity in A B at the angle of inclination A B C, when A C = h_f, and A B = C D, is therefore such that it remains unaffected by the length A E or A *e*, to which it may be extended or cut short; and *at this inclination the water in the pipe* A B *is said to be "in train."* In like manner a river or stream is said to be "*in train*" when the inclination of its surface bears such a relation to the cross section that the mean velocity is neither accelerated nor retarded by the length of the channel; and it can be perceived from this *that the acceleration that would be caused by the inclination is exactly counterbalanced by the resistances to the motion when the moving water in a pipe or river channel is in train.*

Some writers and engineers appear to confound the inclination of a pipe, simply so called, or the head divided by the length, with the hydraulic inclination; and consequently have fallen into error in applying such of the known formulæ as take into consideration only the head due to the resistance of friction. When pipes are of considerable length, and the water is supplied from a reservoir at one end, the inclination, found as above, and the hydraulic inclination, may be taken equal to each other without sensible error; but for shorter pipes, of say up to 800 or 1000 diameters long, the greater number of formulæ, as Du Buât's and others, do not directly apply; and it is necessary to take into consideration the head due to the orifice of entry, the velocity in the tube, and also to the impulse of supply when there are junctions. These separate elements, and their effects, will be now considered; but it will be of use to refer a little farther on to some experiments, and the imperfect application of formulæ to them, first premising that a *pipe may be horizontal, or even turn upwards, and yet have a considerable hydraulic inclination.*

As $h = (1 + c_r)\dfrac{v^2}{2\,g}$ where c_r is the coefficient of the height due to the resistance at the orifice of entry A or C, and $h_f = c_f \dfrac{v^2\,l}{2\,g\,r}$, therefore

$$(73.)\quad \mathrm{H} = (1 + c_r)\frac{v^2}{2\,g} + c_f \times \frac{l\,v^2}{2\,g\,r} = \left(1 + c_r + c_f \frac{l}{r}\right)\frac{v^2}{2\,g},$$

and hence the mean velocity of discharge is found to be

$$(74.)\quad v = \left\{\frac{2\,g\,\mathrm{H}}{1 + c_r + c_f\,\dfrac{l}{r}}\right\}^{\frac{1}{2}} = \left\{\frac{\dfrac{r}{c_f} \times 2\,g\,\mathrm{H}}{(1 + c_r)\dfrac{r}{c_f} + l}\right\}^{\frac{1}{2}} *;$$

or,

$$(74\mathrm{A}.)\quad v = \left\{\frac{2\,g\,\mathrm{H}}{\dfrac{1}{c_d^2} + c_f\,\dfrac{l}{r}}\right\}^{\frac{1}{2}} = \left\{\frac{2\,g\,\mathrm{H}\,r}{\dfrac{r}{c_d^2} + c_f\,l}\right\}:$$

as $c_d^2 = \dfrac{1}{c_r + 1}$, equation (65). Also this last equation by another change of form becomes

$$(74\mathrm{B}.)\quad v = \sqrt{2\,g\,\mathrm{H}} \times \left\{\frac{1}{\dfrac{1}{c_d^2} + c_f \times \dfrac{l}{r}}\right\}^{\frac{1}{2}};$$

---

* See equations (152) and (152A) for a still more general expression for the velocity; and page 229, for the value of c_f suited to various velocities.

the values of the second member on the right-hand side of this equation, or of

$$\left\{\frac{1}{\frac{1}{c_d^2}+c_f\times\frac{l}{r}}\right\}^{\frac{1}{2}}$$

are given, for different values of c_f, c_d, and $\frac{l}{r}$, in the small table at p. 146, and below at p. 199.

When h is small compared with h_f, or, which comes to the same thing, $1 + c_r$ small compared with $c_f \times \frac{l}{r}$,

$$\text{(75.)}\qquad \mathrm{H} = c_f \times \frac{l\,v^2}{2\,g\,r},$$

and

$$\text{(76.)}\qquad v = \left\{\frac{2\,g\,r\frac{\mathrm{H}}{l}}{c_f}\right\}^{\frac{1}{2}}.$$

In the last equation, if s be substituted for $\frac{\mathrm{H}}{l}$, equal the sine of the angle of inclination A B C, then

$$\text{(77.)}\qquad v = \left\{\frac{2\,g\,r\,s}{c_f}\right\}^{\frac{1}{2}}.$$

The average value of c_f for all pipes with straight channels, with velocities of about 1·5 foot per second, may be taken at ·0069914, from which equation (77) becomes, for measures in feet,

$$\text{(78.)}\qquad v = 96\sqrt{r\,s}.$$

As the mean value of the coefficient of resistance c_r for the entrance into a tube is ·508, and as $2\,g = 64·403$, and $c_f =$ ·0069914, equation (74), for measures in feet, becomes

$$(79.)\left\{\begin{aligned} v &= \left\{\frac{64{\cdot}403\,\text{H}}{1{\cdot}508 + {\cdot}0069914\,\frac{l}{r}}\right\}^{\frac{1}{2}}, \text{ or} \\ v &= \left\{\frac{\text{H}\,r}{{\cdot}0234r + {\cdot}0001085\,l}\right\}^{\frac{1}{2}}, \text{ or} \\ v &= 100\left\{\frac{\text{H}\,r}{234r + 1{\cdot}085\,l}\right\}^{\frac{1}{2}}, \text{ or} \\ v &= 50\left\{\frac{\text{H}\,d}{58d + 1{\cdot}085l}\right\}^{\frac{1}{2}}. \end{aligned}\right.$$

This, multiplied by the section, gives the discharge.

For velocities between 2 and $2\frac{1}{2}$ feet per second, $c_f = {\cdot}0064403$, and therefore

$$v = \left\{\frac{\text{H}\,r}{{\cdot}0234\,r + {\cdot}0001\,l}\right\}^{\frac{1}{2}} = 50\left\{\frac{\text{H}\,d}{58\,d + l}\right\}^{\frac{1}{2}},$$

in which $d = 4\,r =$ diameter of a pipe.

The following table is calculated from equation (74B) *for a velocity of about* 20 *feet per second,*[*] when $c_f = {\cdot}004556$, and for different orifices of entry, in which c_d varies from ·986 for a rounded orifice, to ·715 when the pipe projects into the vessel. It gives directly the coefficient, which, multiplied by $\sqrt{2\,g\,\text{H}}$, gives the velocity in the pipe, taking friction into account.

The small table SECTION VI., p. 146, gives the like coefficients of $\sqrt{2\,g\,\text{H}}$ in equation (74B), when $c_f = {\cdot}00699$ suited to a velocity of about 18 inches per second, and can be applied in like manner. The value of $\sqrt{2\,g\,\text{H}}$ is given, in inches, in column 2, TABLE II. For feet it is equal $8\sqrt{\text{H}}$ nearly.

Mr. Provis's valuable experiments[†] with $1\frac{1}{2}$-inch

---

[*] See p. 146.

[†] Transactions of the Institution of Civil Engineers, vol. ii. pp. 201—210.

VALUES OF $\left\{\dfrac{1}{\dfrac{1}{c_d^2}+c_f\times\dfrac{l}{r}}\right\}^{\frac{1}{2}}$.

FOR VELOCITIES OF ABOUT 20 FEET PER SECOND.*

Number of diameters in the length of the pipe.	Corresponding coefficients of discharge.			Number of diameters in the length of the pipe.	Corresponding coefficients of discharge.		
2 diameters	·986	·814	·715	900 diameters	·239	·236	·233
5 ,,	·957	·791	·698	950 ,,	·234	·230	·227
10 ,,	·919	·769	·683	1000 ,,	·228	·225	·222
15 ,,	·886	·749	·669	1050 ,,	·233	·220	·317
20 ,,	·855	·731	·656	1100 ,,	·218	·215	·213
25 ,,	·828	·713	·643	1200 ,,	·209	·207	·205
30 ,,	·804	·698	·632	1400 ,,	·194	·192	·191
35 ,,	·781	·683	·620	1600 ,,	·182	·180	·179
40 ,,	·760	·668	·610	1800 ,,	·172	·171	·170
45 ,,	·741	·655	·600	2000 ,,	·163	·162	·161
50 ,,	·723	·643	·590	2200 ,,	·156	·155	·154
55 ,,	·706	·632	·580	2400 ,,	·149	·149	·148
100 ,,	·595	·548	·514	2600 ,,	·144	·143	·142
150 ,,	·518	·485	·462	2800 ,,	·139	·138	·137
200 ,,	·464	·440	·422	3000 ,,	·134	·133	·133
250 ,,	·424	·405	·391	3200 ,,	·130	·129	·129
300 ,,	·392	·378	·366	3400 ,,	·126	·125	·125
350 ,,	·367	·356	·345	3600 ,,	·122	·121	·121
400 ,,	·346	·336	·329	3800 ,,	·119	·119	·118
450 ,,	·329	·319	·314	4000 ,,	·116	·116	·115
500 ,,	·314	·307	·300	4210 ,,	·113	·113	·113
550 ,,	·301	·295	·289	4400 ,,	·111	·111	·111
600 ,,	·289	·283	·278	4600 ,,	·108	·108	·108
650 ,,	·279	·273	·269	4800 ,,	·106	·106	·106
700 ,,	·269	·265	·261	5000 ,,	·104	·104	·104
750 ,,	·261	·257	·253	5200 ,,	·102	·102	·102
800 ,,	·253	·249	·246	5400 ,,	·100	·100	·100
850 ,,	·246	·242	·239	5600 ,,	·098	·098	·098

pipes, from 20 to 100 feet long, have been used in a published work† for the purpose of testing the accuracy of Du Buât's and some other formulæ; but the head divided by the length is assumed to be the hydraulic inclination throughout, and no allowance is made for the head due to the orifice of entry and velocity in the pipe. Of course the writer's conclusions are erroneous. It is shown, SECTION I., page 16, how very nearly the formulæ and experiments agree.

---

* See p. 146.

† Researches in Hydraulics. Weale's Quarterly Papers on Engineering.

The formulæ appear to have been also misunderstood by the surveyor who experimented for the General Board of Health; for the inclination of the pipe in itself is assumed to be the hydraulic inclination, and no allowance is made for the head due to the impulse of supply. In the CIVIL ENGINEER AND ARCHITECT'S JOURNAL, Vol. XV., page 366, it is stated that "the chief results as respect the house drains are thus described in the examination of the surveyor appointed to make the trials."*

"What quantity of water would be discharged through a 3-inch pipe on an inclination of 1 in 120?—Full at the head it would discharge 100 gallons in three minutes, the pipe being 50 feet in length. This is with stone-ware pipe manufactured at Lambeth. This applies to a pipe receiving water only at the inlet, the water not being higher than the head of the pipe.

"What water was this?—Sewage-water of the full consistency, and it was discharged so completely that the pipe was perfectly clean.

"At the same inclination what would a 4-inch pipe discharge with the same distances?—Twice the amount (that I found from experiment); or, in other words, 100 gallons would be discharged in half the time. This likewise applies to a pipe receiving water only at the inlet, and of not greater height than the head. In these cases the section of the stream is diminished at the outlet to about half the area of the pipe.

"Before these experiments were made, were there not various hypothetical formulæ† proposed for general use?—Yes.

---

* Minutes of Information with reference to Works for the removal of Soil, Water, or Drainage, &c., &c. Presented to both Houses of Parliament, 1852.

† It is a mistake to call those formulæ hypothetical, unless so far as

"What would these formulæ have given with a 3-inch pipe, and at an inclination of 1 in 100? and what was the result of your experiments with the 3-inch pipe?—The formulæ would give 7 cubic feet, the actual experiment gave 11⅕ cubic feet; converting it into time, the discharge, according to the formulæ, compared with the discharge found by actual practice, would be as 2 to 3.

"How would it be with a 4-inch pipe?—The formulæ would give about 14·7 cubic feet per minute, whereas practice gave 23 cubic feet per minute.

"Take the case of a 6-inch pipe of the same inclination?—The results, according to Mr. Hawksley's formula, would be 40⅓ cubic feet per minute; from experiment it was found to be 63½ cubic feet per minute.

"Then with respect to mains and drainage over a flat surface, the result of course becomes of much more value, as the difference proved by actual practice increases with the diminution of the inclination?—Certainly, to a very great extent. For example, the tables give only 14·2 cubic feet per minute as the discharge from a pipe 6 inches diameter, with a fall of 1 in 800; practice shows that, under the same conditions, 47·2 cubic feet will be discharged.

"Will you give an example of the practical value of this when it is required to carry out drainage works over a very flat surface?—An inclination of 1 in 800 gives only 14 cubic feet per minute, according to theory, while, according to actual experiment, and with the same inclination, 47 cubic feet are given.

"Then this difference may be converted either into a saving of water to effect the same object, or into power of water to remove feculent matter from beneath the site of any houses or town?—It may be so.

"And also the power of small inclinations properly managed?—Yes; for example, if it was required to construct a water course that

---

the hypothesis is founded on observed facts. Every formula in practical use is founded on experiments, and has been deduced from them, but those formulæ are too often hypothetically applied to short tubes without the necessary corrections. It will be seen in this SECTION that the experiments from which the formulæ given were derived, were in every way greatly more extensive than those made by the directions of the Board of Health. The formula named as Mr. Hawksley's is, substantially, Eytelwein's algebraically transformed.

should discharge, say 200 feet per minute, the formula would require an inclination of 1 in 60 = 2 inches in 10 feet; whereas, experiment has shown that the same would be discharged at an inclination of 1 in 200 = $\frac{5}{8}$ inch in 10 feet, thus effecting a considerable saving in excavation, or a smaller drain would suffice at the greater inclination."

The results given above are calculated in the following Table, and also eight of the experiments made for the Metropolitan Commissioners of Sewers*; assuming for the present, with the surveyor, examined by the Commissioners, that the inclinations of the pipes and hydraulic inclinations of the formulæ are the same, *which is incorrect*, the calculated discharges, found by means of TABLES VIII. and IX., are given in the last column of the Table.

Diameter of pipe in inches.	Inclination of pipe.	Discharge in cubic feet per minute by experiment.	Hypothetical discharge by Du Buât's formula.
3	1 in 120	5·3	6·6
4	1 in 120	10·7	14
3	1 in 100	11·2	7·5
4	1 in 100	23	15·6
6	1 in 100	63·5	43·8
6	1 in 800	47·2	13·3
6	1 in 60	75	59·3
6	1 in 100	63	43·8
6	1 in 160	54	33·4
6	1 in 200	52	29·2
6	1 in 320	49	21·8
6	1 in 400	48·5	19·6
6	1 in 800	47·2	13·3
6	Level.	46	00

Du Buât's formula, therefore, gives larger results than the experiments in the two first cases, because the water received at one end only barely filled it, and the pipe was not full at the lower end; but less in the others. If in these the head due to the impulse of entry, at the upper end, and at the side junctions, were known, and the proper hydraulic inclination determined by the experiments, the formulæ would be found to give larger approximate results in every case, as might have been expected from the sewage-water used. In the last eight experiments it is stated,* that "the water was admitted at the head of the pipe, and at *five junctions or tributary pipes* on each side, so regulated as to keep the main pipe full," and that "without the addition of junctions the transverse sectional area of the stream of water near the discharging end was reduced to one-fifth of the corresponding area of the pipe, *and that it required a simple head of water of about* 22 *inches to give the same result as that accruing under the circumstances of the junctions*." It is also stated, that "in the case of the 6-inch pipe, which discharged 75 cubic feet per minute, the lateral streams had a velocity of a few feet per minute."

---

* Adcock's Engineer's Pocket Book, 1852, pp. 261 and 262.

Now, the head of "about 22 inches" is wholly neglected in the foregoing calculations, *though in a pipe* 100 *feet long it would be equal to an inclination of* 1 *in* 55! It however includes three elements at least, viz., the portion due to the orifice of entry, the portion due to the velocity in the pipe, and the portion due to friction. Assume the case of *the horizontal pipe, which discharged* 46 *cubic feet per minute*.† This is equal to a mean velocity of 46·9 inches per second; with this velocity, we find from TABLE VIII. the hydraulic inclination of a 6-inch pipe to be 1 in 94, and, therefore, the head due to friction in a pipe 100 feet long is 12·7 inches. Assuming the coefficient for the orifice of entry and velocity to be ·815, we also find from TABLE II. a head of $4\frac{1}{4}$ inches due to these. We then have,

---

* Adcock's Engineer's Pocket Book, 1852, pp. 261 and 262.

† The horizontal pipe would discharge equally at both ends, unless there was a head of water at either, or an equivalent in the velocity of

Head due to the velocity and orifice of entry	4·25	inches
Head due to the resistance of friction	12·70	,,
Radius of pipe	3·00	,,
Total . . .	19·95	

which is about 2 inches less than the observed head: this, however, is not stated definitely. *It is therefore evident, that the formula gives, if anything, larger results than these experiments,*[*] *as might have been expected, instead of less in the ratio of* 2 *to* 3, *as is stated in the Report.*

Wherever junctions are applied, as in the examples above referred to, the formulæ in general use require correction; for the quantity of water then flowing below each junction is increased. A certain amount of error is, perhaps, inseparable from every calculation of this kind; but before formulæ deduced from experiment by men every way qualified for the task are condemned, it would be well that their critics should learn to understand and properly apply them.

---

approach. Of course, a smaller pipe with a fall, must be better than the larger one with none at all, in preventing deposits.

[*] This is also true of the other formulæ, for finding the discharge from pipes, given in this work.

The diameter of a short pipe gives in itself the means of increasing very considerably the surface inclination of the fluid stream, by reducing the section at the lower end. Assume a horizontal pipe 50 feet long and 6 inches in diameter, then if the receiving end be full, and the discharging end one-third full, this inclination will be $\frac{6-2}{50 \times 12} = \frac{1}{150}$; and that the discharging end cannot be kept full unless a head of several inches be maintained at the receiving end, or an equivalent from a lateral supply. When the pipe is about two diameters long it becomes a short tube; and when the length vanishes, the transverse section becomes, simply, a discharging orifice.

DU BUÂT'S FORMULA.

The coefficient of friction c_f is not, however, constant, as it varies with the velocity. That given, p. 198, viz., $c_f = \cdot 004556$ answers for pipes when the velocity is 20 feet per second. For pipes and rivers it is found to increase as the velocity decreases; that is, the loss of head is proportionately greater for small than for large velocities. Du Buât found the loss of head to be also greater for small than large channels, and applied a correction accordingly in his formula. This, expressed in French inches, is

$$(80.) \quad v = \frac{297\,(r^{\frac{1}{2}} - \cdot 1)}{\left(\frac{1}{s}\right)^{\frac{1}{2}} - \text{hyp. log.}\left(\frac{1}{s} + 1\cdot 6\right)^{\frac{1}{2}}} - \cdot 3\,(r^{\frac{1}{2}} - \cdot 1),$$

maintaining the preceding notation, in which $s = \frac{h}{l}$.

In this formula ·1, in the numerator of the first term, is deducted as a correction due to the hydraulic mean depth, as it was found that 297 $(r^{\frac{1}{2}}-0{\cdot}1)$ agreed more exactly with experiment than $297r^{\frac{1}{2}}$ simply. The second term hyp. log.$\left(\frac{1}{s}+1{\cdot}6\right)^{\frac{1}{2}}$, of the denominator is also deducted to compensate for the observed loss of head being greater for less velocities, and the last term ·3 $(r^{\frac{1}{2}}-{\cdot}1)$ is a deduction for a general loss of velocity sustained from the unequal motions of the particles of water in the cross section as they move along the channel. These corrections are empirical; they were, however, determined separately, and after being tested by experiment, applied, as above, to the radical formula $v = 297\sqrt{r\,s}$.

Du Buât's formula was published in his Principes d'Hydraulique, in 1786. It is, as we have seen, partly empirical, but deduced by an ingenious train of reasoning and with considerable penetration from about 125 experiments, made with pipes from the 19th part of an inch to 18 inches in diameter, laid horizontally, inclined at various inclinations, and vertical; and also from experiments on open channels with sectional areas from 19 to 40,000 square inches, and inclinations of from 1 in 112 to 1 in 36,000. The lengths of the pipes experimented with varied from 1 to 3, and from 3 to 3,600 feet.

In several experiments by which the author has tested this formula, the resulting velocities found from it were from 1 to 5 per cent. too large for small pipes, and too small for straight rivers in nearly the same proportion. As the experiments from which it was derived were made with great care, those with pipes particularly so, this was to be expected. Experiments with pipes of moderate or short lengths should have the circumstances of the orifice of entry from the reservoir duly noted; for the close agreement of this formula with them must depend a great deal, in such pipes, on the coefficient due to the height h, which must be deducted from the whole head H before the hydraulic inclination, $\frac{h_f}{l} = s$, can be obtained; but for very long pipes and uniform channels this is not necessary.

The experiments from which Du Buât's formula was constructed are given in full by the late Dr. Robinson in his able article on "rivers" in the Encyclopædia Britannica, pp. 268, 269, and 270, where the calculated and observed velocities are placed side by side in French inches per second. In all these experiments Du Buât carefully deducted the head due to the velocity and orifice of entry before finding the hydraulic inclination s, and those who attempt to calculate the velocity from the head and length of the channel only, without making this deduction, will find their calculated results very different from those there given. If there were bends, curves, or contractions, deductions would have to be made for these in like manner before finding s.

Under all the circumstances, and after comparing the results obtained from various other formulæ, the author originally preferred calculating tables for the values of v from this formula reduced for measures in English inches, which is

$$v = \frac{306{\cdot}596\,(r^{\frac{1}{2}} - {\cdot}1032)}{\left(\frac{1}{s}\right)^{\frac{1}{2}} - \text{hyp. log.}\left(\frac{1}{s} + 1{\cdot}6\right)^{\frac{1}{2}}} - {\cdot}2906\,(r^{\frac{1}{2}} - {\cdot}1032),$$

or more simply,

$$(81.)\quad v = \frac{307\,(r^{\frac{1}{2}} - {\cdot}1)}{\left(\frac{1}{s}\right)^{\frac{1}{2}} - \text{hyp. log.}\left(\frac{1}{s} + 1{\cdot}6\right)^{\frac{1}{2}}} - {\cdot}3\,(r^{\frac{1}{2}} - {\cdot}1).$$

This gives the value of v a little larger than the original formula, but the difference is immaterial. For measures in English feet it becomes

$$(82.)\ v = \frac{88{\cdot}51\,(r^{\frac{1}{2}} - {\cdot}03)}{\left(\frac{1}{s}\right)^{\frac{1}{2}} - \text{hyp. log.}\left(\frac{1}{s} + 1{\cdot}6\right)^{\frac{1}{2}}} - {\cdot}084\,(r^{\frac{1}{2}} - {\cdot}03).$$

The results of equation (81·) are calculated for different values of s and r, and tabulated in TABLE VIII.,* the first eight pages of which contain the velocities for values of r varying from $\frac{1}{16}$th inch to 6 inches; or if pipes, diameters from $\frac{1}{4}$ inch to 2 feet, and of various inclinations from horizontal to vertical. The last five pages contain the velocities for values of r from 6 inches to 12 feet, and with falls from 6 inches to 12 feet per mile.

EXAMPLE VIII. *A pipe,* 1½ *inch diameter and* 100 *feet long, has a constant head of* 2 *feet over the discharging extremity; what is the velocity of discharge per second?*

---

* When this TABLE was first calculated, the author's formula (119A) was not known, and as a development of Du Buât's valuable but complex expression the table is retained. Others have since given TABLES calculated from (119A), but the formula itself is easily remembered, and results for any particular case easily calculated; especially so by using the last column of the table attached to it.

The mean radius $r = \frac{1\frac{1}{2}}{4} = \frac{3}{8}$ inches, and $\frac{100}{2} = 50 = \frac{1}{s}$, is the approximate hydraulic inclination. At page 2 of TABLE VIII., in the column under the mean radius $\frac{3}{8}$, and opposite to the inclination 1 in 50, is found 30·69 inches for the velocity sought. This, however, is but approximative, as the head due to the velocity should be subtracted from the whole head of 2 feet, before finding the true hydraulic inclination. This head depends on the coefficient of resistance at the entrance orifice, or the coefficient of discharge for a short tube. In all Du Buât's experiments this latter was taken at ·8125, but it will depend on the nature of the junction, as, if the tube runs into the cistern, it will become as small as ·715; and, if the junction be rounded into the form of the contracted vein, it will rise to ·974, or 1 nearly. In this case, the coefficient of discharge may be assumed ·815,* from which, in TABLE II., the head due to a velocity of 30·69 inches is $1\frac{7}{8} = 1·87$ inch nearly, which is the value of h; and hence, $\text{H} - h = h_f = 24 - 1·87 = 22·13$ inches; and $\frac{l}{h_f} = \frac{100 \times 12}{22·13} = 54·2 = \frac{1}{s}$, the hydraulic inclination, more correctly. With this new inclination and the mean radius $\frac{3}{8}$, the velocity by interpolating between the inclinations 1 in 50 and 1 in 60, given in the table, is $30·69 - 1·34 = 29·35$ inches per second. This operation may be repeated until v is found to any degree of accuracy according to the formula; but it is, practically, unnecessary to do so. The discharge per minute in cubic feet, is now easily found from Table IX., in which, for an inch and a half pipe,

---

* See EXAMPLE 16, pp. 14, 15.

	Inches.		Cubic feet.	
For a velocity of	20·00	per second,	1·22718	per minute.
,, ,,	9·00	,, ,,	·55223	,, ,,
,, ,,	·30	,, ,,	·01841	,, ,,
,, ,,	·04	,, ,,	·00245	,, ,,
,, ,,	29·34	,, ,,	1·80027	,, ,,

The discharge found experimentally by Mr. Provis, for a tube of the same length, bore, and head, was 1·745 cubic foot per minute.

If the coefficient of discharge due to the orifice of entry and stop-cock in Mr. Provis's 208 experiments* with $1\frac{1}{2}$ inch lead pipes of 20, 40, 60, 80, and 100 feet lengths, be ·715, the results calculated by the tables will agree with the experimental results with very great accuracy, and it is very probable, from the circumstances described, that the ordinary coefficient ·815 due to the entry was reduced by the circumstances of the stop-cock and fixing to about ·715; but even with ·815 for the coefficient, the difference between calculation and experiment is not much, the calculation being then in excess in every experiment, the average being about 5 per cent., and not so much in the example we have given.

Table VIII. gives the velocity, and thence the discharge, immediately, for long pipes, and Table X.

---

* Transactions of the Institution of Civil Engineers, vol. ii. pp. 201, 210,

enables us to calculate the cubic feet discharged per minute, with great facility. For rivers, the mean velocity, and thence the discharge, is also found with quickness. See also TABLES XI., XII., and XIII., and the TABLES at pp. 28 and 29.

EXAMPLE IX. *A watercourse is* 7 *feet wide at the bottom, the length of each sloping side is* 6·8 *feet, the width at the surface is* 18 *feet, the depth* 4 *feet, and the inclination of the surface* 4 *inches in a mile; what is the quantity flowing down per minute?*

Here $\frac{(18 + 7) \times \frac{4}{2}}{7 + 2 \times 6{\cdot}8} = \frac{50}{20{\cdot}6} = 2{\cdot}4272$ feet $= 29{\cdot}126$ inches $= r$, is the hydraulic mean depth; and as the fall is 4 inches per mile, at the 11th page of TABLE VIII., the velocity $v = 12{\cdot}03 - {\cdot}16 = 11{\cdot}87$ inches per second; the discharge in cubic feet per minute is, therefore,

$$50 \times \frac{11{\cdot}87}{12} \times 60 = 2967{\cdot}5.$$

If $94{\cdot}17 \sqrt{r\,s} = v$, then $v = 94{\cdot}17 \sqrt{2{\cdot}427 + \frac{1}{15840}}$ $= 94{\cdot}17 \times \sqrt{\frac{1}{6526}} = \frac{94{\cdot}17}{80{\cdot}7} = 1{\cdot}17$ feet $= 14{\cdot}04$ inches.

Watt, in a canal of the fall and dimensions here given, found the mean velocity about 13½ inches per second. This corresponds to a fall of 5 inches in the mile, according to the formula. Du Buât's formula is less by 12½ per cent. or ⅛th; the common formula too much by 5 per cent.

In one of the original experiments with which the formula was tested on the canal of Jard, the measurements accorded very nearly with those in this example, viz. $\frac{1}{s} = 15360$, and $r = 29{\cdot}1$ French inches; the observed velocity at the surface was 15·74, and the calculated mean velocity, from the formula, 11·61 French inches.* Table VII. will give 12·29 inches for the mean velocity, corresponding to a superficial velocity of 15·74 inches. This shows that the formula also gives too small a value for v in this case, by about $\frac{1}{17}$th of the result, it being about $\frac{1}{8{\cdot}3}$ part in the other. The probable error in the formula *applied to straight clear rivers* of about 2 feet 6 inches hydraulic mean depth is nearly $\frac{1}{12}$th or 8 per cent. of the tabulated velocity, and this must be added for the more correct result; the watercourse being supposed straight and free from aquatic plants.

Notwithstanding the differences above remarked on, the results of this formula, as calculated and tabulated, may be pretty safely relied on when applied to *general practical purposes.* Many of the others which we shall proceed to lay before our readers are more partial in their application. Rivers or watercourses are seldom straight or clear from weeds, and even if the sections, during any improvements, be made uniform, they will seldom continue so, as "*the regimen,*" or adaptation of the velocity to the tenacity of the banks, must vary with the soil and bends of the channel, and can seldom continue permanent for any length of time unless protected. From these causes a loss of velocity takes place, difficult, if not impossible, to estimate accurately, but which may be taken at from 10 to 100 per cent. of that in the clear unobstructed direct channel; but be this as it may, *it is safer to calculate the drainage or mechanical results obtainable from a given fall and river channel, from formulæ which give lesser, than from those which give larger velocities.* This is a principle engineers cannot too much observe.

---

* These measures reduced to inches, give $r = 31{\cdot}014$, $v = 12{\cdot}374$; and the surface velocity 16·775 inches; reduced for mean velocity 13·101 inches.

It was before remarked, that for both pipes and rivers the coefficient of resistance increases as the velocity decreases. This is as much as to say, in the simple formula for the velocity, $v = m \sqrt{r s}$, that m must increase with v, and as some function of it. This is the case in TABLE VIII., throughout which the velocities increase faster than $\sqrt{r}$, the $\sqrt{s}$, or the $\sqrt{r s}$. In all formulæ made use of by engineers, but the author's, Weisbach's, Du Buât's and Young's, *the velocity found is constant when* $\sqrt{r s}$ *or* $r \times s$ *is constant.* In Du Buât's formula for $r \times s$ constant, v obtains maximum values between $r = \frac{3}{8}$ inch and $r = 1$ inch; the differences of the velocities for different values of r above 1 inch, $r \times s$ being constant, are not much. The maximum value, or nearly so, may always be found by assuming $r = \frac{3}{4}$ inch, and finding the corresponding inclination from the formula $\frac{4 r s}{3}$, which is equal to it. For example, if $r = 12$ inches, and $s = \frac{1}{10560}$, the velocity is found equal 9·52 inches; but when $r s$ is constant, the inclination s corresponding to $r = \frac{3}{4}$ inch is $\frac{4 \times 12''}{3 \times 10560}$ $= \frac{1}{660}$, from which, is found from the table, $v = 10{\cdot}25$

inches, for the maximum velocity, making a difference of fully 7 per cent.

When $r =$ ·01 of an inch, or a pipe is $\frac{1}{25}$th part of an inch in diameter, Du Buât's formula fails, but it gives correct results for pipes $\frac{1}{8}$th of an inch in diameter, and two of the experiments from which it was derived were made with pipes 12 inches long and only $\frac{1}{18}$th part of an inch in diameter.

Table VIII. is extended so as to make it directly available for hydraulic mean depths, from $\frac{1}{16}$th of an inch to 12 feet, and for various hydraulic inclinations, even up to vertical, for pipes. The fall in rivers seldom exceeds 2 or 3 feet per mile, or the velocity 5 or 6 feet per second. The extension of the Table for great inclinations, and consequently great velocities, was made for the purposes of calculation, and to include pipes. It must be understood throughout this Table that the velocities are those which continue unchanged for any length of channel, viz., when the resistance of friction is equal to the acceleration of gravity, the moving water and channel being then *in train*. Several of Du Buât's experiments were made with small vertical pipes. This Table is equally applicable to pipes and rivers, and gives directly either the hydraulic inclination, the hydraulic mean depth, or the velocity when any two of them are known.

The mean velocity is given in preference to the discharge itself in Table VIII., because, while an infinite number of channels having the same hydraulic inclination (s) and the same hydraulic mean depth (r) must have the same velocity (v), yet the sectional areas, and consequently the discharges, may vary upwards from $6{\cdot}2832r^2$, the area of a semicircular channel, to any extent; and the operation of multiplying the area by the mean velocity, to find the discharge, is so very simple that any tabulation for that purpose is unnecessary. Besides this, the banks of rivers, unless artificially protected, remain very seldom at a constant slope, and therefore any TABLES of discharge for particular side slopes are only of use so far as they apply to hypothetical cases. Indeed, in new river cuts, the banks, cut first to a given slope, alter very considerably in a few months; while the necessary regimen between the velocity of the water and the channel is in the course of being established. The velocity suited to the permanency of any proposed river channel, though too often entirely neglected, is the very first element to be considered.

COULOMB having shown that the resistance opposed to a disc revolving in water increases as the function $a\,v + b\,v^2$ of the velocity v, we may assume that the height due to the resistance of friction in pipes and rivers is also of this form; and that

$$(83.) \qquad h_f = (a\,v + b\,v^2)\,\frac{l}{r},$$

and consequently,

$$(84.) \quad r\,s = a\,v + b\,v^2, \text{ and } v = \left\{\frac{r\,s}{b} + \frac{a^2}{4\,b^2}\right\}^{\frac{1}{2}} - \frac{a}{2\,b}.$$

GIRARD first gave values to the coefficients a and b. He assumed them equal, and each equal to $\cdot0003104$ for measures in metres, and thence the velocity in canals, (85.) $v = (3221{\cdot}016\, r\, s + {\cdot}25)^{\frac{1}{2}} - {\cdot}5$;*

which reduced for measures in English feet becomes

(86.) $$\begin{cases} v = (10567{\cdot}8\, r\, s + 2{\cdot}67)^{\frac{1}{2}} - 1{\cdot}64, \text{ or} \\ v = 103 \sqrt{r\, s} - 1{\cdot}64, \text{ nearly.} \end{cases}$$

The value of $a = b = {\cdot}0003104$ was obtained by means of twelve experiments by Du Buât and Chezy. Of course the value is four times this in the original, as the mean radius is used in all the formulæ instead of the diameter. This formula is only suited for very small velocities in canals, between locks, containing aquatic plants; it is inapplicable to rivers and channels in which the velocity exceeds an inch per second.

Prony found from thirty experiments on canals, that $a = {\cdot}000044450$ and $b = {\cdot}000309314$,† for measures in metres, from which

(87.) $v = (3232{\cdot}96\, r\, s + {\cdot}00516)^{\frac{1}{2}} - {\cdot}0719$;

this reduced for measures in English feet is,

(88.) $$\begin{cases} v = (10607{\cdot}02\, r\, s + {\cdot}0556)^{\frac{1}{2}} - {\cdot}236; \text{ or} \\ v = 103 \sqrt{r\, s} - {\cdot}24 \text{ nearly:} \end{cases}$$

the velocities did not exceed 3 feet per second in the experiments from which this was derived. See also note, p. 192.

For pipes, Prony found,‡ from fifty-one experiments made by Du Buât, Bossut, and Couplet, with pipes from 1 to 5 inches in diameter, from 30 to 7,000 feet in length, and one pipe 19 inches diameter and nearly

---

* See Brewster's Encyclopædia, Article Hydrodynamics, p. 259.

† Recherches Physico-Mathématiques sur la Théorie des Eaux Courantes.

‡ Recherches Physico-Mathématiques sur la Théorie du Mouvement des Eaux Courantes, 1804.

TABLE of the fifty-one Experiments referred to in Equation (89), *the value of g in the sixth being taken at* 9·8088 *metres.*

It will be perceived that Prony did not take into calculation, in framing his formula, the head due to the velocity in the pipe and to the orifice of entry.

Number of selected experiments.	Names of Experimenters.	Heads measured to the lower orifice in metres.	Diameters of pipes in metres.	Length of the pipes in metres.	Values of $\frac{g r s}{v}$ in metres.	Experimental values of the velocity v in metres.	Calculated velocity from formula (89) in metres.
1	Du Buât	·0041	·0271	19·95	·000314	·0430	·0427
2	Couplet	·1511	·1353	2280·37	·000404	·0544	·0591
3	Couplet	·3068	·1353	2280·37	·000523	·0854	·0921
4	Du Buât	·0135	·02707	19·95	·000459	·0980	·0926
5	Couplet	·4534	·1333	2280·37	·000590	·1117	·1263
6	Couplet	·5105	·1333	2280·37	·000638	·1301	·1330
7	Couplet	·6497	·1333	2280·37	·000670	·1411	·1433
8	Couplet	·6767	·1333	2280·37	·000683	·1441	·1467
9	Du Buât	·0189	·0271	3·75	·001426	·2352	·2895
10	Du Buât	·1137	·0271	3·75	·001138	·2826	·3088
11	Du Buât	·1137	·0271	3·75	·001309	·2888	·3088
12	Bossut	·1083	·0271	16·24	·001337	·3308	·3359
13	Bossut	·3248	·0361	58·47	·001446	·3400	·3553
14	Du Buât	·1605	·0271	19·95	·001482	·3604	·3713
15	Bossut	·3248	·0361	48·75	·001549	·3807	·3915
16	Du Buât	·2106	·0271	19·95	·001713	·4091	·4287
17	Bossut	·3248	·0361	38·98	·001687	·4366	·4402
18	Du Buât	·2425	·0271	19·95	·001830	·4408	·4618
19	Bossut	·3248	·0544	58·47	·001672	·4433	·4416
20	Du Buât	·2425	·0271	19·95	·001793	·4500	·4618
21	Bossut	·3248	·0544	48·73	·001795	·4955	·4860
22	Bossut	·6497	·0361	58·47	·001922	·5115	·5122
23	Bossut	·3248	·0361	29·23	·001918	·5128	·5122
24	Du Buât	·3335	·0271	19·95	·002050	·5411	·5450
25	Bossut	·3248	·0544	38·98	·001981	·5605	·5458
26	Du Buât	·3709	·0271	19·95	·002174	·5676	·5766
27	Bossut	·6497	·0361	48·73	·002073	·5693	·5634
28	Du Buât	·3952	·0271	19·95	·002223	·5916	·5961
29	Bossut	·3248	·0271	26·24	·002201	·6032	·5990
30	Bossut	·3248	·0361	19·49	·002333	·6323	·6327
31	Bossut	·3248	·0544	29·23	·002300	·6444	·6344
32	Bossut	·6497	·0361	38·98	·002267	·6498	·6323
33	Bossut	·6497	·0544	58·47	·002214	·6695	·6344
34	Bossut	·6497	·0544	48·73	·002392	·7436	·6972
35	Bossut	·6497	·0361	29·23	·002588	·74	·7343
36	Du Buât	·6416	·0271	19·95	·002750	·7761	·7660
37	Bossut	·3248	·0544	19·49	·002812	·7908	·7823
38	Du Buât	·1624	·0271	3·75	·003620	·7943	·8930
39	Bossut	·6497	·0544	38·98	·002656	·8363	·7810
40	Bossut	·3248	·0361	9·74	·003287	·8976	·9048
41	Bossut	·65	·0361	19·49	·003161	·9332	·9048
42	Bossut	·65	·0544	29·23	·003062	·9681	·9071
43	Couplet	3·9274	·4873	1169·42	·003785	1·0600	1·0592
44	Bossut	·3248	·0544	9·74	·004073	1·0915	1·1164
45	Bossut	·6497	·0544	19·49	·003821	1·1640	1·1164
46	Bossut	·6497	·0361	9·74	·004491	1·3138	1·2896
47	Du Buât	·4873	·0271	3·17	·006470	1·5784	1·7043
48	Du Buât	·5671	·0271	3·75	·006307	1·5919	1·6898
49	Bossut	·6497	·0544	9·74	·005578	1·5945	1·5890
50	Du Buât	·7219	·0271	3·17	·007838	1·9301	2·0798
51	Du Buât	·9745	·0271	3·17	·008882	2·2994	2·4205

4,000 feet long, that $a = \cdot00001733$, and $b = \cdot0003483$, from which values

(89.) $v = (2871\cdot09\ r\ s + \cdot0006192)^{\frac{1}{2}} - \cdot0249$,

for measures in metres, and for measures in English feet,

(90.) $\begin{cases} v = (9419\cdot75\ r\ s + \cdot00665)^{\frac{1}{2}} - \cdot0816; \text{ or} \\ v = 97\sqrt{r\ s} - \cdot08 \text{ nearly.} \end{cases}$

Prony also gives the following formula applicable to pipes and rivers. It is derived from fifty-one selected experiments with pipes, and thirty-one with open channels:

(91.) $v = (3041\cdot47\ r\ s + \cdot0022065)^{\frac{1}{2}} - \cdot0469734$,*

for measures in metres, which, reduced for measures in English feet, is

(92.) $\begin{cases} v = (9978\cdot76\ r\ s + \cdot02375)^{\frac{1}{2}} - \cdot15412; \text{ or} \\ v = 100\sqrt{r\ s} - \cdot15 \text{ nearly.} \end{cases}$

Eytelwein, following the method of investigation pursued previously by Prony, found from a large number of experiments, $a = \cdot0000242651$, and $b = \cdot000365543$ in rivers, for measures in metres; and, therefore,

(93.) $v = (2735\cdot66\ r\ s + \cdot001102)^{\frac{1}{2}} - \cdot0332$.†

This reduced for measures in English feet, is

---

* Recherches Physico-Mathématiques sur la Théorie des Eaux Courantes. A reduction of this formula into English feet is given at page 6, Article Hydrodynamics, Encyclopædia Britannica; at page 164, Third Report, British Association, by Rennie, and at pages 427 and 533, Article Hydrodynamics, Brewster's Encyclopædia. This reduction $v = -0\cdot1541 + (\cdot02375 + 32806\cdot6\ r\ s)^{\frac{1}{2}}$ is entirely incorrect; and being the same in each of those works, appears to have been copied one from the other.

† Mémoires de l'Académie de Berlin, 1814 et 1815. See equation (110).

$$(94.)\ \begin{cases} v = (8975{\cdot}43\ r\ s + {\cdot}0118858)^{\frac{1}{2}} - {\cdot}1089;\ \text{or} \\ v = 94{\cdot}5\ \sqrt{r\ s} - {\cdot}11\ \text{nearly, or} \\ v = \sqrt{1{\cdot}7 f r} - {\cdot}11 = 1{\cdot}3\ \sqrt{f r} - {\cdot}11 \end{cases}$$

when f is the fall in feet per mile. He also shows,* that $\frac{10}{11}$ths of a mean proportional between the fall in two English miles and the hydraulic mean depth, gives the mean velocity very nearly. This rule for measures in inches is equivalent to

$$(95.)\qquad v = 324\ \sqrt{r\ s};$$

and for measures in feet

$$(96.)\qquad v = 93{\cdot}4\ \sqrt{r\ s}.$$

For the velocity of water in pipes he found,† from the fifty-one experiments of Du Buât, Bossut, and Couplet, that $a = {\cdot}0000223$, and $b = {\cdot}0002803$, from which for measures in metres,

$$(97.)\qquad v = (3567{\cdot}29\ r\ s + {\cdot}00157)^{\frac{1}{2}} - {\cdot}0397;$$

which reduced for measures in English feet becomes

$$(98.)\ \begin{cases} v = (11703{\cdot}95\ r\ s + {\cdot}01698)^{\frac{1}{2}} - {\cdot}1303;\ \text{or} \\ v = 108\ \sqrt{r\ s} - {\cdot}13\ \text{nearly.} \end{cases}$$

Another formula given by Eytelwein for pipes, which includes the head due to the velocity for the orifice of entry, is reducible to

---

* Handbuch der Mechanik und der Hydraulik, Berlin, 1801.

† Mémoires de l'Académie des Sciences de Berlin, 1814 et 1815. Eytelwein's formula is $v = 90{\cdot}8\ \sqrt{r\ s}$ for Prussian feet, and for pipes $v = 6{\cdot}42\sqrt{\frac{50\ d\ h}{l + 50\ d}} = 46{\cdot}1\ \sqrt{\frac{d\ h}{l + 50\ d}}$; which he changed afterwards to $v = 6{\cdot}41\sqrt{\frac{54\ d\ h}{l + 54\ d}} = 47{\cdot}9\ \sqrt{\frac{d\ h}{l + 54\ d}}$. The Prussian foot is here equal to $1{\cdot}0297$ English feet.

(99.) $$v = 50 \left(\frac{dh}{l + 50\,d}\right)^{\frac{1}{2}};$$

nearly, in which h is the head, l the length, and d the diameter of the pipe, all expressed in English feet. This is a particular value of equation (74) suited to velocities of about 2½ feet per second. It must be here mentioned, that much of the valuable information presented by Prony and Eytelwein is but a modification of what Du Buât had previously given, and to whom, only, for much that is attributed to the two former, we are primarily indebted.

In the foregoing as well as in the following equations for the velocity, unless otherwise stated, one class of standards has been maintained. It is evident, if these standards be changed in part, or in whole, that apparently different forms of the equations will arise; thus—if for s, the hydraulic inclination, we substitute $\frac{m}{5280}$, the fall m in feet per mile is then used in place of the inclination s; so that equation (94), for instance, would become

$$v = (1{\cdot}7\, m\, r + {\cdot}012)^{\frac{1}{2}} - {\cdot}11 = (1{\cdot}7\, m\, r)^{\frac{1}{2}} - {\cdot}11 \text{ nearly},$$

in which v is the velocity, in feet per second, m the fall in feet per mile, and r the "hydraulic mean depth" in feet. In like manner equation (98) would become

$$v = (2{\cdot}2\, m\, r + {\cdot}02)^{\frac{1}{2}} - {\cdot}13 = (2{\cdot}2\, m\, r)^{\frac{1}{2}} - {\cdot}13.$$

The first of these reductions, viz.:—

$$v = (1{\cdot}7\, m\, r + {\cdot}0119)^{\frac{1}{2}} - {\cdot}109,$$

is given in a book of tables calculated for river channels for the Commissioners of Public Works, Ireland, the original equation being Eytelwein's, and not D'Aubuisson's, who merely copied it. It is suited for velocities averaging about 1·3 foot per second. Again

Mr. Hawksley, by changing the form of an old result, gives for pipes the formula

$$v_y = {\cdot}77 \left\{ \frac{d\,\mathrm{H}}{l + 1{\cdot}5\,d\cdot} \right\}^{\frac{1}{2}},$$

in which l is the length in yards, H the head in inches, d the diameter in inches, and v the velocity in yards per second. For uniform feet measures, for, v, d, and H, this becomes

$$v = 48{\cdot}045 \left\{ \frac{d\,\mathrm{H}}{l + 54\,d} \right\}^{\frac{1}{2}},$$

which is only an alteration in form of Eytelwein's equation, note to (93). Eytelwein's equation expressed in the measures used by Mr. Hawksley would be very nearly

$$v_y = {\cdot}8 \left\{ \frac{d\,\mathrm{H}}{l + 1\frac{1}{2}\,d} \right\}^{\frac{1}{2}},$$

which is the simpler of the two; both, however, are but particular cases of the general equation (74), and only suited for velocities of about 2½ feet per second.

DR. THOMAS YOUNG* also derives his formula from the supposition, that the head due to the resistance of friction assumes the form of equation (83); calling the diameter of a pipe d, he takes

$$h_f = (a\,v + b\,v^2)\,\frac{l}{d},$$

---

* Philosophical Transactions for 1808. Young also translated Eytelwein's Handbuch.

and the whole height $H = h_f + \frac{v_2}{586}$, expressed in inches, which corresponds with a coefficient of ·871, nearly, for the orifice of entry. He found from some experiments of his own, those collected by Du Buât, and some of Gerstner's, that

$$(100.)\quad a = \cdot 0000002 \left\{ \frac{900\, d^2}{d^2 + 1136} + \frac{1}{d^{\frac{1}{2}}}\left(1085 + \frac{13\cdot 21}{d} + \frac{1\cdot 0563}{d^2}\right) \right\};$$

and

$$(101.)\quad b = \cdot 0000001 \left\{ 413 + \frac{75}{d} - \frac{1440}{d + 12\cdot 8} - \frac{180}{d + \cdot 355} \right\};$$

then as $\frac{1}{586} = \cdot 00171$, the value of the velocity becomes

$$(102.)\quad v = \left\{ \frac{H\, d}{b\, l + \cdot 00171\, d} + \left(\frac{a\, l}{2\, b\, l + \cdot 00341\, d}\right)^2 \right\}^{\frac{1}{2}} - \frac{a\, l}{2\, b\, l + \cdot 00341\, d}.$$

When the length l of the pipe is very great compared with the head due to the orifice of entrance and velocity, $\cdot 00171\, v^2$, then

$$(103.)\qquad v = \left\{ \frac{H\, d}{b\, l} + \frac{a^2}{4\, b^2} \right\}^{\frac{1}{2}} - \frac{a}{2\, b};$$

or by substituting for $\frac{H}{l}$ its value s, equal the sine of the inclination,

$$(104.)\qquad v = \left\{ \frac{s\, d}{b} + \frac{a^2}{4\, b^2} \right\}^{\frac{1}{2}} - \frac{a}{2\, b}.$$

The values of a and b are for measures in inches. For most rivers he finds for French inch measures, $v = \sqrt{20000\, d\, s}$, *in which d must be taken equal* $4\, r$; this reduced for English inches is

TABLE OF THE VALUES OF a, b, $\frac{a}{2b}$, AND $\frac{a^2}{4b^2}$, IN EQUATION (104).

d in inches.	r in inches.	a.	b.	$\frac{a}{2b}$.	$\frac{a^2}{4b^2}$.	d in inches.	r in inches.	a	b	$\frac{a}{2b}$.	$\frac{a^2}{4b^2}$.
·1	·025	·000837	·000066	6·341	40·207	6	1·5	·000094	·000032	1·468	2·155
·2	·05	·000536	·000035	7·657	58·629	7	1·75	·000090	·000033	1·363	1·857
·3	·075	·000406	·000028	7·250	52·562	8	2	·000087	·000033	1·318	1·737
·4	·100	·000356	·000025	7·120	50·694	9	2·25	·000084	·000034	1·235	1·526
·5	·125	·000316	·000025	6·320	39·942	10	2·50	·000083	·000034	1·220	1·489
·6	·150	·000287	·000024	5·979	35·750	11	2·75	·000083	·000034	1·220	1·489
·7	·175	·000264	·000024	5·500	30·250	12	3·00	·000084	·000035	1·200	1·440
·8	·200	·000247	·000025	4·940	24·403	15	3·75	·000086	·000035	1·228	1·509
·9	·225	·000232	·000025	4·640	21·529	20	5	·000096	·000036	1·333	1·777
1	·250	·000220	·000025	4·400	19·360	40	10	·000140	·000038	1·842	3·393
1·5	·375	·000179	·000027	3·315	10·988	50	12·5	·000154	·000039	1·974	3·898
2	·500	·000155	·000028	2·767	7·661	60	15	·000165	·000039	2·115	4·474
2·5	·625	·000139	·000028	2·482	6·161	80	20	·000177	·000040	2·212	4·895
3·0	·750	·000127	·000029	2·189	4·794	100	25	·000184	·000040	2·300	5·290
3·5	·875	·000119	·000030	1·983	3·933	300	75	·000190	·000041	2·317	5·369
4	1·000	·000111	·000031	1·790	3·205	500	125	·000189	·000041	2·305	5·312
5	1·250	·000101	·000031	1·629	2·653	Infinite.	Infinite.	·000180	·000041	2·195	4·818

(105.) $$v = 292 \sqrt{r\,s};$$
which again reduced for feet measures, becomes
(106.) $$v = 84{\cdot}3 \sqrt{r\,s};$$
These latter values, for rivers, are even smaller than those found from Du Buât's formula; less than the observed velocities, and less than those found from any other formula, with the exception of Girard's. The values of the coefficients a and b vary in this formula with the value of $d = 4\,r$; they are expressed generally in equations (101) and (102), from which the preceding table for different values of d and r has been calculated.

An examination of this table will show that a obtains a minimum value when d is between 10 and 11 inches; and b when the diameter is between ½ and ¾ of an inch. Now, it appears from equation (102), that v increases with $\sqrt{\frac{\text{H}\,d}{b\,l}}$ nearly, or, which is the same thing, as b decreases, there must, *cæteris paribus*, be a maximum value of v for a given value of $\frac{\text{H}\,d}{l}$, or $r\,s$, when d is between ½ and ¾ inch; but as $\frac{a}{2\,b}$ has a minimum value when d is nearly 12 inches, the maximum value of v referred to will be found between values of d from ¾ inch to 12 inches; in fact, when d = 10 inches nearly. A similar peculiarity has already been pointed out in Du Buât's general theorem, at page 213. *It will not be necessary to take out the values of* $\frac{a}{2\,b}$ *and* $\frac{a^2}{4\,b^2}$ *to more than one place of decimals.*

The values of $\frac{a}{2\,b}$ are also given in the table, and may be used in equation (104) for finding the discharge from long pipes. It is, however, necessary to remark, that this equation is sometimes misapplied in finding the velocity from short pipes, and those of moderate lengths. It is necessary to use equation (102), which takes into consideration the head due to the velocity and orifice of entry for such pipes.

For a pipe 11 inches in diameter, the expression for the velocity, equation (104), becomes for inch measures,

$$v = \left\{ \frac{s\,d}{\cdot 000034} + 1\cdot 49 \right\}^{\frac{1}{2}} - 1\cdot 22:$$

and for feet measures, also substituting 4 r for d,

(106A.) $$v = \left\{ \frac{s\,r}{\cdot 000102} \right\}^{\frac{1}{2}} - \cdot 1 = 100\,(r\,s)^{\frac{1}{2}} - \cdot 1$$

very nearly. For a pipe ·7 inch in diameter would be found in a like manner for feet measures,

(106B.) $$v = 118\,(r\,s)^{\frac{1}{2}} - \cdot 5,$$

which is only suitable for very high velocities.

Sir John Leslie states,* that the mean velocity of a river in miles per hour, is $\frac{15}{16}$ths of the mean proportional between the hydraulic mean depth and the fall in two miles in feet. This rule is equivalent, for measures in feet, to

(107.) $$v = 100\,\sqrt{r\,s};$$

and is applicable to rivers with velocities of about $2\frac{1}{6}$ feet per second.

D'Aubuisson, from an examination of the results obtained by Prony and Eytelwein, assumes† for measures in metres that $a = \cdot0000189$, and $b = \cdot0003425$ for pipes, substituting these in equation (84) and resolving the quadratic

---

* Natural Philosophy, p. 423.

† Traité d'Hydraulique, p. 224.

(108.) $v = (2919{\cdot}71\, r\, s + \cdot00074)^{\frac{1}{2}} - \cdot027$;
which reduced for measures in English feet becomes

(109.) $\begin{cases} v = (9579\, r\, s + \cdot00813)^{\frac{1}{2}} - \cdot0902, \text{ or} \\ v = 98\, \sqrt{r\, s} - \cdot1 \text{ nearly.} \end{cases}$

For rivers he assumes with Eytelwein,* $a = \cdot000024123$ and $b = \cdot0003655$, for measures in metres, and hence

(110.) $v = (2735{\cdot}98\, r\, s + \cdot0011)^{\frac{1}{2}} - \cdot033$;
which for measures in English feet is

(111.) $\begin{cases} v = (8976{\cdot}5\, r\, s + \cdot012)^{\frac{1}{2}} - \cdot109, \text{ or} \\ v = 94{\cdot}5\, \sqrt{r\, s} - \cdot11 \text{ nearly.} \end{cases}$

When the velocity exceeds two feet per second, he assumes, from the experiments of Couplet, $a = 0$, and $b = \cdot00035875$; these values give

(112.) $v = \sqrt{2787{\cdot}46\, r\, s}$,
for measures in metres, and

(113.) $v = 95{\cdot}6\, \sqrt{r\, s} = \sqrt{9145\, r\, s}$
for measures in English feet. Equations (110) and (111) are the same as (93) and (94), found from Eytelwein's values of a and b, and it may be remarked that D'Aubuisson's equations for the velocity generally, are simply those of Prony and Eytelwein.

The values which are found to agree best with the general run of experiments on clear straight rivers of uniform section are $a = \cdot0000035$, and $b = \cdot0001150$ for measures in English feet, from which we find

(114.) $\begin{cases} v = (8695{\cdot}6\, r\, s = \cdot00023)^{\frac{1}{2}} - \cdot0152, \text{ or} \\ v = 93\, \sqrt{r\, s} - \cdot02, \end{cases}$

---

* Traité d'Hydraulique, p. 133. See Equation (93).

which for an average velocity of $1\frac{1}{2}$ foot per second will give $v = 92{\cdot}3\sqrt{r\,s}$ nearly, and for larger velocities $v = 93{\cdot}3\sqrt{r\,s}$; for smaller velocities than $1\frac{1}{2}$ foot per second, the coefficients of $\sqrt{r\,s}$ decrease pretty rapidly. This formula will be found to agree more accurately with observation and experiment than any other of this form.*

Weisbach is perhaps the only writer who has modified the form of the equation $r\,s = a\,v + b\,v^2$. In Dr. Young's formula, a and b vary with r, but Weisbach assumes that $h_f = \left(a + \frac{b}{v^{\frac{1}{2}}}\right)\frac{l}{a} \times \frac{v^2}{2\,g}$, and finds from the fifty-one experiments of Couplet, Bossut, and Du Buât, before referred to, one experiment by Guemard, and eleven by himself, all with pipes varying from an inch to five and a half inches in diameter, and with velocities varying from $1\frac{1}{2}$ inch to 15 feet per second, that $a = {\cdot}01439$, and $b = {\cdot}0094711$ for measures in metres; hence for the metrical standard

$$(115.) \qquad h_f = \left({\cdot}01439 + \frac{{\cdot}0094711}{v^{\frac{1}{2}}}\right)\frac{l}{d} \times \frac{v^2}{2\,g}.$$

This reduced for the mean radius r is

$$(116.) \qquad h_f = \left({\cdot}003597 + \frac{{\cdot}0023678}{v^{\frac{1}{2}}}\right)\frac{l}{r} \times \frac{v^2}{2\,g};$$

---

* In a stream (the Muddock), with a curved channel, jagged and irregular banks, variable depths averaging about 1 foot, aquatic plants growing on the bed, varying velocity, and an average cross section of about 16·50 feet, the flow was only 18 cubic feet per second, the formula giving 35 cubic feet. In such cases the application of a formula investigated for a uniform channel and a uniform slope is inadmissible; yet we have heard evidence in the Four Courts, Dublin, founded on such mistaken applications. See note, p. 192.

from which for measures in English feet

$$(117.) \qquad h_f = \left(\cdot 003597 + \frac{\cdot 0042887}{v^{\frac{1}{2}}}\right)\frac{l}{r} \times \frac{v^2}{2\,g},$$

and thence

$$(118.) \qquad r\,s = \left(\cdot 003597 + \frac{\cdot 0042887}{v^{\frac{1}{2}}}\right)\frac{v^2}{2\,g};$$

and by substituting for $2\,g$, its value 64·403,

$$(119.) \qquad r\,s = \left(\cdot 00005585 + \frac{\cdot 00006659}{v^{\frac{1}{2}}}\right)v^2.$$

In equation (117), $\left(\cdot 003597 + \frac{\cdot 0042887}{v^{\frac{1}{2}}}\right) = c_f$ is the coefficient of the head due to friction. The equation does not admit of a direct solution, but the coefficient should be first determined for different values of the velocity v and tabulated, after which the true value of v can be determined by finding an approximate value, and thence taking out the corresponding coefficient from the table, which does not vary to any considerable extent for small changes of velocity. In the following small table the author has calculated the coefficients of friction, and also those of v^2, in equation (119), for different values of the velocity v.

TABLE OF THE COEFFICIENTS OF FRICTION IN PIPES.

Velocity in feet.	c_f	$\frac{c_f}{64{\cdot}4}$	$\frac{64{\cdot}4}{c_f}$	$\sqrt{\frac{64{\cdot}4}{c_f}}$	Velocity in feet.	c_f	$\frac{c_f}{64{\cdot}4}$	$\frac{64{\cdot}4}{c_f}$	$\sqrt{\frac{64{\cdot}4}{c_f}}$
·1	·017159	·0002664	3078·07	55·5	2·4	·006365	·0000988	10121·5	100·5
·2	·013186	·0002047	4885·2	69·9	2·5	·006309	·0000979	10214·5	101·0
·3	·011427	·0001774	5636·9	75·08	2·6	·006257	·0000972	10288·1	101·4
·4	·010378	·0001611	6270·3	78·8	2·7	·006207	·0000964	10373·4	101·8
·5	·009662	·0001500	6666·6	81·6	2·8	·006160	·0000956	10460·2	102·2
·6	·009133	·0001418	7052·2	84·0	2·9	·006115	·0000949	10537·4	102·6
·7	·008723	·0001354	7385·5	85·9	3·	·006073	·0000943	10604·4	102·9
·8	·008391	·0001303	7674·6	87·6	3·5	·005890	·0000914	10940·9	104·6
·9	·008117	·0001260	7936·5	89·1	4·	·005741	·0000891	11223·3	105·9
1·0	·007886	·0001224	8169·2	90·4	5·	·005514	·0000856	11682·2	108·0
1·1	·007686	·0001193	8382·2	91·5	6·	·005348	·0000830	12048·2	109 7
1·2	·007512	·0001166	8576·3	92·6	7·	·005218	·0000810	12345·6	111·1
1·25	·007433	·0001154	8665·5	93·1	8·	·005113	·0000794	12632·2	112·4
1·3	·007358	·0001142	8756·5	93·5	9·	·005026	·0000780	12820·5	113·3
1·4	·007221	·0001121	8920·6	94·4	10·	·004953	·0000769	13003·9	114·0
1·5	·007098	·0001102	9074·4	95·2	15·	·004704	·0000730	13698·6	117·0
1·6	·006987	·0001085	9216·5	96·0	16·	·004669	·0000725	13793·1	117·4
1·7	·006886	·0001069	9354·5	96·7	20·	·004556	·0000707	14144·2	118·9
1·75	·006839	·0001062	9416·2	97·03	25·	·004455	·0000691	14471·7	120·3
1·8	·006794	·0001054	9487·6	97·4	30·	·004380	·0000680	14705·9	121·2
1·9	·006715	·0001042	9596·9	97·9	35·	·004322	·0000671	14903·1	122·0
2·	·006629	·0001029	9718·2	98·5	40·	·004275	·0000664	15060·2	122·7
2·1	·006556	·0001018	9823·2	99·1	45·	·004236	·0000658	15197·5	123·3
2·2	·006488	·0001007	9930·5	99·6	50·	·004203	·0000653	15313·8	123·7
2·3	·006424	·0000997	10003·	100·	100·	·004200	·0000625	16000·0	126·4

If the value of $\frac{64{\cdot}4}{c_f}$ here found, be substituted in the equation $v = \sqrt{\frac{64{\cdot}4}{c_f} r\, s}$, we shall have the value of v. According to this table the coefficient of friction for a velocity of six inches is more than twice that for a velocity of twenty feet, and the velocity is less in the proportion of 81·6 to 118·9, or of 81·6 $(r\,s)^{\frac{1}{2}}$ to 118·9 $(r\,s)^{\frac{1}{2}}$. On comparing these coefficients and those for pipes in the preceding formulæ, with those for rivers of the same hydraulic depth, it will be perceived that the loss from friction is greatest in the latter, as might have been anticipated; but this evidently arises from lesser velocities.

It has been remarked that the coefficient of friction decreases as the velocity increases. The only general formula which properly meets this defect in the common formulæ is Weisbach's, but it does not give the velocity v directly, as this quantity is involved in both sides of his equation. As for several hydraulic works it is necessary to convey water through pipes to work machines under high heads, and for which the common formula would give results considerably under the true ones, it appeared to the author desirable to obtain some simple expression for the velocity which might be easily remembered and applied, which would be equally correct with other formulæ for medium velocities of from one to two and a half feet, and which at the same time would give practically correct results for lesser and greater velocities within the limits of experiment. By reducing the velocity found from experiment to the form $v = m \sqrt{r\,s}$ for every case, and afterwards applying a correction of the form $n \sqrt[x]{r\,s}$ to meet the increasing value of m as v increased, the following expression was discovered:—

(119 A) $v = 140\,(r\,s)^{\frac{1}{2}} - 11\,(r\,s)^{\frac{1}{3}}$

which gives results not differing more from experiments than these frequently do from each other. The following table exhibits the velocities compared with those obtained from the experiments made by Du Buât, Couplet, Watt, Mr. Provis, and Mr. Leslie, in the Minutes of the Institution of Civil Engineers for February 1855. The last experiment was furnished by Mr. Hodson of Lincoln. Numbers 34 and 35 were made as stated, and give the mean results of several experiments made with great care; the coefficient of

TABLE showing the Experimental Results of observed Velocities in Water Channels, with the Author's general formula for Pipes and Rivers (119A) *viz.*

$$v=140\,(r\,s)^{\frac{1}{2}}-11\,(r\,s)^{\frac{1}{3}}.^{*}$$

No.	Heads in feet (H).	Lengths in feet (l).	Values of r.	Values of s.	Values of r s.	Velocities from experiment.	Velocities from the formula.	Velocities expressed in the form $v = m\sqrt{r\,s}$.	Experimenter's Names.
1	·08333	1086·	·052083	·000076	·00000396	·100	·105	52·6 $\sqrt{r\,s}$	Mr. Leslie
2	·01332	65·37	·022204	·000196	·00000434	·140	·113	54·0 ,,	Couplet
3	·14583	1086·	·052083	·000133	·00000693	·118	·157	60·0 ,,	Mr. Leslie
4	·49586	7482·	·111000	·000066	·00000134	·178	·167	61·5 ,,	Couplet
5	·20833	1086·	·052083	·000190	·00000989	·217	·206	65·0 ,,	Mr. Leslie
6	·45833	1086·	·052083	·000417	·00002170	·361	·345	74·1 ,,	,,
7	1·48768	7482·	·111000	·000198	·00002220	·366	·348	74·1 ,,	Couplet
8	1·44800	1086·	·052083	·001321	·0000688	·715	·711	85·7 ,,	Mr. Leslie
9	2·78125	1086·	·052083	·002538	·0001322	1·085	1·050	91·3 ,,	,,
10	,,	,,	2·427200	·000063	·0001532	1·166	1·143	92·4 ,,	Watt
11	·50000	100·	·031250	·004741	·0001482	1·023	1·122	92·2 ,,	Mr. Provis
12	2·78125	1086·	·052083	·004348	·0002265	1·461	1·438	95·5 ,,	Mr. Leslie
13	4·76042	1086·	·052083	·006410	·0003340	1·725	1·796	98·3 ,,	,,
14	1·06580	127·9	·044630	·007748	·0003458	1·839	1·840	98·9 ,,	Bossut
15	·50000	40·	·031250	·010810	·0003378	1·711	1·816	98·7 ,,	Mr. Provis
16	1·06580	95·92	·044630	·010050	·0004485	2·111	2·124	100·3 ,,	Bossut
17	1·5	100·	·031250	·014156	·0004422	2·005	2·103	100·5 ,,	Mr. Provis
18	9·9896	1086·	·052083	·009174	·0004779	2·095	2·185	100·6 ,,	Mr. Leslie
19	·8575	40·	·031250	·018042	·0005638	2·380	2·414	101·7 ,,	Mr. Provis
20	2·1316	191·9	·044630	·010548	·0004708	2·463	2·183	100·6 ,,	Bossut
21	2·1316	159·9	·044630	·012524	·0005589	2·440	2·404	101·7 ,,	,,
22	,,	,,	·052083	·014286	·0007440	2·800	2·823	103·5 ,,	Mr. Leslie
23	2·1316	127·9	·044630	·015350	·0006851	2·744	2·696	103·0 ,,	Bossut
24	,,	,,	·044630	·027921	·0012465	3·819	3·760	106·5 ,,	,,
25	,,	,,	·052083	·025000	·0013021	3·783	3·852	106·7 ,,	Mr. Leslie
26	3·27416	40·	·031250	·018040	·002093	5·054	5·006	109·3 ,,	Mr. Provis
27	2·3684	10·39155	·022204	·133689	·0029679	6·322	6·048	111·0 ,,	Du Buât
28	3·27416	20·	·031250	·111200	·0034750	6·723	6·572	111·5 ,,	Mr. Provis
29	3·4525	20·	·031250	·113900	·0035594	7·086	6·668	111·9 ,,	,,
30	7·135	62·8822	·029605	·098861	·0029268	6·157	5·999	110·9 ,,	Couplet
31	14·270	125·7644	·029605	·106151	·0031426	6·151	6·239	111·3 ,,	,,
32	21·405	188·6466	·029605	·108579	·00321455	6·145	6·316	111·4 ,,	,,
33	3·1974	10·39155	·022204	·176991	·0039292	7·544	7·039	112·3 ,,	Du Buât
34	11·125	9·292	·021250	·713000	·01515125	14·583	14·513	117·9 ,,	Mr. Neville
35	20·8	19·2	·021250	·814000	·01729750	15·667	15·617	118·4 ,,	,,
36	150·	100·	·020833	1·400000	·0291667	21·7	20·6	120·3 $\sqrt{r\,s}$	Mr. Hodson

---

* The form in which this formula was first found was as follows :—

$$v=\left\{140-\frac{10\cdot 6}{(r\,s)^{\frac{1}{6}}}\right\}\times\sqrt{r\,s}.$$ For measures in metres it becomes $77\cdot 3\,(r\,s)^{\frac{1}{2}}-4\cdot 9\,(r\,s)^{\frac{1}{3}}$.

the orifice of entry was found to be ·860.* The measures have been all reduced to English feet. The results found by the same experimenters, at the same time, with the same apparatus, sometimes differ by three or four per cent., as may be seen by referring to Mr. Provis' experiments (Transactions of the Institution of Civil Engineers, vol. II., p. 203), and the difference in the experiments shown in the table are apparent. The difference in the velocities found from the experiments, do not exceed those inseparable from practical investigations, and they differ as much in themselves as from the formula, which for cylindrical pipes of diameter d may be thus expressed,

$$(119\text{B.})\qquad \begin{cases} v = 70\,(d\,s)^{\frac{1}{2}} - 6{\cdot}93\,(d\,s)^{\frac{1}{3}}, \text{ or} \\ v = 70\,(d\,s)^{\frac{1}{2}} - 7\,(d\,s)^{\frac{1}{3}} \text{ nearly.} \end{cases}$$

The expression fails when $70\,(d\,s)^{\frac{1}{2}}$ is equal to or less than $6{\cdot}93\,(d\,s)^{\frac{1}{3}}$, but as this only happens when $r s = \left(\frac{11}{140}\right)^6 = {\cdot}000000235$, and for velocities below one inch per second, its practical value is not thereby affected. The expression of Du Buât fails with a tube of one twenty-fifth part of an inch in diameter, no matter what the head may be, as it then makes the velocity equal to nothing, although some of the experiments from which it was derived were made with tubes but the eighteenth part of an inch diameter. The following expression is free from this defect:

$$(119\text{c.})\qquad v = 60\,(r\,s)^{\frac{1}{2}} + 120\,(r\,s)^{\frac{2}{3}},$$

---

* The coefficient for the orifice of entry was found by cutting off the pipe at two diameters from the cistern at the conclusion of the experiments, and finding the time of emptying. *Vide* p. 172.

and will give results approximating very closely to those found from Du Buât's formula, and, therefore, with those experiments with which it most nearly coincides, but agreeing much more closely with Watt's and other experiments, on rivers. It gives higher results than the previous formula for velocities below six inches, but the results found by different experimenters differ very much in those. For higher velocities it appears to differ occasionally only about one-twentieth from observation, being in general less, as far as twenty feet per second, where it coincides very closely with Mr. Hodson's experiment. As the errors appear to be of an opposite kind generally, in the two last expressions, combining them is found

(119 D.) $v = 100\ (r\,s)^{\frac{1}{2}} + 60\ (r\,s)^{\frac{2}{3}} - 5{\cdot}5\ (r\,s)^{\frac{1}{3}}$,

an expression which, however, wants simplicity for ready practical application. When the length of the pipe does not exceed from 1000 to 2000 diameters, a correction is due to the velocity in it, and to the orifice of entry before finding the "hydraulic inclination" (s). The coefficient used in reducing the foregoing experiments for the orifice of entry was ·815, which gives $1{\cdot}508\ \frac{v^2}{2\,g}$ for the height due to the joint effects of velocity and orifice. This must be deducted from the head (H) before dividing it by the length (l) to find the inclination (s) in our table.

The following table, calculated from the formula (119 A), $v = 140\ (r\,s)^{\frac{1}{2}} - 11\ (r\,s)^{\frac{1}{3}}$, gives the corresponding values of $r\,s$ and v, so that when one is known the other is immediately found from inspection. Thus, if $r\,s = {\cdot}03125$, then we shall have

TABLE for finding the Velocity in feet per second, from the product of the hydraulic mean depths and hydraulic inclinations, and the reverse calculated from the Author's formula $v = 140\ (r\,s)^{\frac{1}{2}} - 11\ (r\,s)^{\frac{1}{3}}$, *in which r, s, and v are feet measures.*

Values of *r s.*	Velocity *v.*	Values of *r s.*	Velocity *v.*	Values of *r s.*	Velocity *v.*	Values of *r s.*	Velocity *v.*
·00000296	·083	·0001302	1·04	·000689	2·70	·003559	6·67
·00000332	·091	·0001322	1·05	·000710	2·75	·003599	6·71
·00000395	·104	·0001420	1·09	·000744	2·83	·003630	6·74
·00000427	·111	·0001482	1·12	·000758	2·85	·003788	6·90
·00000543	·133	·0001532	1·14	·000789	2·91	·003929	7·04
·00000592	·142	·0001578	1·16	·000805	2·94	·003946	7·05
·00000690	·158	·0001610	1·17	·000833	3·00	·003977	7·08
·00000734	·167	·0001657	1·19	·000852	3·04	·004104	7·20
·00000947	·198	·0001736	1·21	·000900	3·13	·004167	7·27
·00000989	·206	·0001776	1·24	·000947	3·22	·004356	7·44
·00001184	·231	·0001815	1·26	·001042	3·40	·004546	7·62
·00001263	·241	·0001894	1·30	·001105	3·51	·004630	7·69
·00001420	·261	·0002052	1·35	·001136	3·57	·004735	7·78
·00001578	·280	·0002131	1·38	·001231	3·73	·005556	8·49
·00001677	·292	·0002265	1·43	·001246	3·76	·006944	9·61
·00001894	·316	·0002367	1·47	·001263	3·78	·007576	10·0
·00001973	·325	·0002552	1·50	·001302	3·85	·008333	10·5
·00002170	·345	·0002604	1·55	·001326	3·89	·009259	11·1
·00002367	·365	·0002652	1·57	·001420	4·04	010417	11·8
·00002565	·385	·0002778	1·61	·001515	4·18	·011905	12·7
·00002841	·411	·0002841	1·63	·001576	4·28	·013889	13·8
·00003255	·448	·0003030	1·69	·001610	4·32	·015151	14·5
·00003354	·457	·0003157	1·73	·001667	4·41	·016667	15·3
·00003551	·473	·0003220	1·75	·001705	4·46	·017297	15·6
·00003748	·489	·0003314	1·79	·001735	4·51	·020833	17·1
·00003946	·505	·0003378	1·80	·001799	4·60	·027778	20·2
·00004143	·521	·0003409	1·81	·001894	4·73	·029167	20·6
·00004340	·536	·0003551	1·85	·001989	4·87	·041666	24·7
·00004632	·558	·0003630	1·89	·002052	4·94	·055556	28·8
·00005130	·594	·0003706	1·90	·002083	4·98	·062500	30·6
·00005327	·608	·0003788	1·92	·002093	5·00	·072916	33·2
·00005524	·622	·0003946	1·98	·002178	5·10	·083333	35·6
·00005919	·648	·0004022	1·10	·002210	5·14	·104167	40·0
·00006314	·674	·0004103	2·02	·002273	5·22	·125	43·9
·00006708	·698	·0004261	2·06	·002375	5·35	·145583	47·6
·0000688	·711	·0004419	2·10	·002462	5·46	·166667	51·1
·00007102	·724	·0004485	2·12	·002533	5·53	·208333	57·3
·00007694	·760	·0004546	2·14	·002652	5·68	·229167	60·2
·00008049	·781	·0004708	2·18	·002683	5·72	·250000	63·0
·00008523	·808	·0004735	2·18	·002841	5·90	·270833	65·7
·00008681	·828	·0004893	2·23	·002968	6·05	·312500	70·7
·00009270	·849	·0005051	2·27	·002999	6·08	·333333	73·2
·00009470	·861	·0005208	2·31	·003030	6·11	·354167	75·5
·00010259	·903	·0005303	2·33	·003143	6·23	·375000	77·7
·00010654	·923	·0005638	2·41	·003157	6·25	·395833	80·0
·00011048	·945	·0006061	2·52	·003214	6·31	·416667	82·1
·00011364	·960	·0006155	2·54	·003220	6·32	·437500	84·2
·00011837	·983	·0006313	2·57	·003314	6·42	·458333	86·2
·00012232	1·00	·0006440	2·60	·003409	6·51	·479166	88·3
·00012627	1·02	·0006629	2·64	·003475	6·58	·500000	90·2

$$\begin{array}{rlr} v = 20{\cdot}6 & \text{when} & r\,s = {\cdot}029167 \\ v = 24{\cdot}7 & \text{when} & r\,s = {\cdot}041666 \\ \hline \text{Difference } 4{\cdot}1 & \text{corresponds to} & {\cdot}012499 \end{array}$$

$$\begin{array}{r} {\cdot}03125 \\ {\cdot}02917 \\ \hline \text{Difference } {\cdot}00208 \end{array}$$

Whence ·0125 : 4·1 : : ·00208 : ·7 nearly, and 20·6 + 7 = 21·3 is the velocity sought; the same practically as found in EXAMPLE 26, p. 24. If allowance is to be made for the head due to the orifice of entry and velocity, this head can be determined from the velocity due to the value of $r\,s$ in the table next less than the given value with sufficient accuracy. In this case, this velocity is 20·6 feet per second = 247 inches nearly. If the orifice of entry be square, the coefficient is ·815, and the head due to the velocity and this coefficient is, TABLE II., 10 feet nearly. If r be known separately, and also s, as well as the head H, and the length of the pipe l, at first

$$\frac{\text{H}}{l} = s, \text{ and, therefore, } \frac{\text{H} - 10}{l} = \frac{h}{l} = s.$$

In EXAMPLE 26, p. 24, H = 150, and l = 100 feet, therefore, the new value of $\frac{h}{l} = \frac{140}{100}$ is 1·4; and as r must be equal ·020833, $r\,s$ = ·02917: the value corresponding to which, in the table, is 20·6, the velocity when allowance is made for the head due to the velocity and orifice of entry.

In general, by taking the value of v for the next less value of $r\,s$ in the table, the velocity will be found with sufficient accuracy, and also the value of $r\,s$ from that of v by taking it as the next greater. If $r\,s = \cdot 0008523$, the table would give $v = 3{\cdot}04$ feet, the same practically as already found in EXAMPLE 27, p. 25.

The value of $r\,s$, when known, determines and fixes the value of v. If r be assumed of any convenient dimensions, s is then determined; and, in like manner, any suitable value of s determines r; thus:

$$\frac{r\,s}{r} = s, \text{ and } \frac{r\,s}{s} = r.$$

It is well to remark, here again, that for pipes the value of r is the fourth part of the diameter d, and that

$$r = \frac{d}{4}, \text{ and } 4\,r = d.$$

M. DARCY in 1857, inspecteur des ponts et chaussées, published his "Recherches expérimentales relatives au Mouvement de l'Eau dans les Tuyaux,"* the result of 198 experiments, in which the velocities varied from ·03 to 5 or 6 metres per second, or from $1\frac{1}{5}$ inch to 16 or 19 feet, and with pipes varying from $\frac{1}{2}$ inch to 20 inches diameter. The formula by which he presents the results is, in metres,

$$(a.) \qquad \text{R J} = b_1 \, \text{U}^2,$$

in which R is the radius of the pipe, J the hydraulic inclination, b_1 a variable coefficient dependent on the circumstances, and U the velocity per second. For wrought and cast iron pipes of the same state of bore, the value of b_1 is expressed by M. Darcy, by the equation

---

* Mémoires présentés par divers savants à l'Académie des Sciences de l'Institut impérial de France, tome XV., Paris, 1858.

$$(b.) \qquad b_1 = \cdot000507 + \frac{\cdot00000647}{\mathrm{R}},$$

the agreement between which and experiment is shown in the following table.

Diameters in English inches.	Diameters in metres.	Value of b_1 from experiments.	Value of b_1 by the formula.	Remarks.
·5	·0122	·001673	·001568	
1·	·0266	·000918	·000993	
1·5	·0395	·000785	·000835	
3·2	·0819	·000695	·000665	
5·4	·1370	·000553	·000601	Well polished bore.
7·4	·1880	·000584	·000576	
11·7	·2970	·000612	·000551	Pipe already in use, but the bore cleaned.
19·7	·5000	·000509	·000532	

For iron coated with bitumen, the value of b_1 in a pipe ·196 metres in diameter was ·0004334; for a newly cast pipe of ·188 metres, b_1 was ·000584; and for a pipe ·2432 metres in diameter, b_1 was ·001168; the relative proportions of b_1 in these three instances, being as

1·1 to 1·5 and to 3;

and, therefore, the velocities, or discharges, would be inversely as the square roots of these, or as

·95 to ·82 and to ·58.

By substituting the notation used in this work for that of M. Darcy, then for measures in metres, from equations (*a*) and (*b*),

$$r\,s = \frac{b_1}{2}\,v^2 = \left\{ \cdot0002535 + \frac{\cdot0000016175}{r} \right\} v^2;$$

which for feet measures becomes (as 1 metre = 3·281 feet)

$$rs = \left\{ \cdot0002535 + \frac{3\cdot281 \times \cdot0000016175}{r} \right\} \times \frac{v^2}{3\cdot281}:$$

hence

$$v^2 = \frac{r\,s}{\cdot00007726 + \frac{\cdot00000162}{r}},$$

and, therefore,

$$v = \left\{ \frac{r\,s}{\cdot00007726 + \frac{\cdot00000162}{r}} \right\}^{\frac{1}{2}}.$$

For all half-inch pipes this becomes

$$v = \left\{ \frac{r\,s}{\cdot00023278} \right\}^{\frac{1}{2}} = 65\cdot5\sqrt{r\,s};$$

for all inch pipes,

$$v = \left\{ \frac{r\,s}{\cdot00015502} \right\}^{\frac{1}{2}} = 80\cdot3\sqrt{r\,s};$$

for all two-inch pipes,

$$v = \left\{ \frac{r\,s}{\cdot00011614} \right\}^{\frac{1}{2}} = 92\cdot8\sqrt{r\,s};$$

for all four-inch pipes,

$$v = \left\{ \frac{r\,s}{\cdot0000967} \right\}^{\frac{1}{2}} = 101\cdot7\sqrt{r\,s};$$

for all six-inch pipes,

$$v = \left\{ \frac{r\,s}{\cdot00009022} \right\}^{\frac{1}{2}} = 105\cdot3\sqrt{r\,s};$$

for all nine-inch pipes,

$$v = \left\{ \frac{r\,s}{\cdot0000859} \right\}^{\frac{1}{2}} = 107\cdot8\sqrt{r\,s};$$

for all twelve-inch pipes,

$$v = \left\{ \frac{r\,s}{\cdot00008374} \right\}^{\frac{1}{2}} = 109\cdot3\sqrt{r\,s};$$

for all eighteen-inch pipes,

$$v = \left\{ \frac{r\,s}{\cdot00008158} \right\}^{\frac{1}{2}} = 110{\cdot}7\,\sqrt{r\,s}\,;$$

for all twenty-four inch pipes,

$$v = \left\{ \frac{r\,s}{\cdot0000805} \right\}^{\frac{1}{2}} = 111{\cdot}5\,\sqrt{r\,s}\,;$$

and when r is large, as for very large pipes and channels, the velocity

$$v = \left\{ \frac{r\,s}{\cdot00007726} \right\}^{\frac{1}{2}} = 113{\cdot}8\,\sqrt{r\,s}$$

is obtained.

There is evidently, on an examination of these results, a great error in the formula of M. Darcy. As long as the diameter of a long pipe continues constant, the velocity is always represented by a given fixed multiple of $\sqrt{r\,s}$, or of the square root of the product of the hydraulic inclination and hydraulic mean depth, no matter how small or great the velocity in the pipe may be. For an inch pipe this multiplier for feet measures is 80·3. Now with a lead pipe the author has found, from several experiments, for a velocity of about 15 feet per second, the multiplier to be 117 or 118; and for a velocity of about 22 feet per second, Mr. Hodson's experiment gives a multiplier of about 120. Taking the other extreme for large pipes, the multiplier derived from M. Darcy's formula is 113·8, no matter how small the velocity may be. But there are experiments in abundance to prove that for velocities of about 12 or 13 inches per second, the multiplier cannot exceed 95. We, therefore, look upon these researches of M. Darcy as partial and defective, and his formula as a representation, at best, of a limited range of velocities, in which those at either side are omitted or not perceived.

For small pipes, any obstruction arising from defective bore, decomposition, encrustation, or from diminished bore, affects the discharge much more considerably than the same obstructions in a large pipe. In order to compare correctly the effects of the state of the bore on the discharge, pipes of exactly the same diameter must be used, and the value of b_1 determined from experiments in which the velocity is the same, otherwise the results, as deduced by M. Darcy and given by Morin, cannot be depended upon.

A few examples, taken at discretion, are given to show how limited this formula must be in its application.

1. Couplet's experiment, No. 43, p. 217, reduced to feet, gives $r = \cdot 3997$ feet, $s = \cdot 0035$, $r\,s = \cdot 001339$, and the observed velocity $v = 3\cdot 478$ feet $= 95\sqrt{r\,s}$ nearly. Darcy's formula would give $v = 110\cdot 8\sqrt{r\,s}$, the author's formula $106\sqrt{r\,s}$ nearly, and Weisbach's $105\sqrt{r\,s}$ nearly. The pipe was probably an old one, and a deduction of about 10 per cent. might be made for the state of the bore. Here, however, there is no means of judging the effect of a change of inclination on the multiplier m, table page, 231.

2. From Du Buât's experiments with an inch pipe, nearly, Nos. 50 and 51, p. 217, after reducing them to feet, in experiment 50, $r = \cdot 0222$, $s = \cdot 228$ and $v = 6\cdot 33$ feet $= 89\cdot 2\sqrt{r\,s}$; or, after making the necessary deductions in the head for the velocity and the orifice of entry with the coefficient $\cdot 815$, $s = \cdot 147$

and $v = 6{\cdot}33$ feet $= 111{\cdot}4 \sqrt{r\,s}$. In experiment 51, in feet $r = {\cdot}0222$, $s = {\cdot}3074$, and $v = 7{\cdot}54 = 92 \sqrt{r\,s}$; or, by making allowance for the head due to the velocity and the orifice of entry, as before, $s = {\cdot}179$, and $v = 7{\cdot}54$ feet $= 119{\cdot}7 \sqrt{r\,s}$. Here it is seen how the velocity, or value of the inclination, s, affects the value of the multiplier, the diameter remaining constant. M. Darcy's formula, in each case, would only make $v = 80{\cdot}2 \sqrt{r\,s}$.

3. In the excerpt proceedings of the Institution of Civil Engineers, p. 4, 6th February, 1855, James Simpson, president, in the chair, there is given for the "Colinton pipe" 16 inches diameter, eight or nine years in use, three observations. First, 29,580 feet long, a head of 420 feet and a discharge of 571 cubic feet per minute: these give $v = 6{\cdot}816$ feet $= 99{\cdot}2 \sqrt{r\,s}$ nearly. Secondly, a length of 25,765 feet a head of 184 feet, and a discharge of 440 cubic feet per minute: these give $v = 5{\cdot}252$ feet $= 96{\cdot}3 \sqrt{r\,s}$. And thirdly, a length of 3,815 feet a head of 184 feet, and a discharge of 1,215 cubic feet per minute: these give $v = 14{\cdot}5$ feet $= 115 \sqrt{r\,s}$ nearly. In these three examples, the diameter, castings, and age of the pipes are the same. Yet it is seen, clearly, that the inclination affects the multiplier of $\sqrt{r\,s}$, which increases with the inclination, s, although M. Darcy's formula would make the multiplier the same in each case, and for all inclinations, viz. $v = 110 \sqrt{r\,s}$. Making those allowances inseparable from the state of the pipe, and all experimental observations, these results, as well as those from Du Buât's experiments, confirm the accuracy

of the author's general formula (119A), page 230, and those others that have been given following it, as well also as that of Weisbach.

Dr. Young's formula, page 222, bears a resemblance to that of M. Darcy, in making the multiplier of $\sqrt{r\,s}$ depend only on the diameter; but it works in a contrary manner: for the high velocities being derived from pipes with small diameters, in the experiments at his command, the value of c in $v = c\sqrt{r\,s}$, reduced from his formula, becomes larger in general for small than for larger diameters. No doubt an allowance should be made in small pipes for a thin film of water adjoining the pipe with little or no velocity; but within the limits with which the engineer has to deal, this may be neglected. Its effect, as well as that of all the other resistances, junctions, contractions, deposits, &c., is greater in pipes of small bore than in larger ones.

COEFFICIENTS DUE TO THE ORIFICE OF ENTRY.—THREE PROBLEMS.

Unless where otherwise expressed, the head due to the velocity and orifice of entry is not considered in the preceding equations. In equation (74), where it is taken into calculation generally,

$$v = \left\{ \frac{2\,g\,\text{H}}{1 + c_r + c_f \times \frac{l}{r}} \right\}^{\frac{1}{2}}$$

in which $1 + c_r$ is equal to $\left(\frac{1}{c_v}\right)^2$, c_r being the coefficient of resistance due to the orifice of entry, and c_v the coefficient of velocity or discharge from a short tube.

If the tube project into the reservoir, and be of small thickness, c_v will be equal ·715 nearly, and therefore $c_r = $ ·956; if the tube be square at the junction, the mean value of c_v will be ·814, and therefore $c_r = $ ·508; and if the junction be rounded in the form of the contracted vein, c_v is equal to unity very nearly, and $c_r = 0$. For other forms of junction the coefficients of discharge and resistance will vary between these limits, and particular attention must be paid to their values in finding the discharge from shorter tubes and those of moderate lengths; but in very long tubes $1 + c_r$ becomes very small compared with $c_f \times \frac{l}{r}$, and may be neglected without practical error. These remarks are necessary to prevent the misapplication of the tables and formulæ, as the height due to the velocity and orifice of entry is an important element in all calculations for short tubes.

It is considered unnecessary to give any formulæ for finding the discharge itself, because, the mean velocity once determined, the calculation of the discharge from the area of the section is one of simple mensuration; and the introduction of this element into the three problems to which this portion of hydraulic engineering applies itself, renders the equations of solution complex, though easily derived; and presents them with an appearance of difficulty and want of simplicity which excludes them, nearly altogether, from practical application. The three problems are as follows:—

I. *Given the fall, length, and diameter of a pipe or hydraulic mean depth of any channel, to find the discharge.*

Here all that is necessary is to find the mean velocity of discharge, which, multiplied by the area of the section (equal $d^2 \times \cdot 7854$ in a cylindrical pipe), gives the discharge sought. TABLE VIII. gives the velocity at once for long channels, according to Du Buât, or it can be found from equation (119A) by calculation. TABLE IX. gives the discharge in cubic feet per minute for different diameters of pipes, and velocities in inches per second, when found from TABLE VIII., or formula (119A). See also TABLES XI. and XII. FOR A PIPE 6 INCHES IN DIAMETER, THE VELOCITY PER SECOND IS PRACTICALLY EQUAL TO THE DISCHARGE IN CUBIC FEET PER MINUTE. See also the tables at pp. 28, 29, 270, and 271.

II. *Given the discharge and cross section of a channel, to find the fall or hydraulic inclination.*

If the cross section be circular, as in most pipes, the hydraulic mean depth is one-fourth of the diameter; in other channels it is found by dividing the water and channel line of the section, wetted perimeter, or border, into the area. The velocity is found by dividing the area into the discharge, and reducing it to inches per second; then in TABLE VIII., under the hydraulic mean depth, find the velocity, corresponding to which the fall per mile will be found in the first column, and the hydraulic inclination in the second. This result can be corrected by trial and error to accord with formula (119A), and the table for the values of r s and v, p. 234, calculated from it. See also the tables, pp. 28, 29, 270, and 271.

III. *Given the discharge, length, and fall, to find* *the diameter of a pipe, or hydraulic mean depth and dimensions of a channel.*

This is the most useful problem of the three. Assume any mean radius r_a, and find the discharge D_a by Problem I. Then for cylindrical pipes

$$r_a^{\frac{5}{2}} : r^{\frac{5}{2}} :: D_a : D :: 1 : \frac{D}{D_a};$$

and as r_a, D, and D_a, are known, $r^{\frac{5}{2}}$ becomes also known, and thence r. Table XIII. will then assist to find r with great facility. Thus, if $r_a = 1$ and D_a was found 15, D being 33, then

$$1 : r^{\frac{5}{2}} :: 1 : \frac{33}{15} :: 1 : 2{\cdot}2, \text{ therefore } r^{\frac{5}{2}} = 2{\cdot}2;$$

and thence by Table XIII., $r = 1{\cdot}37$, the mean radius required, four times which or 5:48 is the diameter of the pipe. For other channels, the quantity thus found must be the hydraulic mean depth; and all channels, however varied in the cross section, will have the same velocity of discharge, when the fall, length, and hydraulic mean depth are constant. If r_a be assumed equal to $1\frac{1}{2}$ inch, the velocity found from Table VIII. will then be the discharge in cubic feet per minute nearly, and this "mean radius" can always be assumed for the first term of the proportion. See also the tables, pp. 28, 29, 270, and 271.

In order to find the dimensions of any polygonal channel whatever, which will give a discharge equal to D, assume any channel similar to that proposed, one of whose known sides is s_a, and find the corresponding discharge, D_a, by Problem I., or from Tables XI. and XII.; then, if the like side of the required channel, be s, there results the equation $\text{s} = s_a\left(\frac{\text{D}}{\text{D}_a}\right)^{\frac{2}{5}}$, and thence the numerical value from TABLE XIII. The result can be corrected, as before, to accord with any of the formulæ by the method of trial and error.

As it frequently happens that deposits in and encrustations on a pipe take place from time to time, which diminish the flowing section considerably, it is always prudent, when calculating the necessary diameter, to take the largest coefficient of friction, c_f, or to double its mean value, particularly for small pipes, when calculating the diameter from any of the formulæ. Some engineers, increase the quantity of water by one-half to find the diameter; but much must depend on the peculiar circumstances of each case, as sometimes less may be sufficient, or more necessary. The discharge increases in similar figures, nearly as $r^{\frac{5}{2}}$ or as $d^{\frac{5}{2}}$, that is, as the square root of the fifth power of the diameter, and the corresponding increase in the diameter for any given or allowed increase in the discharge can be easily found by means of TABLE XIII., as shown above. If the dimensions be increased by one-sixth, the discharge will be increased by one-half nearly; and by doubling the dimensions the discharge is increased in the proportion of $5\frac{2}{3}$ to 1.

For shorter pipes, it is necessary to take into consideration the head due to the velocity and orifice of entry. Taking the mean coefficient of velocity or discharge, the head due to the velocity and orifice of entry, if it be known is found from TABLE II.; this subtracted from the whole head, H, leaves the head, h_f, due to the hydraulic inclination, which is that to be made use of in TABLE VIII. If the velocity be not given, it can be found approximately; then the head found for this velocity, due to the orifice of entry, when deducted, as before, will give a close value of h_f, from which the velocity may be determined with greater accuracy, and so on to any degree of approximation. In general, one approximation to h_f will be sufficient, unless the pipes be very short, in which case it is best to use equation (74). EXAMPLE VIII., p. 208, and the explanation of the use of the tables, SECTION I., may be usefully referred to.

TABLES XI., XII., and XIII. assist to solve with considerable facility all questions connected with discharge, dimensions of channel, and the ordinary surface inclinations of rivers. The discharge corresponding to any intermediate channels or falls to those given in TABLES XI. or XII., will be found with abundant accuracy, by inspection and simple interpolation; and in the same manner the channels from the discharges. Rivers have seldom greater falls than those given in TABLE XII., but in such a case, it is only to divide the fall by 4, then twice the corresponding discharge will be that required. TABLE XIII. gives the comparative discharging powers of all similar channels, whether pipes or rivers, and the comparative dimensions from the discharges. It will be perceived from it, that an increase of one-third in the dimensions doubles, and a decrease of one-fourth reduces the discharge to one-half. By means of this table, and by a simple proportion, the dimensions of any given form of channel when the discharge is known can be determined. See EXAMPLE 17, p. 17. See also the tables pp. 28, 29, 270, and 271.

The mean widths in Tables XI. and XII. are calculated for rectangular channels, and those having side slopes of 1⅓ to 1. Both these tables are, however, practically, equally applicable to any side slopes from 0 to 1 up to 2 to 1, or even higher when the mean widths are taken and not those at top or bottom. A semihexagon of all trapezoidal channels of equal area has the greatest discharging power, and the semisquare and all rectangles exactly the same as channels of equal areas and depths with side slopes of 1⅓ to 1. The maximum discharge is obtained between these for the semihexagon with side slopes of nearly ½ to 1, but for equal areas and depths *the discharge decreases afterwards as the slope flattens.* The question of "HOW MUCH?" is here, however, a very important one; for, as already pointed out in equations (28) and (31), the differences for any practical purposes may be immaterial. This is particularly so in the case of channels with different side slopes, if, instead of the top or bottom, the mean width is made use of to calculate from. Then it is only to subtract the ratio of the slope multiplied by the depth to find the bottom, and add it to find the top. If the mean width be 50 feet, the depth 5 feet, and the side slopes 2 to 1, then $50 - (2 \times 5) = 40$ for the bottom, and $50 + (2 \times 5) = 60$ for the top width.

Side slopes of 2 to 1 present a greater difference from the mean slope of 1⅓ to 1, than any others in general practice when new cuts are to be made. A triangular channel having slopes of 2 to 1, and bottom equal to zero, differs more in its discharging power from the half square, equal to it in depth and area, than if the bottom in each was equally increased, yet even here it is easy to show that this maximum difference is only 5½ per cent. If the bottom be increased so as to equal the depth, it is only 4½ per cent.; when equal to twice the depth, 3·8 per cent.; and when equal to four times the depth, to 2 per cent.; while the differences in the dimensions taken in the same order are only 2·2, 1·8, 1·5, and 0·8 per cent. For greater bottoms in proportion to the depth the differences become of no comparative value. It therefore appears pretty evident, that TABLES XI. and XII. will be found *equally applicable to all side slopes from* 0 *to* 1 *up to* 2 *to* 1, *by taking the mean widths.* When new cuts are to be made, there is no reason whatever in starting from bottom rather than mean widths, to calculate the other dimensions; indeed the necessary extra tables and calculations involved ought entirely to preclude us from doing so. Besides, the formulæ for finding the discharge vary in themselves, and for different velocities the coefficient of friction also varies.* Added to which the inequalities in every river channel, caused by bends and unequal regimen, preclude altogether any regularity in the working slopes and bottom, though the mean width would continue pretty uniform under all circumstances.

---

* The coefficient m in the formula $v = m\,(r\,s)^{\frac{1}{2}}$ in rivers for velocities from 3 inches to 3 feet per second, varies from about 72 to 103; yet, strange to say, most tables are calculated from one coefficient alone; or, rather, from a formula equivalent to $94{\cdot}17\,(r\,s)^{\frac{1}{2}}$, which gives results suited to a velocity of 16 inches only. Dimensions of channels calculated by means of this formula are too small in one case, and too large in the other. In pipes, the variation of the coefficients is shown in the small tables, pp. 229 and 231.

The quantities in Table XII. are calculated, from the velocities found from Table VIII., to correspond to a channel 70 feet wide and of different depths, the equivalents to which are given in Table XI. In order to apply these tables generally to all open channels, the latter are to be reduced to rectangular ones of the same depth and mean width, or the reverse, as already pointed out. If the dimensions of the given channel be not within the limits of Table XI., divide the dimensions of the larger channels by 4, and multiply the corresponding discharge found in Table XII. by 32; for smaller channels, multiply the dimensions by 4, and divide by 32. In like manner, if the discharge be given and exceed any to be found in Table XIII., divide by 32, and multiply the dimensions of the suitable equivalent channel found in Table XI. by 4. If it be desirable to find equivalent channels of less widths than 10 feet for small discharges, multiply the discharge by 32, and divide the dimensions of the corresponding equivalent by 4. Many other multipliers and divisors as well as 4 and 32 may be found from Table XIII., such as 3 and 15·6, 6 and 88·2, 7 and 130, 9 and 243, 10 and 316, 12 and 499, &c. The differences indicated at pages 212 and 213, must be expected in the application of these rules, which will give, however, dimensions for new channels which can be depended on for doing their duty. The Tables, pp. 270, 271, are also applicable.

It will be seen from Table XIII. that a very small increase in the dimensions increases the discharging power very considerably. Table XII. also shows that a small increase in the depth alone adds very much to the discharge. If in this latter case *a small increase* in the depth, d, be expressed by $\frac{d}{n}$, then it is easy to prove that the corresponding increase in the velocity, v, will be $\frac{v}{2\,n}$; and that in the discharge D, $\frac{3\,\text{D}}{2\,n}$, if the surface inclination continue unchanged; but as it is always observable in rivers that the surface inclinations increase with floods, the differences in practice will be found greater than these expressions make it. As in a large river the surface inclination must be very small, four times the fall will add very little to the sectional area; yet this increase of fall would double the discharge, and thence may be perceived how tributaries can be absorbed into the main channel without any great increase to its depth.

SECTION IX.

BEST FORMS OF THE CHANNEL.—REGIMEN.—VELOCITY.—EQUALLY DISCHARGING CHANNELS.

THE determination of the hydraulic mean depth does not necessarily determine the section of the channel. If the form be a circle, the diameter is four times the mean radius; but, though this form be almost always adopted for pipes, the beds of rivers take almost every curvilineal and trapezoidal shape. Other things being the same, that form of a river channel, in which the area of the cross section divided by the border is a maximum, is the best. This is a

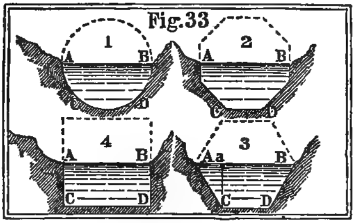


Fig. 33

semicircle having the diameter for the surface line, and in the same manner, half the regular figures, an octagon, hexagon, and square, in Fig. 33, are better forms for the channel, the areas and side slopes being constant, than any others of the same number of sides. Of all rectangular channels, Diagram 4, in which A B C D is half a square, is the best cross section; and in Diagram 3, A C D B, half a hexagon, is the best trapezoidal form of cross section. When the width of the bottom, C D, Diagram 3, is given, and the slope $\frac{\text{A a}}{\text{C a}} = n$, then, in order that the discharge may be the greatest possible,

$$\text{C a} = \left\{ \frac{\text{A}}{2\,(n^2 + 1)^{\frac{1}{2}} - n} \right\}^{\frac{1}{2}}, \text{ and}$$

$$\text{C D} = \frac{\text{A}}{\text{C a}} - n \times \text{C a}$$

$$= \{[2\,(n^2 + 1)^{\frac{1}{2}} - n] \times \text{A}\}^{\frac{1}{2}} - n \left\{ \frac{\text{A}}{2\,(n^2 + 1)^{\frac{1}{2}} - n} \right\}^{\frac{1}{2}},$$

in which A is the given area of the channel. As, however, a river has never been known in which the slope of the natural banks continued uniform, even although made so for any improvements, it is not necessary to give tables for different values of n. If, notwithstanding, ϕ be put for the inclination of the slope A C, equal angle C A a; then as $\cot.\ \phi = n$, and $\sqrt{n^2 + 1} = \frac{1}{\sin.\ \phi}$, the foregoing equations become

(120.) $$\text{C a} = \left\{\frac{\text{A} \sin. \phi}{2 - \cos. \phi}\right\}^{\frac{1}{2}} = \frac{\text{C D}}{2\left\{(n^2 + 1)^{\frac{1}{2}} - n\right\}};$$

and

(121.) $$\text{C D} = \frac{\text{A}}{\text{C a}} - \text{C a} \times \cot. \phi,\text{*}$$

which will give the best dimensions for the channel when the angle of the slope for the banks is known.

When the discharge from a channel of a given area, with given side slopes, is a maximum, it is easy to prove that THE HYDRAULIC MEAN DEPTH MUST BE HALF OF THE CENTRAL OR GREATEST DEPTH. This simple principle gives the construction of the best form of channel with great facility. *Describe any circle on the drawing-board; draw the diameter and produce it on both sides, outside the circle; draw a tangent to the lower circumference parallel to this diameter, and draw the side slopes at the given inclinations, touching the circumference also on each side and terminating on the parallel lines: the trapezoid thus formed will be the best form of channel, and the width at the surface will be equal to the sum of the two side slopes.* It is easy to perceive that this construction may be, simply, extended for finding the best form of a channel having any polygonal border whatever of more sides than three and of given inclinations.

Commencing with the best discharging form of channel, which in practice will have the mean width, about double the depth; an equally discharging section of double the width of the first will have the contents one-eleventh greater, and the depth less in the proportion of 1 to 1·85. A channel of double the mean width of the second must have the sectional area further increased by about one-fifth, and a further decrease in the depth from 1·67 to 1 nearly. The greater expanse of the excavation at greater depths will, in general, more than counterbalance these differences in the contents of the channel. When the banks rise above the flood line, and are unequal in their section, the wider channel involves further upper extra cutting, but there is greater capacity to discharge extra and extraordinary flooding, the banks are less liable to slip or give way, the slopes may be less, and the velocity being also less, the regimen will, in general, be better preserved. The table of equally discharging channels, p. 270, will afford the means of calculating the difference of the cubical contents.

---

* When $\text{C D} = 0$. The channel is triangular; and $\text{A} = \text{C a}^2 \times \cot. \phi$ and $\text{C a} = \left(\frac{\text{A}}{\cot. \phi}\right)^{\frac{1}{2}}$.

When the sectional area is given, the following table shows that the semicircle is the best discharging channel, and the complete circle the worst; the latter is so, however, only compared with the *open* channels given in the table, it being the best form for an *enclosed* channel flowing full. *The best form of an open channel is particularly suited for new cuts in flat, marsh, callow, and fen lands, in which it is also often advisable to cut them with a level bed, up from the discharging point,* in order to increase the hydraulic mean depth, and consequently the velocity and discharge.

As the quantity of water coming down a river channel in a season varies very considerably,—the author has observed it in one case to vary from one to

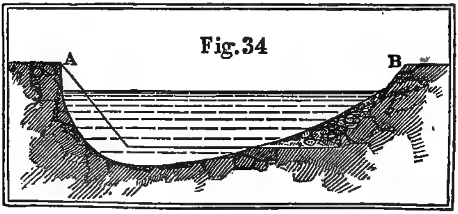
TABLE OF THE RELATIVE DIMENSIONS OF MAXIMUM DISCHARGING CHANNELS.

Angle of slope.	Engineering slope.	Depth in terms of the area.	Bottom in terms of the area.	Top in terms of the area.	Hydraulic mean depth in terms of the area.	Ratio of bottom to depth.	Ratio of top to depth.	Area in terms of the depth *d*.
90° 0′	0 to 1	·707$\sqrt{A}$	1·414$\sqrt{A}$	1·414$\sqrt{A}$	·354$\sqrt{A}$	2· to 1	2· to 1	$2d^2$
63 26	½ to 1	·759$\sqrt{A}$	·938$\sqrt{A}$	1·697$\sqrt{A}$	·379$\sqrt{A}$	1·236 to 1	2·236 to 1	1·736d^2
48 34½	¾ to 1	·748$\sqrt{A}$	·675$\sqrt{A}$	1·996$\sqrt{A}$	·374$\sqrt{A}$	·902 to 1	2·667 to 1	1·784d^2
55 0	1 to 1	·740$\sqrt{A}$	·613$\sqrt{A}$	2·093$\sqrt{A}$	·370$\sqrt{V}$	·828 to 1	2·828 to 1	1·828d^2
36 52	1⅓ to 1	·707$\sqrt{A}$	·471$\sqrt{A}$	2·357$\sqrt{A}$	·354$\sqrt{A}$	·667 to 1	3·333 to 1	$2d^2$
33 41½	1½ to 1	·689$\sqrt{A}$	·417$\sqrt{A}$	2·484$\sqrt{A}$	·345$\sqrt{A}$	·605 to 1	3·605 to 1	2·105d^2
30 58	1⅔ to 1	·671$\sqrt{A}$	·372$\sqrt{A}$	2·608$\sqrt{A}$	·336$\sqrt{A}$	·554 to 1	3·888 to 1	2·221d^2
26 34	2 to 1	·636$\sqrt{A}$	·300$\sqrt{A}$	2·844$\sqrt{A}$	·318$\sqrt{A}$	·472 to 1	4·472 to 1	2·472d^2
Semicircle	Curved	·798$\sqrt{A}$	·000	1·596$\sqrt{A}$	·399$\sqrt{A}$	·000 to 1	2 to 1	1·571d^2
Circle	Curved	1·128$\sqrt{A}$	·000	·000	·282$\sqrt{A}$	·000 to 1	0 to *d*	·785d^2

NOTE.—Half the top will give the length of the side slope. The top itself is the sum of the side slopes, and the border the sum of the top and bottom. The sum of the side slopes in any channel whatever is equal to the number corresponding to the slope in the eighth column multiplied by the depth ; and the sum of the triangular side areas, corresponding to the slopes, is equal to the square of the depth multiplied by the numbers corresponding to the ratio in column 2. The bottom multiplied by the depth added to this gives the area of the section. The top of the best form of channel should be at the level of the extraordinary flood line, and not at that of the mean annual surface.

thirty, and occasionally in the same channel from one to seventy-five,—the proportion of the water section to the channel itself must also vary, and those relations of the depth, sides, and width to each other, above referred to, cease to hold good, and be the best under such circumstances. If the object be to construct a mill-race, temporary drain for unwatering a river, or other small channel, in which the depth remains nearly constant, channels of the form of a half hexagon, diagram 3, Fig. 33, will be, perhaps, the best, if the tenacity of the banks permit the slope; but rivers, in which the quantity of water varies considerably, require wider channels in proportion to the depth; and also, that the velocity be so proportioned to the tenacity of the soil, or as it is termed "*the regimen*," that the banks and bed shall not vary from time to time to any injurious extent, and that any deposits made during their summer state, and during light freshes, shall be carried off periodically by floods. Another circumstance, also, modifies the effects of the water on the banks. It is this, that at curves, and turns, the current acts with greatest effect against the bank, concave to the direction in which it is moving; deepening the channel there; undermining also the bank, as at A, Fig. 34; and raising the bed to the opposite side B. The reflection of the current to the opposite bank from A acts also

Fig. 34



in a similar manner, lower down, upon it; and this natural operation proceeds, until the number of turns, increased length of channel, and loss of head from reflexion and unequal depths, bring the currents into regimen with the material in the bed and banks. At all bends it is, therefore, prudent to widen the channel on the convex side B, and protect the concave side A, Fig. 34, in order to reduce the velocity and its effects; and if the bed be here also sunk below its natural inclination, as it may be seen in most rivers at bends, the velocity will be farther reduced, and the permanence of the bed better established.

The circumstances to be considered in deciding on the dimensions and fall of a new river course, after the depth to which the surface of the water is to be brought has been decided on, are the following:—

The mean velocity must not be too slow, or aquatic plants will grow, and deposits take place, reducing the sectional area until a new and smaller channel is formed within the first with just sufficient velocity to keep itself clear. This velocity should not in general be less than from ten to fourteen inches per second. The velocity in a canal or river is increased very considerably by cutting or removing reeds or aquatic plants growing on the sides or bottom.*

---

* "M. Girard a fait observer, avec raison, que les plantes aquatiques, qui croissent toujours sur le fond et sur les berges des canaux, augmentent considérablement le perimètre mouillé, et par suite la résistance; il a rapellé que Du Buât, ayant mesuré la vitesse de l'eau dans le canal du Jard, avant et après la coupe des roseaux dont il était garni, avait trouvé un resultat bien moindre avant qu'après. En conséquence, il a presque doublé la pente donnée par le calcul . . ."—Traité

The mean velocity must not be too quick, and should be so determined as to suit the tenacity and resistance of the channel, otherwise the bed and banks will change continually, unless artificially protected; it should not exceed

25 feet per minute in	soft alluvial deposits.
40 ,, ,,	clayey beds.
60 ,, ,,	sandy and silty beds.
120 ,, ,,	gravelly.
180 ,, ,,	strong gravelly shingle.
240 ,, ,,	shingly.
300 ,, ,,	shingly and rocky.
400 and upwards in	rocky and shingly.*

---

d'Hydraulique, p. 135. When the fall does not exceed a few inches per mile, the velocity, as determined from the inclination, is very uncertain, and for this reason it is always prudent to increase the depths and sectional areas of channels in flat lands, as far as the regimen will permit. In such cases the section of the channel should approximate towards the best form. See pp. 192 and 255.

* TABLE OF VELOCITIES OF SOME MOVING BODIES COMPARED WITH THOSE OF RIVERS

Objects in motion.	Miles per hour.	Feet per second.	Objects in motion.	Miles per hour.	Feet per second.
Current of slow rivers .	$\frac{6}{13}$	⅔	Railway trains, German	24	53¼
Currents of ordinary rivers up to . .	1½	2¼	Sound when atmosphere is at 32° Fahr. .	743	1,090
Currents of rapid rivers	7	10¼	Ditto 60° Fahr. . .	765	1,122
Man walking . . .	3	4½	Air rushing into vacuum	850	1,247
Horse trotting . .	7	10¼	Ditto when the barometer stands at 30 inches. . . .	917	1,344
Swiftest race-horse . .	60	88	Common musket-ball .	850	1,247
Moderate winds .	7	10¼	Rifle-ball	1,000	1,467
Storms	36	52¾	Cannon-ball . . .	1,091	1,600
Hurricanes . . .	80	117¼	Bullet discharged from air-gun, air being compressed into the hundredth part of its volume	477	700
Swift English steamboats navigating the channels . . .	14	20½	A point on earth's surface at the equator moving round the axis	1,040	1,525
Swift American river steamers . . .	18	26½	Earth moving round sun	68,182	100,000
Fast sailing vessels . .	12	17½			
Railway trains, English	32	47			
,, ,, American	18	26½			
,, ,, Belgian.	25	36¾			
,, ,, French.	27	39½			

A velocity of 180 feet per minute will remove angular stones the size of an egg. Mr. Phillips, under the Metropolitan Commissioners of Sewers, states that 2½ feet per second, or 150 feet per minute, is sufficient to prevent soil depositing in sewers.

The fall per mile should decrease as the hydraulic mean depth increases, and both be so proportioned that floods may have sufficient power to carry off the deposits, if any, periodically. The proportion of the width to the depth of the channel should not be derived, for new cuts or river courses, from any formula, but taken from such portions of the old channel as approximate in depth and in the inclination of the surface to that proposed. When the depth is nearly half the width, the formula shows, *cæteris paribus*, that the discharge will be a maximum; but as (altogether apart from the question of expense) the quantity of water discharged daily, at different seasons, may vary from one to seventy, and more, and "*the regimen*," has to be maintained, the best proportion between the width and depth of a new cut should be obtained, as stated, from some selected portion of the old channel, whose general circumstances and surface inclination approximate to those of the one proposed; and the side slopes of the banks must be such as are best suited to the soil. The resistance of the banks to the current being in general less than that of the beds, which get covered with gravel, and the necessary provision required for floods, appears to be the principal reason why rivers are in general so very much wider than about twice the depth, the relation which gives the minimum of friction.

The following Table is given by Rennie, as an approximation, generally, to the actual state of rivers.* The surface inclinations, however, given in this table for the first and second classes, are very considerable for large rivers, and would give velocities which would effectually scour them. For a hydraulic mean depth of 12 feet, the velocity, with a fall of $\frac{1}{12000}$, would be 2 feet 8 inches per second by Du Buât's formula; and 3·3 feet per second by our formula. The description, therefore, can only apply to smaller channels. In fact, 4 inches to a mile, or $\frac{1}{15740}$, is a considerable inclination for a large river. From Carrick-on-Shannon to Killaloe, a distance of 110 miles, the average fall is only about 4 inches per mile on the river Shannon; and the portion between Athlone and the river Suck below Shannon bridge the fall varies from ·7 to $1\frac{1}{3}$ inch per mile. The Table of the "Falls on the Shannon" (p. 262) explains practically the defects in Rennie's Table, or of any tabular arrangement that omits the size and hydraulic mean depth of the river channel. The mean velocity and quantity flowing remaining the same, the hydraulic mean depth increases as the surface inclination decreases, and in the same ratio. The increase of surface inclination and of velocity are the indices of obstructions in the channel, with this difference, that the obstructions are caused by the velocity where the surface inclination is generally steep; but the obstructions cause the increase of velocity where the inclination is generally flat.

---

* Report to the British Association, 1834

DISTINCTIVE ATTRIBUTES OF THE VARIOUS KINDS OF RIVERS.	Rates of classes of rivers and flowing waters.	Comparative degrees of the mean velocities of currents.	Seconds of time in which currents run 20 fathoms.	Fathoms run by the current per minute of time.	Ratios of declivity compared with horizontal length.	Fathoms of length for each one-twelfth inch of declivity.
Channels wherein the resistance from the bed, and other obstacles, equal the quantity of current acquired from the declivity; so that the waters would stagnate therein, were it not for the compression and impulsion of the upper and back waters .	1st.	0	0″	0	$\frac{1}{12000}$	14
Artificial canals in the Dutch and Austrian Netherlands . .	2nd.	$\frac{2}{3}$	180	$6\frac{2}{3}$	$\frac{1}{7000}$	8
Rivers in low, flat countries, full of turns and windings, and of a very slow current, subject to frequent and lasting inundations	3rd.	1	120	10	$\frac{1}{5200}$	6
Rivers in most countries that are a mean between flat and hilly, which have good currents, but are subject to overflow; also the upper parts of rivers in flat countries	4th.	$1\frac{1}{2}$	80	15	$\frac{1}{4000}$	$4\frac{2}{3}$
Rivers in hilly countries with a strong current, and seldom subject to inundations; also all rivers near their sources have this declivity and velocity, and often much more . . .	5th.	$2\frac{1}{6}$	55	$21\frac{2}{3}$	$\frac{1}{3200}$	$3\frac{2}{3}$
Rivers in mountainous countries having a rapid current and straight course, and very rarely overflowing	6th.	3	40	30	$\frac{1}{2600}$	3
Rivers in their descent from among mountains down into the plains below, in which plains they run torrent-wise . .	7th.	5	24	50	$\frac{1}{2000}$	$2\frac{1}{3}$
Absolute torrents among mountains	8th.	8	15	80	$\frac{1}{1700}$	2

FALLS THE 22ND AUGUST, 1861 (BY MR. BATEMAN), ON THE SHANNON BETWEEN ATHLONE AND VICTORIA LOCK, MEELICK.

Report, May, 1863.

	Height over upper sill of Victoria Lock.	Fall.	Distance.	Fall per Mile.
	ft. in.	ft. in.	miles	inches
Athlone	16 2	..	..	..
Shannon bridge	14 8	1 6	14¼	1·263
Banagher	12 3	2 5	8½	3·411
Victoria Lock, Meelick	8 10	3 5	4½	9·111

NATURAL FALLS ON THE RIVER SHANNON (BY MR. LYNAM).

Report, April, 1867.

Names of Places.	Falls.		Distances.	Fall per mile.
	In the river.	Over the weirs in moderate floods.		
	ft. in.	ft. in.	miles	inches
From Carrick on Shannon to Jamestown Bridge	0 9	..	5½	1·64
Thence to Jamestown Weir	1 0	..	½	6·00
Fall over Jamestown Weir	..	2 4	..	..
Thence to Albert Lock	1 11	..	3½	6·57
Thence to the head of Roosky Fall	0 2½	..	7	0·36
Thence to Roosky Weir	0 6	..	¾	4·00
Fall over Roosky Weir	..	1 5	..	..
From Roosky Weir to Tarmonbarry Weir	2 6	..	7½	4·00
Fall over Tarmonbarry Weir	..	5 6	..	..
From Tarmonbarry Weir to Lanesboro	1 3	..	7¼	2·07
Thence to Athlone, head of the Fall	0 3	..	17½	0·17
Thence to the Weir	0 4	..	2	2·00
Fall over Athlone Weir	..	2 4	..	..
From Athlone Weir to River Suck.	0 10½	..	15	0·70
Thence to Banagher	1 11½	..	8	2·94
Thence to Counsellor's Ford	0 9	..	2	4·50
Thence to Meelick Weir	1 1	..	2	6·50 See above.
Fall over Meelick Navigation and Eel Weirs	..	6 8	½	..
From Meelick to Portumna	0 7	..	8	0·87
Thence to Killaloe Pier head	0 5½	..	23¾	0·23
Thence to Killaloe Weir head	0 3	..	⅙	18·00
Fall over Killaloe Weir at the head, 2 ft. 6 in. Fall over Killaloe Weir at the lower end, 3 ft. 3 in.	..	2 9	..	..
Thence to Killaloe Bridge	1 1	..	⅙	6·50
Total	15 9	21 0	111	..
In river	..	15 9	..	..
Additional fall at Tarmonbarry left out above to suit the heights of the water there	..	1 1	..	
Whole fall	..	37 10	..	..

The fall from Killaloe to Limerick is about 97 feet in about 15 miles.

The following information with reference to the surface inclinations of the Thames, is also from Rennie's Report on Hydraulics,* as a branch of engineering science.

Names of places	Length.		Fall.		Fall in feet per mile.	Ratio of inclinations.
	miles	fur.	feet	in.		
From Lechlade at St. John's Bridge to Oxford at Folly Bridge	28	0	47	0	1·68	$\frac{1}{3,143}$
From Oxford to Abingdon Bridge	9	0	13	11	1·73	$\frac{1}{3,052}$
From Abingdon to Wallingford Bridge. . . .	14	0	27	4	1·95	$\frac{1}{2,708}$
From Wallingford to Reading Bridge . . .	18	0	24	1	1·31	$\frac{1}{4,030}$
From Reading to Henley Bridge	9	0	19	3	2·14	$\frac{1}{2,467}$
From Henley to Marlow Bridge	9	0	12	2	1·35	$\frac{1}{3,911}$
From Marlow to Maidenhead Bridge	8	0	15	1	1·86	$\frac{1}{2,839}$
From Maidenhead Bridge to Windsor Bridge . .	7	0	13	6	1·93	$\frac{1}{2,736}$
From Windsor to Staines Bridge	8	0	15	8	1·96	$\frac{1}{2,694}$
From Staines to Chertsey Bridge	4	6	6	6	1·44	$\frac{1}{3,667}$
From Chertsey to Teddington-Lock	13	6	19	8	1·45	$\frac{1}{3,641}$
From Teddington-Lock to London Bridge . .	19	0	2	9	·145	$\frac{1}{36,414}$
From London to Yantlet Creek.	40	0	2	1	·052	$\frac{1}{101,537}$
From Lechlade to Yantlet Creek	186	4	218	0		
Deduct. . . .	40	0				
From Lechlade to London .	146	4				

For enclosed channels, the circular form of sewer will have the largest scouring power, at a given hydraulic inclination. For then the area of the sections being the same, the velocity in the circular channel will be a maximum. When the supply is intermittent, and the channel too large, the egg-shaped form with the smaller end for the bottom,—or the sides vertical with an inverted ridge-tile or V bottom for drains,—will have a hydrostatic flushing power to remove soil and obstructions, which a cylindrical channel, only partly full, does not possess; because a given quantity of water rises higher against the same obstruction, or obstacle, to the flow in the pipe. It must be confessed, however, that for small drains and house-sewage, this gain is immaterial, and is at best but effected by a sacrifice of space, material, and friction in the upper part of drains, from 6 to 12 inches in diameter. Besides this, the mere hydrostatic pressure is only intermittent, and during an ordinary, or heavy, fall of rain, the hydrodynamic power is always more efficient in scouring properly-proportioned cylindrical drains; and the workmanship in the form and joints is less imperfect than for more compound forms, as those with egg-shaped and inverted tile bottoms. The moulds and joints of cylindrical stone-ware drains, exceeding 12 inches in diameter, are seldom, however, in large quantities perfect; and the expense would exceed that of brick, stone, or other sufficient drains in many localities.

---

* Report, for 1834, of the British Association.

As to the increased discharging power which it is asserted by some, stone-ware cylindrical drains possess over other ordinary drains, no doubt it is true for small sizes, because the form, jointing, and surface are in general more smooth and circular; and *for sewage matter*,* the friction and adherence to the sides and bottom is less; any advantage from these causes becomes, however, immaterial for the larger sizes, as these can be constructed of brick or stone abundantly perfect to any form, and sufficiently smooth for all practical purposes, for in the larger properly-proportioned sizes the same amount of surface roughness opposed to the sewage matter is, comparatively, of no effect. The judicious inclination and form of the bottom, and properly curved junctions, are the principal points to be attended to. Smaller drains tile-bottomed, with brick or stone sides, and flat-covered, have one great advantage over circular pipes.† They can be opened up, for examination and repairs at any time with facility, and at the smallest expense; but greater certainty must be attached to the working of *small* stone-ware drains than to equally-sized small brick or stone drains, and they will be found, in general, also cheaper. This, however, depends on the locality.

It may be observed in numerous experiments, that

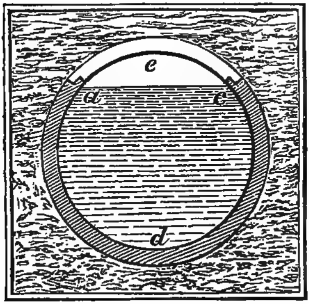
---

* Weisbach found the coefficient of resistance 1·75 times as great for small wooden as for metallic pipes. All *permeable* pipes present greater resistance than *impermeable* ones; hence the principle advantage derived from glazing.

† Half-socket joints at bottom would remedy this imperfection in small pipes, and they could be better laid and cemented. A semi-circular flange laid on at top would effectually protect the joint on the upper side. Latterly Doulton has cut off an upper segment from the pipe, which can be removed for cleaning. And it may be demonstrated, that when this is a segment of 78½ degrees, the lower portion will discharge more than a full pipe at the same inclination.

water flowing from a pipe does not entirely fill the orifice of exit, when the velocities are not considerable, and yet the results are found to be but slightly affected if a little more than three-fourths of the circumference be full. It is easy to demonstrate that the full circle does not give the maximum discharging velocity as has been generally believed, but when filled to the height of the chord *a c* of arc *a e c* of 78½ degrees, and where the velocity is 9½ per cent. over that due to the full circle, for then the $\frac{\text{area } a\,d\,c}{\text{arc } a\,d\,e}$ is a maximum, and the length of the arc *adc* is equal to the tangent of the supplemental arc *a e c*, as may be without difficulty demonstrated. The hydraulic mean depths of the circle and larger segment are to each other as ·5 to ·6, and their square roots, which are as the velocities or scouring powers, are as 1 to 1·095. The discharging powers are to each other as 1 × 3·1416 to 1·095 × 2·946, or as 1 to 1·026, which shows that the segment *a d c* has also a greater discharging power than the whole circle of nearly three per cent. These facts, which were first pointed out by the author, are not unimportant in matters connected with drain-pipes and sewerage. The effects of greater velocity and discharge here pointed out, are sometimes increased, in short pipes, from the fall between the surface *a c*, and the surface from which

Fig. 34*a*.



the head is measured, being greater than the fall to the top of the pipe at *e*, or from the inclination of the surface of the water in the pipe being greater than the inclination of the pipe itself.

EQUALLY DISCHARGING CHANNELS.

In order that different channels should have the same discharging power, the inclination of the surface being the same, the areas must be inversely as the square roots of the hydraulic mean depths. The channel $a\,d\,c\,\mathrm{B}$, Fig. 35, will have the same discharge as the channel A D C B if they be to each other as $\left\{\dfrac{\mathrm{ADCB}}{\mathrm{AD}+\mathrm{DC}+\mathrm{CB}}\right\}^{\frac{1}{2}}$ to $\left\{\dfrac{a\,d\,c\,\mathrm{B}}{a\,d+d\,c+c\,\mathrm{B}}\right\}^{\frac{1}{2}}$,

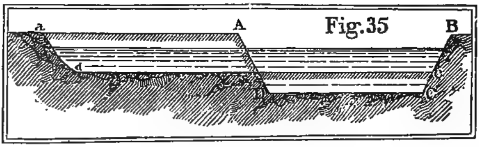


Fig.35

and hence the square root of the cube of the channel area, divided by the border, must be constant. With a fall of one or more feet to a mile, two channels, one 70 feet wide and 1 foot deep, and the other 20 feet wide and 2½ feet deep, will have the same discharge. If *w* be put for the width and *d* for the depth of any rectangular channel, then

$\left\{\dfrac{w^3\,d^3}{w+2\,d}\right\}^{\frac{1}{2}}=m$; and thence the cubic equation

$$d^3-\frac{2\,m^2}{w^3}d=\frac{m^2}{w^2}. \tag{122.}$$

for finding the depth d of any other rectangular channel whose width is w, of the same discharging power. The depths d for different widths of channel have been calculated from this equation, assuming a width of 70 feet and different depths to find m from. The results are given in TABLE XI., which will be found sufficiently accurate for all practical purposes, when the banks are sloped, by taking the mean width. This table is equally applicable to any measures whatever, to their multiples, and sub-multiples.

If the hydraulic inclinations vary, then the $\sqrt{r\,s}$ must be inversely as the areas of the channels when $\sqrt{r\,s} \times$ channel or the discharge is constant; and if the area of the channel and discharge be each constant, r must vary inversely as s; and $r\,s$ be also constant. For instance, a channel which has a fall of four feet per mile, and a hydraulic mean depth of one foot, will have the same discharge as another channel of equal area, having a hydraulic mean depth of four feet, and a fall per mile of only one foot. If in TABLE XII. the same discharge be taken from the columns for different inclinations, the mean rectangular dimensions corresponding to them in the first column will be found, and thereby an engineer be enabled to select an equally discharging channel from TABLE XI., suited to an increase or decrease of the hydraulic inclination.*

The next table at p. 270, of equally discharging river channels, with a primary channel having a mean width of 100, instead of 70, as in Table XI. has been calculated; and in the table at p. 271 are given the discharges at different inclinations from this new primary channel, to find those from its equivalents. The tables at pp. 28, 29, 271, and Table VIII., have also been calculated from Du Buât's formula. For slow velocity of only a few inches per second, the dimensions should be increased by about one-sixth, and the discharges by about one-half.

---

* Tables by the Author similar to numbers XI., XII., and XIII. but on a much more extended scale, have been printed and published on a separate sheet for office use, and may be had from the publisher.

With reference to pipes, it is apparent that a given depth of roughness or contraction arising from any cause will have a greater effect the smaller the diameter becomes. Now in practice, it is necessary to increase the diameter beyond what is found by calculation. For small service pipes half-an-inch is the smallest diameter in general use. For mains and sub-mains the value of c in equation (74B), or at p. 196, should at least be doubled, or the discharge taken at one and a half times its amount to find the diameter. By enlarging the diameter by one-seventh, one-half the amount will be added to the discharge, very nearly; and by increasing the diameter by one-third, the discharge will be doubled. In a broad and practical sense, and considering the losses arising from depositions,* pipes under two inches should have one-third

---

* Mr. Bateman formerly in giving evidence, says :—"He wished to mention a circumstance which might be useful with regard to the spongillæ found in the Dublin water pipes. At Manchester, before the introduction of soft water, the city was supplied with hard water, which favoured the growth of a small fresh-water mussel, which thickly line the reservoirs and pipes. There were myriads of them, and they lay in the pipes as thick as paving stones. These were

TABLE of mean widths and depths of equally discharging trapezoidal River-channels, or Sewers, with side slopes up to 2½ to 1. Practically all river-channels may be reduced to rectangular sections of equal areas and depths to find the discharge. See TABLE XI.

Primary Channel.	Mean rectangular dimensions of equally discharging water-channels or sewers, in any measures whatever, inches, feet, yards, fathoms, or their aliquot parts, or multiples.										Primary Channel.
Mean width 100	Mean width 90	Mean width 80	Mean width 70	Mean width 60	Mean width 50	Mean width 40	Mean width 30	Mean width 20	Mean width 15	Mean width 10	Mean width 100
·1	·11	·12	·13	·14	·16	·18	·22	·29	·35	·47	·1
·125	·13	·14	·16	·17	·20	·23	·28	·37	·45	·60	·125
·2	·21	·23	·25	·28	·32	·37	·45	·60	·73	·98	·2
·25	·27	·29	·32	·35	·40	·46	·56	·75	·92	1·25	·25
·3	·32	·35	·38	·42	·48	·56	·68	·90	1·11	1·52	·3
·375	·40	·44	·48	·53	·60	·70	·85	1·13	1·40	1·94	·375
·4	·43	·46	·51	·56	·64	·74	·91	1·21	1·50	2·08	·4
·5	·54	·58	·64	·71	·80	·93	1·14	1·53	1·90	2·67	·5
·6	·64	·70	·76	·85	·96	1·12	1·37	1·85	2·31	3·28	·6
·625	·67	·73	·79	·88	1·00	1·16	1·43	1·93	2·42	3·44	·625
·7	·75	·81	·89	·99	1·12	1·31	1·61	2·17	2·73	3·92	·7
·75	·80	·87	·95	1·06	1·20	1·41	1·73	2·34	2·95	4·25	·75
·8	·86	·93	1·02	1·13	1·29	1·50	1·85	2·51	3·17	4·59	·8
·875	·94	1·02	1·12	1·24	1·40	1·64	2·02	2·76	3·50	5·10*	·875
·9	·97	1·05	1·15	1·27	1·45	1·69	2·08	2·84	3·61	5·28	·9
1·0	1·07	1·16	1·27	1·42	1·61	1·88	2·32	3·18	4·07	5·99	1·0
1·125	1·21	1·31	1·43	1·60	1·81	2·13	2·63	3·62	4·64	6·92	1·25
1·2	1·29	1·40	1·53	1·70	1·94	2·27	2·81	3·88	5·00	7·50	1·2
1·25	1·35	1·46	1·60	1·78	2·02	2·37	2·94	4·06	5·24	7·89	1·125
1·3	1·40	1·51	1·66	1·85	2·10	2·47	3·06	4·24	5·48	8·29	1·3
1·375	1·48	1·60	1·76	1·96	2·23	2·62	3·25	4·51	5·85	8·89	1·375
1·4	1·50	1·63	1·79	1·99	2·27	2·66	3·31	4·60	5·97	9·10	1·4
1·5	1·61	1·75	1·92	2·14	2·43	2·86	3·56	4·97	6·47	9·92	1·5
1·6	1·72	1·86	2·05	2·28	2·60	3·06	3·81	5·34	6·98	10·78	1·6
1·625	1·75	1·89	2·08	2·32	2·64	3·11	3·87	5·43	7·11	11·00	1·625
1·7	1·83	1·98	2·17	2·43	2·76	3·26	4·06	5·72	7·50*	11·66	1·7
1·75	1·88	2·04	2·24	2·50	2·85	3·36	4·19	5·91	7·77	12·10	1·75
1·8	1·93	2·10	2·30	2·57	2·93	3·45	4·32	6·09	8·03	12·54	1·8
1·875	2·02	2·19	2·40	2-68	3·05	3·60	4·51	6·38	8·43	13·23	1·875
1·9	2·04	2·22	2·43	2·71	3·10	3·65	4·57	6·48	8·57	13·46	1·9
2·0	2·15	2·33	2·56	2·86	3·26	3·86	4·83	6·87	9·11	14·39	2·0
2·1	2·26	2·45	2·69	3·01	3·43	4·06	5·09	7·27	9·67	15·35	2·1
2·2	2·37	2·57	2·82	3·15	3·60	4·26	5·36	7·66	10·23	16·32	2·2
2·3	2·47	2·69	2 95	3·30	3·77	4·46	5·62	8·07	10·80	17·31	2·3
2·4	2·58	2·80	3·08	3·44	3·94	4·87	5·89	8·48	11·38	18·33	2·4
2·5	2·69	2·92	3·21	3·59	4·11	4·67	6·16	8·79	11·97	19·35	2·5
2·6	2·80	3·04	3·34	3·74	4·28	5·08	6·42	9·31	12·57	20·40	2·6
2·7	2·91	3·16	3·47	3·88	4·55	5·28	6·69	9·73	13·17	21·46	2·7
2·8	3·01	3·27	3·60	4·03	4·62	5·49	6·97	10·16*	13·78	22·52	2·8
2·9	3·12	3·39	3·73	4·18	4·79	5·70	7·24	10·59	14·40	23·63	2·9
3·0	3·23	3·51	3·86	4·42	4·96	5·91	7·52	11·02	15·03	24·75	3·0
3·1	3·34	3·63	3·99	4·47	5·13	6·12	7·79	11·46	15·68		3·1
3·2	3·45	3·75	4·13	4·62	5·30	6·33	8·07	11·90	16·32		3·2
3·3	3·55	3·86	4·26	4·77	5·48	6·54	8·35	12·35	16·97		3·3
3·4	3·66	3·98	4·39	4·92	5·65	6·75	8·64	12·80	17·63		3·4
3·5	3·77	4·10	4·52	5·06	5·82	6·96	8·92	13·26	18·29		3·5
3·6	3·88	4·22	4 65	5·21	6·00	7·18	9·21	13·71	18·96		3·6
3·7	3·99	4·34	4·78	5·36	6·17	7·39	9·49	14·18	19·65		3·7
3·8	4·09	4·46	4·91	5·51	6·35	7·60	9·78	14·65	20·34		3·8
3·9	4·20	4·58	5·05	5·66	6·52	7·82	10·07	15·12			3·9
4·0	4·31	4·69	5·18	5·81	6·70	8·04	10·36	15·59			4·0
4·1	4·42	4·81	5·31	5·96	6·87	8·25	10·66	16·07			4·1
4·2	4·53	4·93	5·44	6·11	7·05	8·47	10·95	16·55			4·2
4·3	4·64	5·05	5·57	6·26	7·23	8·69	11·25	17·04			4·3
4·4	4·74	5·17	5·71	6·41	7·40	8·91	11·55	17·53			4·4
4·5	4·85	5·29	5·84	6·56	7·58	9·13	11·85	17·02			4·5
4·6	4·96	5·47	5·97	6·72	7·76	9·35	12·15	18·52			4·6
4·7	5·07	5·53	6·10	6·87	7·94	9·57	12·45	19·02			4·7
4·8	5·18	5·64	6·24	7·02	8·12	9·79	12·75	19·53			4·8
4·9	5·29	5·76	6·37	7·17	8·29	10·02	13·06	20·04			4·9
5·0	5·40	5·88	6·50	7·32	8·47	10·24	13·37				5·0

* A semi-hexagon has the greatest discharging power of all trapezoids of equal depth and area. All rectangles have the same discharging power as trapezoids of the same depth and area, with side slopes of 1½ to 1. The differences for other side slopes are immaterial in practice.

primary Channel in the opposite page, taken in feet; and from the corresponding equivalent Channels, also taken in feet. See TABLE XII.

Discharges in cubic feet per minute. Interpolate for intermediate falls or inclinations: divide greater falls or inclinations by 4, and double the corresponding discharges. If the dimensions be in inches, divide the discharges here given by 500; if in tenths, by 316; if in eighths, by 181; if in sixths, by 88; if in fifths, by 56; if in quarters, by 32; if in thirds, by 15·6; and if in halves, by 5·66. Reverse the operation and multiply for like multiples.

Depths of the primary channel in opposite table, having a mean width of 100; in feet.	4 inches per mile, 1 in 15840.	6 inches per mile, 1 in 10560.	9 inches per mile, 1 in 7040.	12 inches per mile, 1 in 5280.	15 inches per mile, 1 in 4224.	18 inches per mile, 1 in 3520.	21 inches per mile, 1 in 3017·1.	24 inches per mile, 1 in 2640.	30 inches per mile, 1 in 2112.	36 inches per mile, 1 in 1760.	Depths of the primary channel in opposite table, having a mean width of 100; in feet.
·1	110	140	176	207	235	260	284	305	345	383	·1
·125	157	198	250	294	332	373	402	433	490	543	·125
·2	325	409	515	606	686	760	828	891	1,009	1,117	·2
·25	455	575	725	853	966	1,070	1,166	1,256	1,422	1,574	·25
·3	601	760	957	1,125	1,275	1,412	1,539	1,658	1,876	2,076	·3
·375	844	1,068	1,344	1,582	1,793	1,985	2,162	2,329	2,637	2,919	·375
·4	931	1,178	1,484	1,745	1,977	2,189	2,386	2,569	2,900	3,220	·4
·5	1,308	1,653	2,081	2,447	2,775	3,071	3,347	3,606	4,083	4,513	·5
·6	1,721	2,178	2,743	3,227	3,657	4,047	4,410	4,752	5,401	5,956	·6
·625	1,830	2,316	2,917	3,431	3,887	4,303	4,690	5,053	5,795	6,332	·625
·7	2,177	2,750	3,463	4,072	4,614	5,109	5,568	5,999	6,936	7,516	·7
·75	2,414	3,029	3,844	4,520	5,123	5,674	6,180	6,660	7,567	8,342	·75
·8	2,660	3,363	4,236	4,982	5,646	6,253	6,811	7,340	8,309	9,194	·8
·875	3,044	3,850	4,846	5,703	6,463	7,157	7,770	8,401	9,513	10,527	·875
·9	3,175	4,017	5,060	5,951	6,743	7,467	8,082	8,765	9,926	10,984	·9
1·0	3,731	4,711	5,933	6,973	7,903	8,750	9,513	10,273	11,634	12,877	1·0
1·125	4,441	5,614	7,071	8,313	9,421	10,430	11,369	12,216	13,867	15,347	1·125
1·2	4,889	6,186	7,791	9,163	10,381	11,494	12,521	13,494	15,280	16,914	1·2
1·25	5,207	6,582	8,291	9,752	11,048	12,232	13,336	14,361	16,261	18,000	1·25
1·3	5,529	6,981	8,793	10,357	11,718	12,974	14,146	15,234	17,246	19,091	1·3
1·375	6,004	7,591	9,561	11,245	12,734	14,107	15,386	16,576	18,752	20,756	1·375
1·4	6,167	7,797	9,821	11,544	13,087	14,491	15,794	17,031	19,262	21,318	1·4
1·5	6,844	8,653	10,898	12,818	14,524	16,081	17,523	18,917	21,376	23,658	1·5
1·6	7,538	9,520	12,002	14,115	15,994	17,709	19,296	20,829	23,539	26,053	1·6
1·625	7,705	9,741	12,272	14,428	16,348	18,102	19,724	21,286	24,061	26,631	1·625
1·7	8,252	10,432	13,139	15,432	17,509	19,360	21,126	22,780	25,769	28,523	1·7
1·75	8,617	10,893	13,719	16,134	18,282	20,241	22,060	23,776	26,907	29,784	1·75
1·8	8,993	11,369	14,318	16,851	19,079	21,124	23,024	24,821	28,081	31,085	1·8
1·875	9,561	12,088	15,226	17,905	20,287	22,463	24,476	26,372	29,860	33,052	1·875
1·9	9,741	12,316	15,515	18,245	20,672	22,890	24,946	26,872	30,426	33,682	1·9
2·0	10,515	13,297	16,753	19,702	22,320	24,718	26,935	29,019	32,852	36,358	2·0
2·1	11,307	14,300	18,020	21,192	23,991	26,561	29,074	31,213	35,334	39,106	2·1
2·2	12,110	15,314	19,297	22,689	25,708	28,467	31,024	33,424	37,836	41,878	2·2
2·3	12,935	16,357	20,608	24,235	27,456	30,407	33,134	35,694	40,410	44,724	2·3
2·4	13,781	17,425	21,954	25,816	29,250	32,302	35,299	38,022	43,048	47,643	2·4
2·5	14,647	18,520	23,332	27,436	31,087	34,425	37,516	40,407	45,750	50,634	2·5
2·6	15,538	19,645	24,747	29,100	32,974	36,514	39,794	42,856	48,526	53,706	2·6
2·7	16,430	20,773	26,167	30,770	34,867	38,610	42,078	45,316	51,311	56,789	2·7
2·8	17,333	21,915	27,605	32,462	36,784	40,733	44,390	47,809	54,131	59,913	2·8
2·9	18,257	23,084	29,076	34,193	38,744	42,905	46,755	50,359	57,017	63,110	2·9
3·0	19,203	24,280	30,581	35,963	40,750	45,127	49,175	52,968	59,968	66,379	3·0
3·1	20,167	25,498	32,120	37,767	42,794	47,392	51,640	55,634	62,980	69,709	3·1
3·2	21,146	26,737	33,673	39,600	44,871	49,692	54,148	58,327	66,033	73,097	3·2
3·3	22,118	27,969	35,225	41,425	46,939	51,978	56,640	61,017	69,077	76,465	3·3
3·4	23,106	29,220	36,798	43,275	49,036	54,302	59,171	63,745	72,164	79,879	3·4
3·5	24,115	30,497	38,407	45,166	51,180	56,675	61,758	66,534	75,322	83,371	3·5
3·6	25,139	31,795	40,040	47,086	53,356	59,084	64,384	69,366	78,526	86,915	3·6
3·7	26,182	33,116	41,702	49,041	55,572	61,532	67,058	72,249	81,789	90,524	3·7
3·8	27,233	34,446	43,379	51,013	57,807	64,009	69,753	75,158	85,078	94,162	3·8
3·9	28,287	35,777	45,060	52,989	60,046	66,489	72,455	78,061	88,371	97,810	3·9
4·0	29,356	37,128	46,766	54,994	62,318	69,006	75,197	81,012	91,710	101,512	4·0
4·1	30,438	38,495	48,492	57,024	64,616	71,553	77,973	83,999	95,093	105,259	4·1
4·2	31,538	39,884	50,246	59,086	66,950	74,141	80,793	87,033	98,535	109,065	4·2
4·3	32,654	41,294	52,027	61,180	69,327	76,769	83,655	90,116	102,025	112,930	4 3
4·4	33,776	42,712	53,816	63,283	71,709	79,406	86,529	93,209	105,531	116,811	4·4
4·5	34,908	44,138	55,613	65,394	74,100	82,052	89,413	96,318	109,054	120,709	4·5
4·6	36,041	45,579	57,429	67,527	76,500	84,725	92,327	99,460	112,614	124,647	4·6
4·7	37,193	47,034	59,262	69,682	78,955	87,426	95,271	102,632	116,209	128,625	4·7
4·8	38,363	48,514	61,128	71,874	81,438	90,173	98,266	105,860	119,866	132,672	4·8
4·9	39,544	50,009	63,011	74,087	83,944	92,946	101,289	109,119	123,559	136,758	4·9
5·0	40,725	51,507	64,895	76,298	86,450	95,720	104,313	112,376	127,248	140,841	5·0

or more added to their calculated dimensions, and larger pipes from one-third to one-seventh — even after making allowance for junctions, bends, and contractions. For large conduits or channels the allowance need not be so large, if the maximum quantity to be conveyed has been duly estimated.

SECTION X.

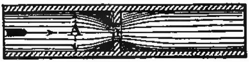
EFFECTS OF ENLARGEMENTS AND CONTRACTIONS. BACK-WATER WEIR CASE.—LONG AND SHORT WEIRS.—THE SHANNON.

When the flowing section in pipes or rivers expands or contracts suddenly, a loss of head always ensues; this is probably expended in forming eddies at the sides, or in giving the water its new section. A side current, moving slowly sometimes *upwards*, may be frequently observed in the wide parts of rivers, when the channel is unequal, though the downward current, at the centre, be pretty rapid; and though it may be assumed generally that the velocities are inversely as the sections, when the channels are uniform, this cannot properly be done when they are not, and the

caused by the large quantity of lime in the water. He was curious to see what would be the effect of passing water without lime. This was done ten or eleven years ago, and the result was that these mussels had entirely disappeared. There was no longer anything from which they could make their shells, and for years, on their discharge, the small pipes were found choked with them. If soft water were supplied to Dublin in place of the present hard water, which probably favoured the growth of spongillæ, they would probably disappear." This has been since done, and Dublin has been supplied, from the Vartry, with most satisfactory results as to quality, quantity, and cost.

motions so uncertain as those referred to. When a pipe is contracted by a diaphragm at the orifice of entry, Fig. 27, it was shown (equation 60), that the loss of head is,

Fig.36



$$(123.)\qquad h = \frac{\left(1 - \frac{\text{A}^2}{\text{C}^2}\right)v^2 + \left(\frac{\text{A}}{a\,c_d} - 1\right)^2 v^2}{2\,g}.$$

When the diaphragm is placed in a uniform pipe, Fig. 36, then A = C, and the loss of head is

$$(124.)\qquad h = \frac{\left(\frac{\text{A}}{a\,c_d} - 1\right)^2 v}{2\,g};$$

and the coefficient of resistance

$$(125.)\qquad c_r = \left(\frac{\text{A}}{a\,c_d} - 1\right)^2,$$

as in equation (67). The coefficient of discharge c_d is here equal to the coefficient of contraction c_c, or very nearly. Now it is shown in equation (45), and the remarks following it, that the value of the coefficient of discharge, c_d, varies according to the ratio of the sections, $\frac{\text{A}}{a}$,* and in TABLE V. we have calculated the new coefficients for different values of the ratios, and different values of the primary coefficient c_d. If c_d, when A is very large compared with a, be ·628, then by attending to the remarks at pp. 101 and 103, it is found, that the different values of c_d corresponding to $\cdot 807 \times \frac{A}{a}$, taken from TABLE V., are those in columns Nos. 2 and 5 of the next small table, the values of the coefficient of resistance, in columns 3 and 6, being calculated from equation (125) for the respective new values of the coefficient of discharge thus found. The table shows that when the aperture in a diaphragm is $\frac{2}{10}$ths of the section of the pipe, that 47 times the head due to the velocity is lost thereby. If the aperture in the diaphragm be rounded at the arrises, the loss will not be so great, as the primary coefficient c_d will then be greater than that due to an orifice in a thin plate: see the TABLE OF COEFFICIENTS, p. 169.

---

* The empirical value of c_d as given by Professor Rankine, is $c_e = \frac{\cdot 618}{\left(1 - \cdot 618\frac{a^2}{\text{A}^2}\right)^{\frac{1}{2}}}$, which is equal to unity when $a = \text{A}$, as it should be; and equal to ·618, when a is very small, compared with A, as it also should be when the diaphragm is a thin plate, *but not otherwise.* If the thickness of the diaphragm be twice the diameter of the orifice a, the coefficient of discharge would be ·815; and if the higher arris be rounded, this would be increased to 1, in which cases the expression would clearly fail; the thickness of the diaphragm and the form of the aperture a must also be considered.

TABLE OF COEFFICIENTS FOR CONTRACTION, BY A DIAPHRAGM IN A PIPE.

$\frac{a}{A}$	c_d	c_r	$\frac{a}{A}$	c_d	c_r
·0	·628	infinite	·6	·713	1·790
·1	·630	221·2	·7	·753	·807
·2	·636	47·1	·8	·807	·301
·3	·647	17·2	·85	·845	·154
·4	·661	7·7	·9	·890	·062
·5	·683	3·7	1	1·000	·000

When there are a number of diaphragms in a tube, the loss of head for each must be found separately, and all added together for the total loss. If the diaphragms, however, approach each other, so that the water issuing from one of the orifices *a*, Fig. 36, shall pass into the next before it again takes the velocity due to the diameter of the pipe, the loss will not be so great as when the distance is sufficient to allow this change to take place. This view is fully borne out by the experiments of Eytelwein with tubes 1·03 inch in diameter, having apertures in the diaphragms of ·51 inch in diameter.

Venturi's twenty-fourth experiment, with tubes varying from ·75 inch to ·934 inch in diameter at the junction with the cistern, so as to take the form of the contracted vein, and expanding and contracting along the length from ·75 to 2 inches and from 2 inches to ·75 inch alternately, shows the great loss of head sustained by successive enlargements and contractions of a channel, even when the junction of the parts is gradual. Calling the coefficient for the short tube, with a junction of nearly the form of the contracted vein, 1, then the following coefficients are derivable from the experiment :—

Short tube with rounded junction	1·
One enlargement	·741
Three enlargements	·569
Five enlargements	·454
Simple tube with a rounded junction of the same length, 36 inches, as the tube with the five enlarged parts	·736

The head, in the experiment, was 32½ inches. Venturi states that no observable differences occurred in the times of discharge when the enlarged portions were lengthened from 3⅙ to 6⅓ inches. See tables, pp. 146 and 199.

With reference to this experiment, so often quoted, it is necessary to remark that the diameters of the enlarged portions were 2 inches each, while the lengths varied only from $3\frac{1}{6}$ to $6\frac{1}{3}$ inches, and consequently were at most only $3\frac{1}{6}$ times the diameter. Now with such a large ratio of the width to the length of the enlarged portions, *a*A B *b*, Fig. 37, it is pretty clear that a good deal of the head is lost by the impact of the moving water on the shoulders at A and B. That this is so is evident from the fact, stated by the experimenter, of the time of discharge remaining the same when *a* A, in five different enlargements, was increased from $3\frac{1}{6}$ to $6\frac{1}{3}$ inches; though this must have lengthened the whole tube from 36 to 50 inches,* thereby increasing the loss from friction proportionately, but which happened to be compensated for by the reduction in the resistances from inpact at A and B, and in the eddies, by doubling the lengths from *a* to A.

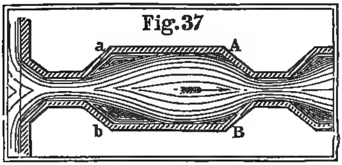


Fig.37

If, however, the length from *a* to A be very large compared with the diameter, and the junctions at *a*, A, B, and *b*, be well grafted, less loss will arise from the enlargement than if the smaller diameter continued all along uniform. The explanation is clear, as the resistance from friction is inversely as the square roots of the mean radii; and the length being the same, the loss must be less with a large than a small diameter.

---

* The dimensions throughout this experiment are given as in the original, viz., in French inches.

These remarks, *mutatis mutandis*, apply equally to rivers as to pipes. The effects of submerged weirs and contracted river channels has been already pointed out at pp. 134 and 141, and formulæ given for calculating them.

BACKWATER FROM CONTRACTIONS IN RIVERS.

A river may be contracted in width or depth, by jetties or by weirs; and when the quantity to be discharged is known, equations from which the increase of head may be found are given in formulæ (9), (55), and (57). The effect of a weir, jetty, or contracted channel of any kind, is to increase the depth of water above; and this is sometimes necessary for navigation purposes, or to obtain a head for mill power. When a weir is to rise over the surface, then from the length, the discharge per minute, the coefficient due to the crest, and the coefficient due to the ratio of the sections, on and above the weir, found from TABLE V., the head can be found from TABLE VI. For submerged weirs and contracted widths of channel, the head can be best calculated, by approximation, from the equations above referred to.

The head once determined, the extent of the backwater is a question of some importance. If F C O D, Fig. 38, be the original surface of a river, and *a* A B F the raised surface by backwater from the weir at *a*, then the extent *a* F of this backwater, in a regular channel, will be from 1·5 to 1·9 times *a* C drawn parallel to the horizon to meet the original surface in C. This rule will be found useful for practical purposes; but in order to determine more accurately the rise for a given

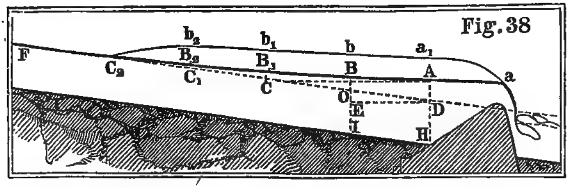


Fig. 38

length, B_1 B_2 or B_1 B, of the channel, it is necessary to commence at the weir and calculate the heights from A to B, B to B_1, and from B_1 to B_2 separately, the distance from A to B_2 being supposed divided into some convenient number of equal parts, so that the lengths A B, B B_1, &c., may be considered free from curvature. Now, as the head A D is known, or may be calculated by some of the preceding formulæ, the section of the channel at the head of the weir also becomes known, and thence the mean velocity in it, by means of the discharge over the weir. Putting A for the area of the channel at A H, d for its depth A H, and v for the mean velocity; also A_1 for the area of the channel at B I, d_1 for its depth, and v_1 for its mean velocity; b_m the mean border between the sections at A H and B I; r_m the mean hydraulic depth; $\frac{v + v_1}{2}$ the mean velocity; $AD = h$; $BO = h_1$; the sine of angle $ODE = s$; and the length $AB = DO$ nearly $= l$; we get $A \times v = A_1 \times v_1$ and $r_m = \frac{A + A_1}{2\, b_m}$; but as, in passing from B to A, the velocity changes from v_1 to v, there is a loss of head equal $\frac{v_1^2 - v^2}{2\,g}$, and if c_f be the coefficient of friction, there is a loss of head from this cause equal $c_f \times \frac{l}{r_m} \times \frac{\left(\frac{v_1 + v}{2}\right)^2}{2\,g}$; hence the whole change of head in passing from B to A is equal to $c_f \times \frac{l}{r_m} \times \frac{(v_1 + v)^2}{8\,g} - \frac{v_1^2 - v^2}{2\,g}$. But this change of head is equal to BE − AD = BO + OE − AD = $h_1 + l\,s - h$, whence

$$(126.)\qquad h_1 - h = d_1 - d = c_f \times \frac{l}{r_m} \times \frac{(v_1 + v)^2}{8\,g} - \frac{v_1^2 - v^2}{2\,g} - l\,s;$$

or as $v_1 = \frac{A\,v}{A_1}$, and $r_m = \frac{A + A_1}{2\,b_m}$, by a few reductions and change of signs,

$$(127.)\qquad h - h_1 = \left(s - c_f \times b_m \times \frac{A + A_1}{2\,A_1^2} \times \frac{v^2}{2\,g}\right) l + \frac{A^2 - A_1^2}{A_1^2} \times \frac{v^2}{2\,g};$$

and therefore

$$(128.)\qquad l = \frac{h - h_1 - \frac{A^2 - A_1^2}{A_1^2} \times \frac{v^2}{2\,g}}{s - c_f \times \frac{b_m \times (A + A_1)}{2\,A_1^2} \times \frac{v^2}{2\,g}},$$

from which the length l corresponding to any assumed change of level between A and B can be calculated. Then, by a simple proportion the change of level for any smaller length can be found. To find the change of level directly from a given length does not admit of a direct solution, for the value of $h - h_1$ in equation (127) involves A_1, which depends again on $h - h_1$, and further reduction leads to an equation of a higher order; but the length corresponding to a given rise, h_1, is found directly by equation (128).

When the width of the channel, w, is constant, and the section equal to $w \times d$ nearly, the above equations admit of a further reduction for $A_1 = d_1 w$ and $A = d w$; by substituting these values in equation (127) it becomes, after a few reductions,

$$(129.)\quad h - h_1 = d - d_1$$
$$= \left(s - c_f \times b_m \times \frac{d+d_1}{2\, d_1^2\, w} \times \frac{v^2}{2\, g}\right) l + \frac{d^2 - d_1^2}{d_1^2} \times \frac{v^2}{2\, g};$$

or, as it may be further reduced,

$$(130.)\quad h - h_1 = \frac{s - c_f \times \frac{b_m}{d_1 w} \times \frac{d + d_1}{2\, d_1} \times \frac{v^2}{2\, g}}{1 - \frac{d + d_1}{d_1^2} \times \frac{v^2}{2\, g}} \times l.$$

Now, in this equation, for all practical purposes,

$$\frac{d + d_1}{2\, d_1} \times \frac{b_m}{d_1\, w} = \frac{b}{d\, w},$$

approximately, b being the border of the section at A H; and also, $\frac{d + d_1}{d_1^2} = \frac{2}{d}$, approximately, therefore

$$(131.)\quad h - h_1 = \frac{s - c_f \times \frac{b}{d\, w} \times \frac{v^2}{2\, g}}{1 - \frac{2}{d} \times \frac{v^2}{2\, g}} \times l;$$

and

$$(132.)\quad l = \frac{(h - h_1) \times \left(1 - \frac{2}{d} \times \frac{v^2}{2\, g}\right)}{s - c_f \times \frac{b}{d\, w} \times \frac{v^2}{2\, g}}.$$

Now, as $\frac{b}{d\,w} = \frac{1}{r}$, $2\,g = 64{\cdot}4$, and the mean value of the coefficient of friction for small velocities $c_f = {\cdot}0078$, then

$$(133.)\qquad h_1 = h - \frac{64{\cdot}4\,d\,s - {\cdot}0078\,\frac{d}{r}\,v^2}{64{\cdot}4\,d - 2\,v^2} \times l;$$

and

$$(134.)\qquad l = \frac{(h - h_1) \times (64{\cdot}4\,d - 2\,v^2)}{64{\cdot}4\,d\,s - {\cdot}0078\,\frac{d}{r}\,v^2},$$

very nearly. Having by means of these equations found A B from B O or B E, and B O from A B, we can in the same manner proceed up the channel and calculate B_1 C, B_2 C_1, &c., until the points B, B_1, B_2 in the curve of the backwater shall have been determined, and until the last nearly coincides with the original surface of the river. When $h_1 = 0$, then

$$h = \frac{64{\cdot}4\,d\,s - {\cdot}0078\,\frac{d}{r}\,v^2}{64{\cdot}4\,d - 2\,v^2} \times l.$$

If equation (134) be examined, it appears that when $64{\cdot}4\;d = 2\,v^2$, l must be equal to zero; or when $\frac{d}{2} = \frac{v^2}{64{\cdot}4}$, equal the height due to the velocity v. When l is infinite, $64{\cdot}4\;d$ must exceed $2\,v^2$, and $64{\cdot}4\;d\,s$ equal to ${\cdot}0078\,\frac{d}{r}\,v^2$;

$$\text{or, } \frac{64{\cdot}4\,r\,s}{{\cdot}0078} = v^2, \text{ and } v = 90{\cdot}9\,\sqrt{r\,s}.$$

This is the velocity due to friction in a channel of the depth d, hydraulic mean depth r, and inclination s;

and, as in wide rivers $r = d$ nearly, $v = 90{\cdot}9 \sqrt{d\,s}$, but when the numerator was zero we had from it $v = \sqrt{32{\cdot}2\,d}$; equating these values of v, $s = {\cdot}0039 = \frac{1}{256}$ nearly: see p. 133. Now, the larger the fraction s is, the larger will the velocity v become; and the larger v becomes, the more nearly, in all practical cases, will the terms

$$64{\cdot}4\,d - 2\,v^2 \text{ and } 64{\cdot}4\,d\,s - {\cdot}0078\frac{d}{r}\,v^2,$$

in the numerator and denominator of equation (134), approach zero; when $64{\cdot}4\,d - 2\,v^2$ becomes zero first, $l = 0$; when $64{\cdot}4\,d\,s - {\cdot}0078\,\frac{d}{r}\,v^2$ becomes zero first, l equals infinity; and when they both become zero at the same time, $l = h - h_1$, and $s = \frac{1}{256}$: see p. 133; if s be larger than this fraction, the numerator in equation (134) will generally become zero before the denominator, or negative, in which cases l will also be zero, or negative; and the backwater will take the form F $c_2\,b_2\,b_1\,b\,a_1\,a$, Fig. 38, with a hollow at c_2. Bidone first observed a hollow, as F $c_2\,b_2$, when the inclination s was $\frac{1}{30}$. When the inclination of a river channel changes from greater to less, the velocity is obstructed, and a hollow similar to F $c_2\,b_2$ sometimes occurs. When the difference of velocity is considerable, the upper water at b_2 falls backwards towards c_2 and F, and forms a *bore,* a splendid instance of which is the *pororoca,* on the Amazon, which takes place where the inclination of the surface changes from six inches to one-fifth of an inch per mile, and the velocity from about 22 feet to 4½ feet per second.

WEIR CASE, LONG AND SHORT WEIRS.

When a channel is of very unequal widths, above a weir, the following simple method of calculating the backwater will be found sufficiently accurate, and the results to agree with observation. *Having ascertained the surface fall due to friction in the channel at a uniform mean section, add to this fall the height which the whole quantity of water flowing down would rise on a weir having its crest on the same level as the lower weir, and of the same length as the width of the channel in the contracted pass. The sum will be the head of water at some distance above such pass very nearly.* A weir was formerly constructed on the river Blackwater, at the bounds of the counties Armagh and Tyrone, half a mile below certain mills, which, it was asserted, were injuriously affected by backwater thrown into the wheel-pits. The crest of the weir, 220 feet long, was 2 feet 6 inches below the pit; the river channel between varied from 50 and 57 feet to 123 feet in width, from 1 foot to 14 feet deep; and the fall of the surface, with 3 inches of water passing over the weir and the sluices down, was nearly 4 inches in the length of half a mile. Having seen the river in this state in summer, the Author had to calculate the backwater produced by different depths passing over the weir in autumn and winter, which in some cases of extraordinary floods were known to rise to 3 feet. The width of the channel about 60 feet above the weir averaged 120 feet. The width, 2050 feet above the weir and 550 feet below the mills, was narrowed by a slip in an adjacent canal bank, to 45 feet at the level of the top of the weir, the average width at this place as the water rose being 55 feet. The channel above and below the slip widened to 80 and 123 feet. Between the mills and the weir there were, therefore, two passes; one at the slip, averaging 55 feet wide; another above the weir, about 120 feet wide. Assuming as above, that the water rises to the heights due to weirs 55 and 120 feet long, at these passes, then, by an easy calculation, or by means of TABLE X., the heads in columns two and four of the table on the next page were found, corresponding to the assumed ones on the weir, given in the first column.

As the length of the river was short, and the hydraulic mean depth pretty large, the fall due to friction for 60 feet above the weir was very small, and therefore no allowance was made for it; even the distance to the slip was comparatively short, being less than half a mile, and as the water approached it with considerable velocity, this was conceived, as the observations afterwards showed, to be a sufficient compensation for the loss of head below by friction. The observations were made by a separate party, over whom the Author had no control, and it is necessary to remark, that with the same head of water on the weir, they often differed more from each other than from the calculation. This, probably, arose from the different directions of the wind, and the water rising during one observation, and falling during another.

The true principle for determining the head at *g*, Fig. 39, apart from that due to friction, is that pointed out at pp. 136 and 141, but when the passes are very near each other, or the depth d_2, Fig. 23, is small, the effect of the discharge through d_2 is inconsiderable in reducing the head, as the contraction and loss of *vis-viva* are then large, and the head d_1 becomes that due to a weir of the width of the contracted channel at A, nearly. The reduction in the extent of the backwater, by lowering the head on a longer weir, is found by taking the difference of the amplitudes due to the heads at *g*, Fig. 39, in both cases, as determined from equations (56), (128), *et seq.* This will seldom exceed a mile up the river, as the surface inclination is found to be considerably greater than that due to mere friction and velocity, and hence the general failure of drainage works designed on the assumption that the lowering

TABLE OF CALCULATED AND OBSERVED HEIGHTS ABOVE M'KEAN'S WEIR, NEAR BENBURB, ON THE RIVER BLACKWATER.

Heights at M'Kean's weir 220 feet long, in inches.	Heights 60 feet above the weir channel 120 feet wide.		Heights 2050 feet above the weir channel 55 feet wide; average.	
	Calculated inches.	Observed inches.	Calculated inches.	Observed inches.
1½	2¼	2¼	4¼	5½
2	...	...	...	...
3	4½	...	7½	7
4	6	...	10	9
5	7½	...	12½	11¼
6	9	9	15	16¼
7	10½	10½	17½	18¾
8	12	—	20	20½
9	13½	12½	22½	20¼
10	15	...	24¼	20
11	16½	...	27¾	24
12	18	17	30¼	31
13	19½	18½	32¾	33
15	22½	21	37¾	40
18	27	25	45¼	46
21	31½	29½	53	54
24	36	34	60½	62

of the head below, by means of long weirs, extends its effects all the way up a channel. It is necessary to treble the length of a weir before the head passing over can be reduced by one-half, TABLE X., even supposing the circumstances of approach to be the same : surely several engineering appliances for shorter weirs, during periods of flood, would be found far more effective and much less expensive than this alternative, with its extra sinking up channel and enlarged weir basin for drainage purposes.*

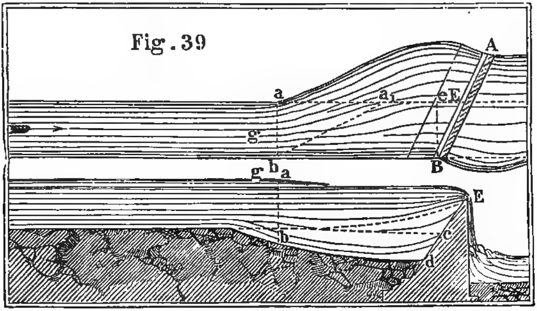
The advocates for the necessity of weirs longer than the width of the channel, for drainage purposes, must show that the reduction of the head and extent of backwater above *g*, Fig. 39, is not small, and that the effects extend the whole way up the channel, or at least as far as the district to be benefited. Practice has heretofore shown, that long weirs have failed (unless after the introduction of sluices or other appliances) in producing the expected arterial drainage results, notwithstanding the increased leakage from increased length, which must accompany their construction.

The deepening in the weir basin *a b* B E A is mostly of use in reducing the surface inclination between *a b* and A B by increasing the hydraulic mean depth ; but, thereby, the velocity of approach is lessened, and therefore the head at E increased. When the length of a weir basin *a* E exceeds that point where these two opposite effects balance each other, there will be a gain by the difference of the surface inclinations in favour of the long weir: but unless *a* E exceeds half a mile, this difference cannot amount to more than one or two inches, unless the river be very small indeed; and *if the channel be sunk for the long weirs* B A *or* $b\ a_1$, *it should also be sunk to at least the same depth and extent for the short weirs* B *e*, *b a*, *otherwise there is no fair comparison of their separate merits.* The effect of the widening between *a b* and A B, the depth being the same, is also to reduce the surface inclination from *a* to E; but, as before, unless *a* E be of considerable length, this gain will also be small. Now A B, at best, is but a weir the direct width of the new channel at A B, and if the length *a* E be considerable, there is an entirely new river channel with a direct weir at the lower end, and the saving of head effected arises entirely from the larger channel, with as it were a *direct* transverse weir at the lower end.

---

* When this was first written, in 1849, the Author was not acquainted with the good common sense appliances of moveable weirs used by the French, which raised the levels at low water to admit of navigation ; and being removed, or falling level with the bottom bed in floods, permitted the full drainage of the upper riparian lands, when most required.

Fig. 39



By referring to TABLE VIII., it will be found that for a hydraulic mean depth of 5 feet a fall of 7½ inches per mile will give a velocity of 2 feet per second; if of double the depth, a fall of 4 inches will give the same velocity; and for a depth of only 2 feet 6 inches, a fall of 12½ inches is necessary. This is a velocity much larger than we have ever observed in a deep weir basin, yet it is easily perceived that the difference in the inclinations for a short distance, E *a* of a few hundred feet, must be small. If one section be double the other, the hydraulic mean depth remaining constant, the velocity must be one-half, and the fall per mile, one-fourth, nearly. This would leave 7¼ − 2 = 5¼ inches per mile, or 1 inch per 1000 feet nearly, as the gain with a hydraulic mean depth of 5 feet for a double water channel. For greater depths the gain would be less, and the contrary for lesser depths.

Is the saving of head and amplitude of backwater here estimated worth the increased cost of long weirs and the consequent necessity and expense of sinking and widening the channels for such long distances? Certainly not; indeed, *any extra sinking in the basin immediately at the weir is absolutely injurious by destroying the velocity of approach*, and increasing the contraction. The gradual approach of the bottom towards the crest, shown by the upper dotted line *b* E in the section, Fig. 39, and a sudden overfall, will be found more effective in reducing the head, unless so far as leakage takes place, that any depth of sinking for nearly 80 or 100 feet above long weirs.

In most instances, the extra head will be only perceived by an increased surface inclination, which may extend for a mile or more up the channel, according to the sinking and widening.

It is a general rule that, for shorter weirs, the coefficients of discharge decrease; this arises from the greater amount of lateral contraction, and is more marked in notches or Poncelet weirs, than for weirs extending from side to side of the channel; but for weirs exceeding 10 feet in length the decrease in the coefficients from this cause is immaterial, unless the head passing over bear a large ratio to the length; and it may be seen from the coefficients, page 68, derived from Mr. Blackwell's experiments, that with 10 inches head passing over a 2-inch plank, the coefficient for a length of 3 feet was ·614; for a length of 6 feet ·539; and for a length of 10 feet ·534; showing a decrease as the weir lengthens, but which may, in the particular cases, be accounted for. Other circumstances which modify the coefficients were before referred to, yet it may be assumed generally, without any error of practical value, that the coefficients are the same for different weirs extending from side to side of a river. If, then, w and w_1 be put for the lengths of two such weirs, the relation of the heads d and d_1 for the same quantity of water passing over is given by the following proportion :—

$$d : d_1 : : w_1^{\frac{2}{3}} : w^{\frac{2}{3}};$$

and therefore

$$(135.) \qquad d_1 = \left(\frac{w}{w_1}\right)^{2} \times d.$$

By means of this equation, Table X. has been calculated; the ratio $\frac{w}{w_1}$ being given in columns 1, 3, 5 and 7, and the value of $\left(\frac{w}{w_1}\right)^{\frac{2}{3}}$, or the coefficient by which

d is to be multiplied, to find d_1 in columns 2, 4, 6 and 8. It appears also, that *if the heads passing over any weir in a river be taken in an arithmetical progression, the heads then passing over any other weir in the same river must also be in arithmetical progression, unless the quantity flowing down varies from erogation or supply, such as drawing off by millraces, &c.* If c_d be the coefficient for a direct weir, $\cdot94\ c_d$ will answer for an obliquity of 45°, and $\cdot91\ c_d$ for an angle of 65°.

In the first edition of this work though not specially mentioned, the observations on this subject had general reference to the weirs constructed across the Shannon at Killaloe, Meelick, and Athlone, and elsewhere. Since then the failure of these works is admitted by all, although previously the author stood alone in asserting that they should fail. To expect that lowering the head by extending the weir would extend its results for miles up the river, showed ignorance of the first principles of river-engineering; but when the surface was shown actually level on the sections from Killaloe to Meelick, and Meelick to Athlone on distances of 32 miles, and 27 miles, without any fall to give velocity to and convey off the waters something like wonder must be felt.* The French had and have several expedients for keeping up the summer levels, all founded on one sound principle, viz.: the removal of these obstructions before and during floods. In *Les Barrages a Hausses Mobiles,*† the separate panels or doors of which the weir is made up, are laid lying at the bottom, during floods; admitting the free navigation of the main channel. These are raised in summer, and the navigation carried on by means of a lateral lock. When the navigation of the main channel cannot be maintained at any time, sluices which lift entirely out of the water are perhaps best, as they can, for most rivers, be made up to 30 feet in width, as in patent of Mr. F. G. M. Stoney. The advantage of both these designs is, that the fall at the site is available in full, not only to discharge the water freely through its own depth, but also to give extra velocity and discharge to the under water thence down to the bottom, as in Fig. 22, if the weir were removed. The use of the syphon (see equations (154 to 154 *b*) *infra*,) assumes the necessity for the fixed weir; but this appliance can at no time discharge a greater amount of water than that due to the head or difference of levels, but always less; and it has no effect in increasing the discharge between the lower surface of the water and the bottom, which both the other designs have, and very considerably augment. See SECTION V.

---

* See Plan 39, Second Report, and Plans Nos. 12 and 13 in Fourth Report of the Shannon Commissioners.

† *Vide* Mémoire sur les barrages a Hausses Mobiles par MM. Chanoine ingénieur en chef et De Lagrêne, ingénieur ordinaire des ponts et chausses, Paris, 1862.

SECTION XI.

BENDS AND CURVES. — BRANCH PIPES. — DIFFERENT LOSSES OF HEAD.—GENERAL EQUATION FOR FINDING THE VELOCITY.—HYDROSTATIC AND HYDRAULIC PRESSURE.—PIËZOMETER.—SYPHONS.

The resistance or loss of head due to bends and curves has now to be considered. If a bent pipe, FBCDEG, Fig. 40, be fixed between two cisterns, so as to be capable of revolving round in collars at F and G, the time the water takes to sink a given distance from *f* to F in the upper cistern is found to be the same, whether the tube occupy the position shown in the figure or the horizontal position shown by the dotted line F *b c d e* G. This shows that the resistances due to friction and to bends are independent of the pressure. If the tube were straight, the discharge would depend on the length, diameter and difference of level between *f* and G, and may be determined from the mean velocity of discharge, found from TABLE VIII. or equation (79). Here, however, it is necessary to take into consideration the loss sustained at the bends and curves, and our illustration shows that it is unaffected by the pressure.

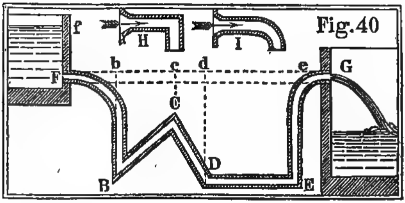


Fig.40

The experiments of Bossut, Du Buât, and others, show that the loss of head from bends and curves—like that from friction—increases as the square of the velocity; but when the curves have large radii, and the bends are very obtuse, the loss is very small. With a head of nearly 3 feet, Venturi's twenty-third experiment, when reduced, gives—for a short straight tube 15 inches long, and $1\frac{1}{4}$ inch in diameter, having the junction of the form of the contracted vein very nearly ·873 for the coefficient of discharge. When of the same length and diameter, but bent as in Diagram

I., Fig. 40, the coefficient is reduced to ·785; and when bent at a right angle as at H, Fig. 40, the coefficient is further reduced to ·560. In these respective cases we have therefore*

1. $v = \cdot 873 \sqrt{2\,g\,h}$, and $h = 1\cdot 312 \times \frac{v^2}{2\,g}$;

2. $v = \cdot 785 \sqrt{2\,g\,h}$, and $h = 1\cdot 623 \times \frac{v^2}{2\,g}$;

3. $v = \cdot 560 \sqrt{2\,g\,h}$, and $h = 3\cdot 188 \times \frac{v^2}{2\,g}$;

showing that the loss of head in the tube H, Fig. 40, from the bend, is $1\cdot 876 \times \frac{v^2}{2\,g}$, or nearly double the theoretical head due to the velocity in the tube. The loss of head by the circular bend is only $\cdot 311 \frac{v^2}{2\,g}$, or not quite one-sixth of the other.

Mr. Mallet's experiments with a syphon tube 6″ × 1½″, about 3 feet long, suited for weir-crests and a straight tube of the same dimensions every way gives coefficients from ·860 to ·874 due to the bend.

Du Buât deduced, from about twenty-five experiments, that the head due to the resistance in any bent tube A B C D E F G H, diagram 1, Fig. 41, depends on the number of deflections between the entrance at A and the departure at H; that it increases at each deflection as the square of the sine of the deflected angle, ABR for instance, and as the square of the velocity; and that if ϕ, ϕ_1, ϕ_2, ϕ_3, &c., be the number of degrees in the angles of deflection at B, C, D, E, &c.,

---

* It is stated that the time necessary for the discharge of a given quantity of water through a straight pipe being 1, the time for an equal quantity through a curve of 90° would be 1·11, with a right angle 1·57; two right angles would increase the time to 2·464, and two curved junctions to only 1·23. *Vide* REPORT ON THE SUPPLY OF WATER TO THE METROPOLIS, P. 237, APPENDIX NO. 3.

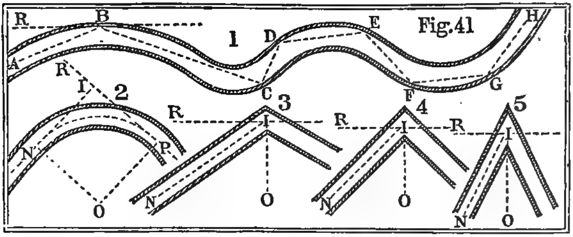


Fig. 41

then for measures in French inches the height h_b, due to the resistance from curves, is

$$(136.) \qquad h_b = \frac{v^2(\sin.^2\phi + \sin.^2\phi_1 + \sin.^2\phi_2 + \sin.^2\phi_3 + \&c.)}{3000}$$

which for measures in English inches becomes

$$(137.) \qquad h_b = \frac{v^2(\sin.^2\phi + \sin.^2\phi_1 + \sin.^2\phi_2 + \sin.^2\phi_3 + \&c.)}{3197}$$

and for measures in English feet,

$$(138.) \qquad h_b = \frac{v^2(\sin.^2\phi + \sin.^2\phi_1 + \sin.^2\phi_2 + \sin.^2\phi_3 + \&c.)}{266{\cdot}4}$$

or, as it may be more generally expressed for all measures,

$$(139.) \qquad h_b = (\sin.^2\phi + \sin.^2\phi_1 + \sin.^2\phi_2 + \sin.\ \phi_3 + \&c.) \times \frac{v^2}{8{\cdot}27\,g},$$

in which $\frac{v^2}{8{\cdot}27\,g} = \frac{v^2}{266{\cdot}4} = {\cdot}00375\ v^2$ in feet.

The angle of deflection, in the experiments from which equation (136) is derived, did not exceed 36°. It has already been shown that the loss of head from the circular bend in diagram I., Fig. 40, where the angle of deflection is nearly 45°, is $\cdot 311 \frac{v^2}{2g} = \cdot 00483\, v^2$, but as the sin. 45° = ·707; sin.^{2}45° = ·5 then $\cdot 00483\, v^2 = \cdot 00966\, v^2 \times \sin.^2 45°$, or more than two and a half times as much as Du Buât's formula would give; and if it be compared with Rennie's experiments,* with a pipe 15 feet long, ½ inch diameter, bent into 15 curves, each 3¼ inches radius, it would be found that the formula gives a loss of head not much more than one half of that which may be derived from the observed change, ·419 to ·370 cubic feet per minute in the discharge. See p. 298.

Dr. Young † first perceived the necessity of taking into consideration the length of the curve and the radius of curvature. In the twenty-five experiments made by Du Buât, he rejected ten in framing his formula, and the remaining fifteen agreed with it very closely. Dr. Young found

$$(140.)\qquad h_b = \frac{\cdot 0000045\, \phi\, \rho^{\frac{1}{8}} \times v^2}{\rho};$$

where ϕ is the number of degrees in the curve N P, diagram 2, Fig. 41, equal the angle N O P; ρ = O N the radius of curvature of the axis; h_b the head due to the resistance of the curve, and v the velocity, all expressed in French inches. This formula reduced for measures in English inches is

$$(141.)\qquad h_b = \frac{\cdot 0000044\, \phi\, \rho^{\frac{1}{8}} \times v^2}{\rho};$$

and for measures in English feet,

---

* Philosophical Transactions for 1831, p. 438.

† Philosophical Transactions for 1808, pp. 173—175.

$$(142.)\qquad h_{\mathrm{b}} = \frac{\cdot 000006\ \phi\ \rho^{\frac{1}{8}} \times v^2}{\rho}.$$

Equation (140) agrees to $\frac{1}{25}$th of the whole with twenty of Du Buât's experiments, his own formula agreeing, so closely, with only fifteen of them. The resistance must evidently increase with the number of bends or curves; but when they come close upon, and are grafted into each other, as in diagram 1, Fig. 41, and in the tube F B C D E G, Fig. 40, the motion in one bend or curve immediately affects those in the adjacent bends or curves, and this law does not hold.

Neither Du Buât nor Young took any notice of the relation that must exist between the resistance and the ratio of the radius of curvature to the radius of the pipe. Weisbach does, and combining Du Buât's experiments with some of his own, found for circular tubes,

$$(143.)\quad h_{\mathrm{b}} = \frac{\phi}{180} \times \left\{ \cdot 131 + 1\cdot 847 \left(\frac{d}{2\,\rho}\right)^{\frac{7}{2}} \right\} \times \frac{v}{2\,g};$$

and for quadrangular tubes,

$$(144.)\quad h_{\mathrm{b}} = \frac{\phi}{180} \times \left\{ \cdot 124 + 3\cdot 104 \left(\frac{d}{2\,\rho}\right)^{\frac{7}{2}} \right\} \times \frac{v}{2\,g};$$

in which ϕ is equal the angle N O P = N I R, diagram 2, Fig. 41; d the mean diameter of the tube, and ρ the radius N O of the axis. When $\frac{d}{2\,\rho}$ exceeds ·2, the value of $\cdot 131 + 1\cdot 847 \left(\frac{d}{2\,\rho}\right)^{\frac{7}{2}}$ exceeds $\cdot 124 + 3\cdot 104 \left(\frac{d}{2\,\rho}\right)^{\frac{7}{2}}$, and the resistance due to the quadrangular tube exceeds that due to the circular one. The author arranged and calculated the following table of the numerical values of these two expressions for the more easy application of equations (143) and (144).

This table will be found of considerable use in calculating the values of equations (143) and (144), as the second and fifth columns contain the values of $\cdot 131 + 1{\cdot}847\left(\frac{d}{2\rho}\right)^{\frac{7}{2}}$, and the third and sixth columns the values of $\cdot 124 + 3{\cdot}104\left(\frac{d}{2\rho}\right)^{\frac{7}{2}}$, corresponding to different values of $\frac{d}{2\rho}$; and it is carried to twice the extent of those given by Weisbach.

TABLE OF THE VALUES OF THE EXPRESSIONS.

$$\cdot 131 + 1{\cdot}147\left(\frac{d}{2\rho}\right)^{\frac{7}{2}} \text{ and } \cdot 124 + 3{\cdot}104\left(\frac{d}{2\rho}\right)^{\frac{7}{2}}.$$

$\frac{d}{2\rho}$.	Circular tubes.	Quadrangular tubes.	$\frac{d}{2\rho}$.	Circular tubes.	Quadrangular tubes.
·1	·131	·124	·6	·440	·643
·15	·133	·128	·65	·540	·811
·2	·138	·135	·7	·661	1·015
·25	·145	·148	·75	·806	1·258
·3	·158	·170	·8	·977	1·545
·35	·178	·203	·85	1·177	1·881
·4	·206	·250	·9	1·408	2·271
·45	·244	·314	·95	1·674	2·718
·5*	·294	·398	1·00	1·978	3·228

For bent tubes, diagrams 3, 4, and 5, Fig. 41, the loss of head is considerably greater than for rounded tubes. If, as before, the angle NIR be put equal to

---

* The values corresponding to $\frac{d}{2\rho} = \cdot 55$ are ·350 and ·507 for circular and quadrangular tubes.

ϕ, I R being at right angles to I O the line bisecting the angle or bend, then, by decomposing the motion, the head $\frac{v^2}{2g}$ becomes $\frac{v^2}{2g} \times \cos.^2 \phi$ from the change of direction; and therefore a loss of head

(145.) $h_b = (1 - \cos.^2 2\phi) \frac{v^2}{2g} = \sin.^2 2\phi \frac{v^2}{2g}$

must take place. When the angle is a right angle, as in diagram 4, $\cos. 2\phi = 0$, and $h_b = \frac{v^2}{2g}$; that is to say, the loss of head is exactly equal to the theoretical head. When the angle or bend is acute, as in diagram 5, the loss of head is $(1 + \cos.^2 2\phi) \frac{v^2}{2g}$, for then $\cos. 2\phi$ becomes negative. Weisbach does not find the loss of head in a right angular bend greater than $\cdot 984 \frac{v^2}{2g}$; while Venturi's twenty-third experiment, made with extreme care, p. 293, shows the loss to be $1\cdot 876 \frac{v^2}{2g}$. When the pipes are long, however, the value of $\frac{v^2}{2g}$ is in general small, and this correction does not affect the final results in any material degree.

Rennie's experiments,* with a pipe 15 feet long, ½ inch in diameter, and with 4 feet head, give the discharge per second

	Cubic Feet.
1. Straight, see table, p. 146 . . .	·00699
2. Fifteen semicircular bends . .	·00617

---

* Philosophical Transactions for 1831, p. 438.

3. One bend, a right angle $8\frac{1}{2}$ inches
   from the end of the pipe . . ·00556
4, Twenty-four right angles . . . ·00253

From these data may be found consecutively, the theoretical discharge being ·021885 cubic feet per second, and the theoretical head $H = \frac{v^2}{2g}$, that

1. $v = \cdot319\sqrt{2gH}$, and therefore $H = 9\cdot82 \times \frac{v^2}{2g}$;

2. $v = \cdot282\sqrt{2gH}$, ,, ,, $H = 12\cdot58 \times \frac{v^2}{2g}$;

3. $v = \cdot254\sqrt{2gH}$, and therefore $H = 15\cdot50 \times \frac{v^2}{2g}$;

4. $v = \cdot116\sqrt{2gH}$, ,, ,, $H = 74\cdot34 \times \frac{v}{2g}$.

The loss of head, therefore, by the introduction of 15 semicircular bends, is $2\cdot76\frac{v^2}{2g}$; by the introduction of one right angle, $5\cdot68\frac{v^2}{2g}$; and by the introduction of 24 right angles, $64\cdot52\frac{v^2}{2g}$, or about 12 times the loss due to one right angle. This shows that the resistance does not increase as the number of bends, as was before remarked, p. 275, when they are close to each other. The loss of head from one right angle, $5\cdot68\frac{v^2}{2g}$, is more than double the loss from 15 semicircular bends, or $2\cdot76\frac{v^2}{2g}$. The loss of head for a right angular bend, determined from Venturi's experiment, is $1\cdot876\frac{v^2}{2g}$; formula (145) makes it $\frac{v^2}{2g}$; and

Weisbach's empirical formula, $(\cdot 9457 \sin. \phi + 2\cdot047 \sin.^4 \phi) \frac{v^2}{2\,g}$, makes it only $\cdot 984 \frac{v^2}{2\,g}$. The formulæ now in use give, therefore, results considerably under the truth. It appears therefore, that the amount of the *velocity* of the water moving directly towards the bend must be taken into consideration, and also the loss of mechanical effect from contraction, and eddies at it, as well as the loss arising from the mere change of direction.

BRANCH PIPES.

When a pipe is joined to another, the quantity of water flowing below the junction B, diagram 1, Fig. 42, must be equal to the sum of the quantities flowing in the upper branches in the case of supply; and when the branch pipe draws off a portion of the water, as in diagram 2, the quantity flowing above the junction must be equal to the quantities flowing in the lower branches. Both cases differ only in the motion being *from* or *to* the branches, which, in pipes, are generally grafted at right angles to the main, for practical convenience, as shown at *b b*, and then carried on in any given direction. The loss of head arising from change

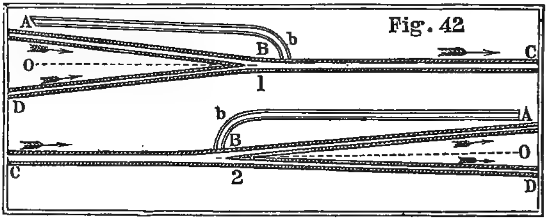


Fig. 42

of direction, equation (145), is $\sin.^2 2\,\phi\,\frac{v^2}{2\,g}$, in which $2\,\phi =$ angle A B O; but as in general $2\,\phi$ is a right angle for branches to mains, this source of loss becomes then simply $\frac{v^2}{2\,g}$. In addition to this, a loss of head is sustained at the junction, from a certain amount of force required to unite or separate the water in the new channel. In the case of drawing off, diagram 2, this loss was estimated by D'Aubisson, from experiments by Génieys, to be about twice the theoretical head due to the velocity in the branch, or $\frac{2\,v^2}{2\,g}$, so that the whole loss of head arising from the junction is $\frac{v^2}{2\,g}+\frac{2\,v^2}{2\,g}=\frac{3\,v^2}{2\,g}$, or three times the theoretical head due to the velocity. In the case of supply, the loss is probably nearly the same. The actual loss is, however, very uncertain; but, as was before observed when discussing the loss of head occasioned by bends, two or three times $\frac{v^2}{2\,g}$ is in general so comparatively small, that its omission does not materially affect the final results. A loss also arises from contraction, &c. See pp. 171, 172.

The calculations for mains and branches become often very troublesome, but they may always be simplified by rejecting at first any minor corrections for contraction at the orifice of entry, bends, junctions, or curves. If, in diagram 2, Fig. 42, h be put for the head at B, or height of the surface of the reservoir over it; h_a for the fall from B to A; h_d for the fall from B to D; l equal the length of pipe from B to the reservoir; l_a equal the length B A; l_d equal the length B D; r equal the mean radius of the pipe B C; r_a the mean radius of the pipe B A; r_d the mean radius of B D; v the mean velocity in B C; v_a the velocity in B A; and v_d the velocity in B D, we then find, by means of equation (73), the fall from the reservoir to A equal to

$$(146.)\quad h+h_a=\left(c_r+c_f\times\frac{l}{r}\right)\frac{v^2}{2\,g}+\left(1+c_f\times\frac{l_a}{r_a}\right)\frac{v_a^2}{2\,g};$$

the fall from the reservoir to D equal to

$$(147.)\quad h+h_d=\left(c_r+c_f\times\frac{l}{r}\right)\frac{v^2}{2\,g}+\left(1+c_f\times\frac{l_d}{r_d}\right)\frac{v_d^2}{2\,g};$$

and, as the quantity of water passing from C to B is equal to the sum of the quantities passing from B to A and from B to D,

$$(148.)\quad v\,r^2=v_a\,r_a^2+v_d\,r_d^2.$$

By means of these three equations any three of the quantities h, h_a, h_d, r, r_a, r_d, b, b_a, b_d, can be found, the others being given. Equations (146) and (147) may be simplified by neglecting c_r, the coefficient due to the orifice of entry from the reservoir, and 1, the coefficient of velocity. They will then become

$$(148\text{A}.)\quad h+h_a=c_f\times\left(\frac{l}{r}\times\frac{v^2}{2\,g}+\frac{l_a}{r_a}\times\frac{v_a^2}{2\,g}\right),$$

and

$$(149.)\quad h+h_d=c_f\times\left(\frac{l}{r}\times\frac{v^2}{2\,g}+\frac{l_d}{r_d}\times\frac{v_d^2}{2\,g}\right).$$

The mean value of c_f for a velocity of 4 feet per second is ·005741, and of $\frac{c_f}{2\,g}$, ·0000891. The values for any other velocities may be had from the table of coefficients of friction given at p. 229. When l, h, and r are given, the velocity v can be had from the equation, $v = \left(\frac{2\,g}{c_f} \times \frac{r\,h}{l}\right)$, or more immediately from TABLE VIII.

GENERAL EQUATION FOR THE MEAN VELOCITY.

A general equation for finding the whole head H, and the mean velocity v, in any channel; and to extend equations (73) and (74) so as to comprehend the corrections due to bends, curves, &c., can now be given. Designating, as before, the height due to the resistance at the orifice of entry by

h_r and the corresponding coefficient by c_r;
h_f the head due to friction, and c_f the coefficient of friction;
h_b the head due to bends, and c_b the coefficient of bends;
h_c the head due to curves, and c_c the coefficient of curves;
h_e the head due to erogation, and c_e the coefficient of erogation;
h_x the head due to other resistances, and c_x their mean coefficient

then evidently

$$(150.)\quad \mathrm{H} = h_r + h_f + h_b + h_c + h_e + h_x + \frac{v^2}{2\,g};$$

that is to say, by substituting for h_r, h_f, &c., their values as previously found,

$$\mathrm{H} = (1 + c_r)\,\frac{v^2}{2\,g} + c_f\,\frac{l}{r} \times \frac{v^2}{2\,g} + c_b \times \frac{v^2}{2\,g}$$
$$+ c_c \times \frac{v^2}{2\,g} + c_e \times \frac{v^2}{2\,g} + c_x \times \frac{v^2}{2\,g};$$

or, more briefly,

$$(151.)\quad \text{H} = \left(1 + c_r + c_f \times \frac{l}{r} + c_b + c_c + c_e + c_x\right)\frac{v^2}{2g};$$

from which may be found

$$(152.)\quad v = \left\{\frac{2g\text{H}}{1 + c_r + c_f \times \frac{l}{r} + c_b + c_c + c_e + c_x}\right\}^{\frac{1}{2}}.$$

It is to be observed here, that for very long uniform channels, the value of the mean velocity will be found in general equal to $\left\{\frac{2gr\text{H}}{c_f l}\right\}^{\frac{1}{2}}$, as the other resistances and the head due to the velocity are all trifling compared with the friction, and may be rejected without error; but, as before stated, it is advisable in practice, when determining the diameter of pipes, p. 246, to increase the value of c_f, table, p. 229, or to increase the diameter found from the formula by one-sixth, which will increase the discharging power by one-half. See TABLE XIII.

In equations (74) and (151), the coefficient of friction c_f depends on the velocity v, and its value can be found from an approximate value of that velocity from the small table at p. 229. If, however, both powers of the velocity be used, as in equation (83), then, when H is the whole head, and h the head from the surface to the orifice of entry

$$(av + bv^2)\frac{l}{r} + (1 + c_r)\frac{v^2}{2g} + h = \text{H},$$

a quadratic equation from which is found

$$v = \left\{\frac{(\text{H} - h)\,2gr}{(1 + c_r)\,r + 2gbl} + \left(\frac{gal}{(1 + c_r)\,r + 2gbl}\right)^2\right\}^{\frac{1}{2}} - \frac{gal}{(1 + c_r)\,r + 2gbl}$$

for a more general value of the velocity than that given in equation (74). If now c_s be put equal to $c_r + c_b + c_c + c_e + c_x$, in equation (151) we shall find

$$(152\text{A.})\quad v = \left\{ \frac{(\text{H} - h)\, 2\, g\, r}{(1 + c_s) r + 2\, g\, b\, l} + \left(\frac{g a l}{(1 + c_s) r + 2\, g\, b\, l} \right)^2 \right\}^{\frac{1}{2}} - \frac{g a l}{(1 + c_s) r + 2\, g\, b\, l}$$

for a more general expression of equation (152), when the simple power of the velocity, as in equation (83), is taken into consideration. For measures in English feet, $a = \cdot 0000223$ and $b = \cdot 0000854$, may be taken, which correspond to those of Eytelwein, in equation (97). The value of a is the same in English as in French measures; but the value of b in equation (83), for measures in metres, must be divided by 3·2809 to find its corresponding value for measures in English feet. In considering the head $\frac{v^2}{2\, g} c_r$, due to contraction at the orifice of entry as not implicitly comprised in the primary values of a and b, equation (83), Eytelwein is certainly more correct than D'Aubuisson, Traité d'Hydraulique, pp. 223 et 224, as this head varies with the nature of the junction, and should be considered in connection with the head due to the velocity, or separately. It can never be correctly considered as a portion of the head due to friction. In all Du Buât's experiments, this head was considered as a portion of that due to the velocity, and the whole head, $(1 + c_r) \frac{v^2}{2\, g}$, deducted to find the head due to friction and thence the hydraulic inclination. The following values of a and b were those taken in the equations referred to.

VALUES OF *a* AND *b* FOR MEASURES IN ENGLISH FEET.

		a.	*b.*
Equation	(88.)	·0000445	·0000944
,,	(90.)	·0000173	·0001061
,,	(94.)	·0000243	·0001114
,,	(98.)	·0000223	·0000854
,,	(109.)	·0000189	·0001044
,,	(111.)	·0000241	·0001114
,,	(114.)	·0000035	·0001150
Mean values for all straight channels, pipes, or rivers .		·0000221	·0001040

These mean values of a and b give the equation

$$r\,s = \cdot0001040\,v + \cdot0000221\,v,$$

from which we find

$$9615\,r\,s = v + \cdot21\,v,$$

and thence

$$(153.)\quad v = (9615\,r\,s + \cdot011)^{\frac{1}{2}} - \cdot105 = 98\sqrt{r\,s} - \cdot1,$$

very nearly, suited to velocities of about 2 feet, p. 231.

HYDROSTATIC AND HYDRAULIC PRESSURE.—PIËZOMETER.

When water is at rest in any vessel or channel, the pressure on a unit of surface is proportionate to the head at its centre,* measured to the surface, and is expressed in lbs. for measures in feet, by 62·5 H S, in which H is the head, and S the surface exposed to the pressure, both in feet measures. This is the *hydrostatic pressure.* In the pipe A B C D F E, Fig. 43, the pressure at the points B, C, D, F, and E, on the sides of the tube will be respectively as the heads B *b*, C *c*, D *d*, F *f*, and E *e*, if all motion in the tube be prevented by stopping the discharging orifice at E. In this case the pressure is a maximum and hydrostatic; but if the discharging orifice at E be partially or entirely open, a portion of each pressure at B, C, D, F, &c., is absorbed in overcoming the different resistances of friction, bends, &c., between it and the orifice of entry at A, and also by the velocity in the tube, and the difference is the *hydraulic pressure.*

---

* This is only correct when the surface is small in depth compared with the head. If H be the depth of a rectangular surface in feet, and also the head of water measured to the lower horizontal edge, then the pressure in lbs. is expressed by $31\frac{1}{4}\,\text{H}^2$; and the centre of pressure is at ⅔rds of the depth.

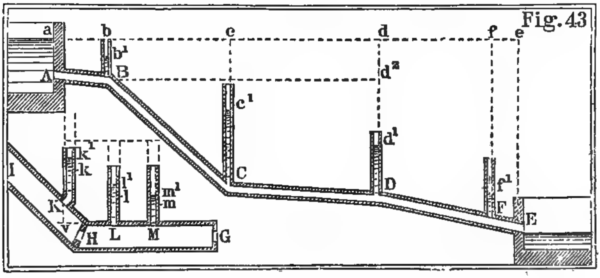


Fig. 43

Bernoulli first showed that *the head due to the pressure at any point, in any tube, is equal to the effective head at that point, minus the head due to the velocity.* When the resistances in a tube vanish, the effective head becomes the hydrostatic head, and by representing the former by h_{ef} and, adopting the notation in equation (150),

$$h_{ef} = \text{H} - (h_r + h_f + h_c + \&c.),$$

and consequently the head due to the hydraulic pressure equal

$$(153\text{A.})\quad h_p = h_{ef} - \frac{v^2}{2g} = \text{H} - (h_r + h_f + h_b + \&\text{c.}) - \frac{v^2}{2g}.$$

If small tubes be inserted, as shown in Fig. 43, at the points B, C, D, and F, the heights B b^1, C c^1, D d^1, F f^1, to which the water rises, will be represented by the corresponding values of h_p in the preceding equation; and the difference between the heights C c^1, F f^1, at C and F, for instance, added to the fall from C to F will, evidently, express the head due to all the resistances between C and F. When H = E e, and the orifice at E is open, from equation (150), $\text{H} = h_r + h_f + h_b + h_c + \&\text{c.} + \frac{v}{2g}$, and therefore $h_p = 0$, that is, the pressure at the discharging orifice is nothing.

The vertical tubes at B, C, D, F, when properly graduated, are termed *piëzometers* or *pressure gauges ;* they not only show the actual pressure at the points where placed, but also the difference between any two; D d^1—B b^1, for instance, added to the difference of head between D and B, or D d^2 will give D d^1 − B b^1 + D d^2 for the head or pressure due to the resistances between B and D. This instrument affords, perhaps, the very best means of determining the loss of head due to bends, curves, diaphragms, &c. The loss of head due to friction, bend, diaphragm, &c., between K and L, Fig. 43, is equal to K k − L l + K v. If M be the same distance from L as K is, L l − M m will be the height due to the friction (L and M being on the same level); therefore K k − L l + K v − L l + M m

$= \text{K}\,k + \text{K}\,v + \text{M}\,m - 2\,\text{L}\,l$ is the head due to the diaphragm and bend both together. If the diaphragm be absent, the head due to the bend is found, and if the bend be absent, the head due to the diaphragm is had in like manner.

When the discharging orifice, as at E, is quite open, we have seen that the pressure there is zero; but when, as at G, it is only partly open, this is no longer the case, and the hydraulic pressure increases from zero to hydrostatic pressure, as the orifice decreases from the full section to one indefinitely small compared with it. A piëzometer, placed a short distance inside G, will give this pressure; and the difference between it and the whole head will be the head due to the resistances and velocity in the pipe: from which, and also the length and diameter, the discharge may be calculated as before shown. Again, by means of the head $\text{M}\,m^1$, and that due to the velocity of approach, the discharge may be found through the diaphragm G; see equation (45) and the remarks following it. This result must be equal to the other; and in this way the formulæ may be tested anew or corrected by the observed results.

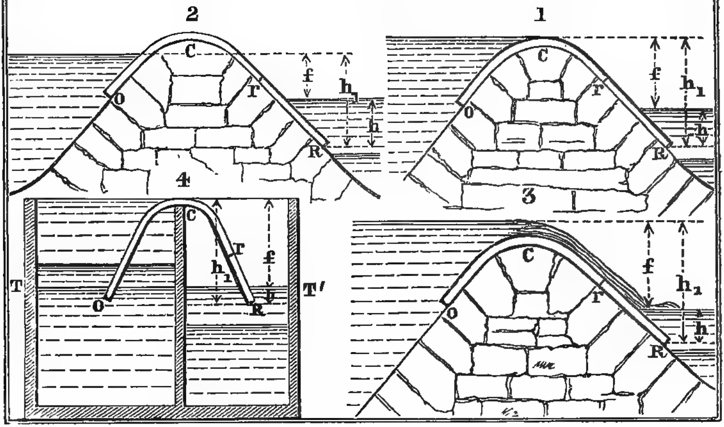
The velocity of discharge of the tube A C D E, may be calculated by means of any piëzometric height $\text{C}\,c^1$; for by putting the whole fall from c^1 to E equal to H^1_c, then, disregarding bends, $v = \left\{\frac{2\,g\,r\,\text{H}^1_\text{c}}{c_\text{f}\,l^1_\text{c}}\right\}^{\frac{1}{2}}$, in which $l^1_\text{c} = \text{C E}$. This is evident from equation (152), as it is supposed that no part of the head is absorbed in generating velocity, or in overcoming the resistance of bends. If the bend at D were taken into consideration, then $v = \left\{ \dfrac{2\,g\,H^1_c}{c_f \times \dfrac{l^1_c}{r} + c_b} \right\}^{\frac{1}{2}}$.

SYPHONS.

If one end of an open tube be placed in water and the air withdrawn, the water will rise and fill the tube to a height corresponding to the atmospheric pressure at the time, but in practice it is advisable to reduce this to about 26 feet. When the tube is bent, as O C R in diagrams 1, 2, 3 and 4, Fig. 43a, water can be raised to this height over the level of that on the side C O and discharged at any lower level on the side C R. The available head or pressure f is the difference of level between the upper and lower water when the latter rises over the end R; and when below R, $h_1 = h + f$, or the difference of level between the upper water and the lower end of the bent tube or syphon is the head. It is advisable to have the lower end R immersed, and hence C R longer than C O, but this is not essential; as long as there is an effective head the arms may be of the same or unequal lengths, and the water will flow off at r, from the syphon O C r, as well as at R from the syphon O C R, but with the difference of velocity due to the heads at r and R. The lengths of the arms C O, C R, are however made to suit the circumstances of each case. In diagram 1 the upper water is at the top of the syphon; in 2 below it; in 3 above it. In diagram 4 the arms of the syphon are of the same length and the orifice of entry

O and of discharge R at the same level. The water in both vessels finally settles at a common level T T^1, and fills the syphon all round in which it is maintained at rest by the atmospheric pressure on the surface of the common level T T^1.

Fig. 43a



In calculating the discharge the circumstances to be considered are: The available or effective head f; the form of the orifices of entry at O and of discharge at R; the length; the hydraulic mean depth of the section,—rectangular or circular; and the bend at C with any smaller bends adjoining O and R to suit the form O C R of the weir. If, as like in formula (68) to (70L), a be put for the area of the section, then

$$(154.)\qquad D = \begin{cases} c_d\, a\, \sqrt{2\, g\, f}, \text{ or} \\ 481{\cdot}5\, c_d\, a\, \sqrt{f}, \text{ feet measures in one minute,} \end{cases}$$

in which the value of c_d depends on the preceding considerations. At pp. 146 and 199 the values of c_d for velocities of about eighteen inches, and twenty feet, are given, and by interpolation intermediate values for intermediate or even higher velocities can be determined; the effects of bends are considered pp. 291 to 300, and taking both together c_d can be found suited to the form and dimensions of the given syphon.

If the length of the syphon—which is generally rectangular for engineering works—be 50 times the hydraulic mean depth, that is equivalent to 12½ mean diameters, then the table p. 199 gives ·759 for the value of c_d in a straight tube. Taking the coefficient due to the bend at about ·867, the value of c_d due to the length of this syphon and to the bend at the crest of the weir, is ·759 × ·867 or ·658. Hence eqn. (154) becomes in round numbers

$$(154\text{A.})\begin{cases} \text{D} = 317\,a\sqrt{f}. & \text{for a length of 50 mean radii; also may be found} \\ \text{D} = 298\,a\sqrt{f} & \text{for a length of 100 mean radii; and} \\ \text{D} = 268\,a\sqrt{f} & \text{for a length of 200 mean radii,} \end{cases}$$

for the discharge in cubic feet per minute. If the syphon were longer in proportion to the hydraulic mean depth, these results would become still less, and more if the orifice of entry were rounded.

In Weale's Quarterly Papers of Engineering, vol. vi., p. 51, Mr. Mallet, in a letter dated 10th Aug., 1843,* addressed to Major-General Sir John F. Burgoyne, R.E., then Chairman of the Board of Public Works in Dublin, gives the result of experiments on a rectangular syphon tube with a section of six inches wide by one-and-a-half inch deep, or area of nine square inches. The length of the syphon is not given, but from the drawings given we infer it to be 30 inches, and of the general form given in diagrams 1, 2 and 3, Fig. 43a, but "slightly bell-mouthed" at the "entrance and discharging terminations." A straight tube of the same length and form was made, and the time of discharging 24 cubic feet of water from each with the same heads was as follows :—

---

* At this time Sir John evidently appears to have lost faith in long solid weirs as a panacea for the Shannon drainage, and in the self-esteem of the Shannon Commissioners on this hobby of theirs.

Heads in inches.	Time in seconds for syphon.	Time in seconds for straight tube.
10½	78	67
13½	67½	59

The head of 10½ inches gives by calculation from these experiments, ·860 for the coefficient of discharge due to the bend over the crest; and the head of 13½ inches, a coefficient of ·874; which shows that the resistance from the bend was less for a greater velocity. Mr. Mallet also found that with an effective head of 8 inches, 24 cubic feet were discharged in 89 seconds. This gives, by calculation, the coefficient ·658 for all the resistances due to the orifice of entry friction in the tube and bends. Taking the coefficient of the bend here at ·850 as due to a lesser head, this gives ·774 for that due to friction and the orifice of entry. Another experiment, with a head of 11½ inches, gave a discharge of 24 cubic feet in 78 seconds. This is equivalent to a coefficient of ·636 for all resistances, and dividing by ·860 for the bend, ·740 is found for the friction and orifice of entry. Again

Experiments, discharge of 24 cubic feet.		Calculated results.		
Heads in inches.	Time in seconds.	General coefficient.	Coefficient for bend.	Coefficient for friction and orifice.
12¼	75½	·632	·860	·738
13	73	·631	·860	·734
14½	67½	·640	·874	·732

The effective head here to calculate the results from, exceeds by ¾ inch or half the depth of the tube, that taken by Mr. Mallet, as in the effective head taken by him this is omitted, but the 8-inch head agrees with that he has taken, the error being eliminated by taking a difference. Taking the mean general coefficient of these results at ·636, the discharge is expressed by the equation

$$(154\text{B.})\ \text{D} = 481{\cdot}5 \times {\cdot}636\, a \sqrt{f} = 306\, a \sqrt{f},$$

the discharge in cubic feet per minute for feet measures. This is less than that before given by our eqn. (154A) by about 3½ per cent.

The application of the syphon for the discharge of surplus waters is of great value, and the head can be increased to any extent not exceeding the difference of level between the upper and lower water. It is not the object here to enter into the question of construction, or the means of withdrawing the air or putting the apparatus to work, but a comparison between its advantages, lifting-sluices, and *Les Barrages Hausses Mobiles* has been already entered into at p. 291.

SECTION XII.

RAIN-FALL.—CATCHMENT BASINS.—DISCHARGES INTO CHANNELS.—DISCHARGE FROM SEWERS.—LOSS FROM EVAPORATION, ETC.

A catchment basin is a district which drains itself into a river and its tributaries. It is bounded generally by the summits of the neighbouring hills, ridges, or high lands forming the water-shed boundary; and may vary in extent from a few square miles to many thousands; that of the Shannon is 4,544 square miles. The average quantity of water which discharges itself into a river will, *cæteris paribus*, depend on the extent of its catchment basin, and the whole quantity of rain discharged on the area of the catchment basin, including lakes and rivers.

THE QUANTITY OF RAIN which falls annually varies with the district and the year; and it also varies at different parts of the same district. The average quantity in Ireland may be taken at about 34 inches deep, that which falls in Dublin being 27 inches, in Armagh, average of 14 years, 35 inches; in Killaloe, average of 17 years, 43 inches; in Galway, average of 11 years, 46 inches; and that in Cork 41 inches nearly. The average yearly fall in Dublin for seven years, ending with 1849, was 26·407 inches; and the maximum fall in any month took place in April, 1846, being 5·082 inches. "The average fall in inches per month for seven years, ending with 1849, was as follows:—October, 3·060; August, 2·936; January, *TABLE of some Catchment Basins in Ireland.*

Names of Drainage districts, or Rivers.	Counties or Towns.	Area of Catchment in acres.	Area of Catchment in square miles.
Avonmore	Wicklow and Wexford	128,000	200·
Avoca River	Wicklow	179,840	281·
Ballinasloe	Mayo	70,000	110·
Barrow, Nore, and Suir	Waterford	2,176,000	3400·
Blackwater and Boyne	Meath, &c.	695,040	1086·
Blackwater	Waterford, Youghal	780,160	1219·
Blackwater	Armagh	336,640	526·
Blackwater	Meath and Kildare	50,000	78·1
Bandon River	Cork	145,920	228·
Bann, Upper and Lower, and the Main	Down, Antrim	810,240	1266·
Boyne	Meath, Westmeath, Kildare, and King's	304,139	478·2
Brusna (Ferbane)	King's	389,120	608·
Ballyteigue	Wexford	26,752	41·8
Ballinamore and Ballyconnel	Cavan, Fermanagh, Leitrim, and Roscommon	101,455	158·5
Breeogue	Sligo	180,408	282·
Ballinhassig	Cork	23,500	36·7
Cappagh	Galway	34,856	54·4
Coolaney	Sligo	90,744	141·8
Camoge	Limerick	61,184	95·6
Dunmore	Galway, Mayo, and Roscommon	96,161	150·2
Dodder	Dublin	35,200	55·
Deel	Meath and Westmeath	64,000	100·
Dee	Louth and Meath	78,000	121·9
Erne	Belturbet, Enniskillen	1,014,400	1585·
Foyle	Londonderry	944,640	1476·
Fergus	Clare and Galway	134,400	210·
Fane	Louth	87,400	136·6
Glyde	Louth, Meath, Monaghan, and Cavan	176,813	276·3
Inny	Meath, Westmeath, Longford, and Cavan	231,116	361·1
Kilbeggan	Westmeath and King's	88,030	137·5
Liffey and Tolka	Dublin, &c.	328,320	513·0
Lee	Cork	470,400	735·
Lough Gara and Mantua	Roscommon, Mayo, and Sligo	128,000	200·
Loughs Oughter and Gowna and River Erne	Cavan, Leitrim, and Longford	260,480	407·
Lough Neagh and Bann	Londonderry, Antrim, Down, and Armagh	1,411,320	2205·2
Lough Mask and River Robe	Mayo and Galway	225,000	351·5
Loughs Corrib, Mask, and Carra	Galway and Mayo	780,000	1218·7
Longford	Longford	72,320	113·0
Moy	Mayo, Ballina	661,120	1033·
Main	Antrim	37,600	90·
Monivea	Galway	54,000	84·4
Maghera	Down	19,000	29·7
Nobber	Meath	40,000	62·3
Quoile	Down	57,000	89·1
Rinn and Black River	Leitrim and Longford	74,000	115·6
Strokestown	Roscommon	70,000	109·4
Shannon above Killaloe	Different counties, towns of Athlone, Limerick	2,908,160	4544·
Slaney	Wexford	521,600	815·

TABLE showing Summer Discharges of some English Rivers, as collected from various authorities, re-arranged, showing to some extent the effect of Springs in supplying Channels in different places.

NAMES OF RIVERS.	Height above sea in feet. Valley. Hill.	Catchment in square miles.	Discharge in cubic feet per minute.	Discharge per square mile in cubic feet per minute.	Representing inches of rainfall per annum.	Total average rainfall in inches per annum.
Gade, at Hunton Bridge, chalk .	150 to 500	69·5	2,500	36·2	8·19	...
Lea, at Lea Bridge, chalk. (Rennie, April, 1796) .	30 to 600	570·0	8,880	15·58	3·53	...
Loddon (Feb. 1850), green sand . .	110 to 700	221·8	3,000	13·53	3·01	25·4
Medway, driest seasons (Rennie, 1787), clay .	...	481·5	2,209	4·59	1·04	...
Medway, ordinary summer run (Rennie, 1787), clay . . .	...	481·5	2,520	5·23	2·19	...
Mimram, at Panshanger, chalk .	200 to 500	29·2	1,500	51·4	11·58	26·6
Nene, at Peterborough, oolites, Oxford clay, and lias . . .	10 to 600	620·0	5,000	8·45	1·88	23·1
Plym, at Sheepstor, granite . .	800 to 1,500	7·6	500	71·4	15·10	45·0
Severn, at Stonebench, silurian .	400 to 2,600	3,900	33,111	8·49	1·98	...
Thames, at Staines, chalk, green sand, Oxford clay, oolites, &c. . .	40 to 700	3,086	40,000	12·98	2·93	24·5
Trent, at its mouth, oolites and Oxford clay . .	100 to 600	3,921	...	...	...	...
Verulam, at Bushey Hall, chalk .	150 to 500	120·8	1,800	14·9	3·37	...
Wandle, below Carshalton, chalk .	70 to 350	41·0	1,800	43·9	9·93	24·0

2·544; April, 2·503; November, 2·300; July, 2·116; June, 2·005; December, 1·938; September, 1·860; May, 1·814; March, 1·739; February, 1·534."* A gauge at Londonderry, 1795 to 1801, gives 31 inches average; one at Belfast, from 1836 to 1841, gives 35 inches; at Mountjoy, Phœnix Park, 182 feet above low water, 1839 and 1840, there is an average of 33 inches; and at the College of Surgeons, 52 feet over low water, the average is 30 inches for the same two years. Sir Robert Kane assumes that 36 inches is the average fall in Ireland, and that out of that depth 12 inches, or one-third, passes on to the sea, two-thirds being evaporated and taken up by plants. The quantity varies a good deal with the altitude of the district. In parts of Westmoreland it rises sometimes to 140 inches; in London, an average of 20 years' observations gives a fall of nearly 25 inches.

The tabular information has been obtained from Mr. Hughes' book, from Rennie's reports, and other sources. The effect of the geology and fissures in the chalk and mountain limestone formations on the springs of a catchment basin, and on the summer discharge, should be carefully noted as one of the elements entering into catchment basin statistics. Indeed, the maximum and minimum discharges from catchments are of as much importance to the engineer as the averages, and, for many purposes, more important. There was abundant opportunity of acquiring this information for all our Irish rivers, but we are not aware if it was turned to any useful account for science.

---

* Proceedings of the Royal Irish Academy, vol. v., p. 18.

Thorough-drainage increases the supply and discharge. *Every catchment basin has, however, its own peculiar data, and a knowledge of these is necessary before we can draw any correct conclusions for new waterworks in connection with it.* It may be remarked, however, that any conclusions drawn from experiments on the supply of tributaries, particularly in high districts, are wholly inapplicable to the main channel into which they flow. The flow into tributaries and mountain streams, or rivers, is always more rapid than into main channels and rivers in flat districts, and the supply from springs often forms a large portion of the water flowing in them. Heretofore, however, little dependence can be placed on gaugings unless the manner in which they were obtained was fully described.

Forty years' observation at Greenwich, Kent, at 155 feet above the level of the sea, gives the following results:—

Description of fall.	Winter.	Spring.	Summer.	Entire years.
	inches	inches	inches	inches
Mean annual fall	7·86	7·25	10·47	25·48 25·58
Maximum fall; being a mean of five of the wettest years during forty years	11·05	10·86	14·96	34·00 36·87
Minimum fall; being a mean of five of the driest years during forty years	5·22	4·05	6·80	18·40 16·07

In this table Winter comprises November, December, January, and February; Spring, the next four months; and Summer, the months of July, August, September, and October. The last column contains means of two classes of years: the first figures showing the ordinary years from January to December, and the second, under the first, years from November to October.* We see here that the mean maximum is fully double of the mean minimum, and about one-and-a-half times the mean annual fall, and therefore the necessity for calculating from the minimum fall for all water works in which it is an element, and from the maximum for sewerage works where it is not intended to pass off a portion on the surface or through other available channels.

In the district surrounding the Upper Bann reservoirs in the County Down, the average fall for thirteen years has been 46 inches at a level of 6 feet over the top water of Lough Island Reavy Reservoir; and Mr. John Smith, the engineer of the work, says that there is a loss of one-third in absorption and evaporation; but as the rainfall is greater on the higher ground than at the gauge, only one-half of the whole rainfall is probably available. Mr. Manning found for the Woodburne river, with a rainfall of 36 inches, a flow of 21·5 inches was produced, or about three-fifths, which was distributed as follows:—

Six months, November to May, 14·766 fall, and 14·351 flow;

Six months, May to November, 21·101 fall, and 7·357 flow.

In Keswick, the average fall is said to be 67½ inches, and in Upminster, Essex, only 19½ inches. Indeed, it is requisite to obtain the fall from observation for any particular district, when it is necessary to apply the results to scientific purposes; and not the mean average fall alone, but also the maximums and minimums in a series of years and months in each year.

---

* See Mr. James Simpson in the Metropolitan Main Drainage Report, 1857, p. 115.

Mr. Symons gives (see *Builder* for 1860, p. 230) the following heavy falls of rain during 1859:—*Wandsworth,* June 12th, 2·17 inches in two hours; *Manchester,* August 7th, 1·849 inches in twenty-four hours; *Southampton,* September 26th, 2·05 inches in two-and-a-quarter hours; *Truro,* October 25th, during the day, 2·4 inches. The mean falls in the *South Western* Counties, 39·1 inches; in the *South Eastern* Counties, 30·2 inches; in the *West Midland* Counties, 28 inches; in the *Eastern* Counties, 25·4 inches; in the *North Midland* Counties, 24 inches; in the *North Western* Counties, 39 inches; in the *Northern* Counties, 55 inches; and the average of all England, 31·857 inches.

As an instance of extraordinary rain-fall, in connection with the sewage question, it is stated that 4 inches of rain fell in one hour in the Holborn and Finsbury sewers' district, on the 1st of August, 1846; at Highgate, 3·5 to 3·3 inches; and at Greenwich, 0·95 inches.* In India, the intensity of the rain-fall varies from half an inch to 5 inches in an hour.

In the upland districts about Manchester, Mr. Homersham † gives the result of observations at Fairfield, Bolton, Rocksdale, Marple, Comlis reservoir, Belmont, Chapel-en-le-Frith, and Whiteholme reservoir, for four years. These give a maximum fall of 61·4 inches at Belmont Sharples in 1847, and a minimum of 24·8 at Whiteholme reservoir in 1844, the general average for the four years being 42·49 inches.

---

* Metropolitan Main Drainage Report, p. 16.

† Report on the Supply of Water to Manchester.—WEALE.

April is the driest month, and October, or about it, the wettest month, and the fall during different years varies sometimes as much as two to one in the same district.

The proportion between the quantity which falls, and that which passes from a catchment basin into its river, also varies very considerably. When the sides of a catchment basin are steep and staunch, and the water passes off rapidly into the adjacent river or tributaries, there is less loss by evaporation and percolation than when they are nearly flat. The soil, subsoil, and stratification have also considerable effect on the proportion. Reservoirs being generally constructed adjacent to steep side falls, give a much larger proportion of the quantity fallen than can be obtained from rivers in flatter districts; besides, the quantity of rain which falls on the high summits, near reservoirs, almost always considerably exceeds the average fall. As 640 acres is equal to 1 square mile, and one acre is equal to 43,560 square feet, a fall of one inch of rain is equal to 3,630 cubic feet per acre, and to 3,630 × 640 = 2,323,200 cubic feet per square mile: the proportion of this fall, for each acre, or square mile of the catchment basin, which enters the river, must depend entirely on the district and local circumstances, the full or maximum quantity being retained on lakes. A stream delivering 53 cubic feet per minute constantly for twelve months supplies an equivalent to 12 inches of rain-fall collected per square mile, and 1 inch of rain collected from each square mile of catchment gives a supply of 4·42 cubic feet per minute, and 6·9 cubic feet for each 1000 acres, flowing in both cases for twelve months.

FLOW EQUIVALENT TO A RAIN-FALL OF ONE INCH ON EACH SQUARE MILE, AND 1000 ACRES, FLOWING REGULARLY, WITHOUT LOSS, FOR ONE MONTH TO ONE YEAR.

For	Sq. mile.	1000 acres.	For	Sq. mile.	1000 acres.	For	Sq. mile.	1000 acres.
One month .	53·	82·8	Five months .	10·6	16·6	Nine months .	5·9	9·2
Two months .	26·5	41·4	Six months .	8·8	13·8	Ten months .	5·3	8·3
Three months.	17·7	27·6	Seven months.	7·6	11·8	Eleven months	4·8	7·5
Four months .	13·2	20·7	Eight months.	6·6	10·3	Twelve months	4·4	6·9

It is too often taken for granted that the discharge from a catchment basin takes place, into the conveying channels, in nearly the same time that a given quantity of rain falls. Perhaps the largest registry on record in Great Britain is a fall of four inches in an hour. The maximum fall in any hour of any year seldom

QUANTITY PER ACRE FOR A GIVEN DEPTH OF FALL.

Fall in inches.	Cubic feet per acre.	Fall in inches.	Cubic feet per acre.	Fall in inches.	Cubic feet per acre.	Fall in inches.	Cubic feet per acre.
2	7260	$\frac{1}{2}$	1815	$\frac{1}{8}$	454	$\frac{1}{20}$	181
$1\frac{3}{4}$	6352	$\frac{3}{8}$	1361	$\frac{1}{9}$	403	$\frac{1}{30}$	121
$1\frac{1}{2}$	5445	$\frac{1}{4}$	907	$\frac{1}{10}$	363	$\frac{1}{40}$	91
$1\frac{1}{4}$	4537	$\frac{1}{5}$	726	$\frac{1}{12}$	302	$\frac{1}{50}$	73
1	3630	$\frac{1}{6}$	605	$\frac{1}{14}$	259	$\frac{1}{60}$	61
$\frac{3}{4}$	2723	$\frac{1}{7}$	519	$\frac{1}{16}$	227	$\frac{1}{70}$	52

exceeds half of this amount, and then perhaps only once in several years. The quantity which falls will not be discharged into the channels in the same time. The quantity discharged, and time, will depend a good deal on the season and district. The arterial channel receives the supply at different places and from different distances, and the water in passing into and from it does not encounter the same amount of resistance as if it all passed first into the upper end. Less sectional area is therefore necessary than if the whole discharge had to pass through the whole length of the channel and during the time of fall. The relation of the quantity of rain-fall to the portion which flows into the main channel, as well as the time which it takes to arrive at it, and the places of arrival, must be known before the proper size of a new channel can be determined, particularly sewers in urban districts. A pipe sufficient to discharge the water from 200 acres need not be 20 times the discharging power of one exactly suited to 10 acres of the same district, for the discharge from the outlying 190 acres will not arrive at the main in the same time as that from the adjacent 10 acres.

The following table of rain-fall, at Athlone, central in Ireland, was furnished to the Royal Irish Academy by General Sir H. D. Jones, and is printed in the Proceedings.* The average for four years gives 29 inches, and the effect on the Upper and Lower Sills of the Lock as affecting the rise and fall of the Shannon, affords valuable data, although not analysed. The rise and fall on the sills is the sum of the

---

* Vol. iv.

YEARS.	RAIN.							RIVER SHANNON.								WIND.							
	Quantity which fell each year.			Greatest fall in one successive Day and Night.	Number of days without rain.	Greatest number of successive Days.		Total Rise and Fall of the Shannon during each year; being the sums of the rise or fall for each month.								Number of days in which the Wind was							
								Upper Sill.				Lower Sill.											
	Days.	Nights.	Total.			With Rain.	Without Rain.	Rise.		Fall.		Rise.		Fall.		North.	North E.	East.	South E.	South.	South W.	West.	North W.
	Inches.	Inches.	Inches.	Inch.	No.	No.	No.	Ft.	in.	Ft.	in.	Ft.	in.	Ft.	in.	No.	No.	No.	No.	No.	No.	No.	No.
1845	14·02	12·97	26·99	1·18	177	9	14	14	11	11	8½	24	4	18	9	60	20	24	47½	52½	31	41	89
1846	14·87	17·50	32·37	1·45	174	15	12	12	8½	15	0½	19	7	24	0	58	15	15	56	58	47	47	69
1847	12·85	10·65	23·50	0·89	193	11	8	17	2	14	8	22	11½	21	1½	51	21	22	49	67	39	49	67
1848	17·66	15·77	33·43	1·11	165	13	14	17	5½	19	0	24	0	27	4	45	27	16	43	67	48	57	63
Amount for four years.	59·40	56·89	116·29	1·45	709			62	3	60	5	90	10½	91	2½	214	83	77	195½	244½	165	194	288
Averages for one year.	14·85	14·22¼	29·07¼		177¼			15	6¾	15	1¼	22	8⅝	22	9⅝	53½	20¾	19¼	48⅞	61⅛	41¼	48½	72

monthly risings and fallings for each year, and must be divided by 12 to get the average monthly rise and fall. In 1845 the greatest rise was in January, 2 feet 9 inches at the upper sill, and 3 feet 11½ inches at the lower sill. In 1846 the greatest rise was 2 feet 5 inches in October, at the upper sill; and 5 feet 6½ inches on the lower sill, in August.

	Upper Sill. Maximum rise in one month.	Lower Sill. Maximum rise in one month.
1845 . . .	2 ft. 9 in. January . . .	3 ft. 11½ in. January.
1846 . . .	2 ft. 5 in. October . . .	5 ft. 6½ in. January.
1847 . . .	3 ft. 1 in. November . . .	4 ft. 6 in. May.
1848 . . .	3 ft. 3 in. February . . .	4 ft. 11 in. February.

The sum of the risings and fallings for each month, taken as a mean of four years, is nearly the same at either sill. The general average of the rise and fall for the upper sill, is about 1 foot 3½ inches each way, and 1 foot 10¾ inches at the lower sill. These would give 2 feet 7 inches for the average difference of level in the Shannon above, and 3 feet 9½ inches for that in the Shannon below. In Lough Allen catchment of 146 square miles, the maximum rise was sometimes 6 inches in 24 hours, calculated at ·568 inch of depth of rain, over the catchment area. Above Killaloe, the catchment is 3611 square miles, and the floods about once a year rose 6 inches in 24 hours, or ·296 inch in depth of rain over the catchment. Once, in 1840, it is reported to have risen 12 inches, or ·6 inch of rain over the catchment in one day. "The greatest observed flood in the Shannon occurred in January, 1853, when the discharge of Killaloe marked 1,617,000 cubic feet per minute, or ·699 cubic foot per acre of catchment. The large floods in the Armagh river, county Galway, yield from 8 to 10 cubic feet, and a summer flood in 1851 gave 13·02 cubic feet per minute for each acre."*

MAXIMUM DISCHARGES OF THE SHANNON AND ERNE, AND A TRIBUTARY OF THE LATTER, THE WOODFORD RIVER.

RIVERS IN IRELAND.	Extent of catchment, statute acres.	Square miles.	Maximum discharge per minute in cubic feet.	Cubic feet per minute from each acre.	Cubic feet per minute from each square mile.
Shannon at Killaloe, measured previous to the commencement of Shannon Works, about	3,000,000	4687·5	1,000,000	0·33	211
Lower Erne, measured during the very high floods of Jan. 1851, at Belleek . . .	974,000	1521·9	657,511	0·67	429
Upper Erne, measured during the very high floods of Jan. 1851, at Belturbet . . .	309,000	482·8	257,771	0·83	531
Woodford River, Counties of Leitrim and Cavan, measured during the very high floods of Jan. 1851, at Ballyconnell .	90,000	140·6	101,035	1·12	717
Yellow River, or upper portion of the Woodford River, measured during the very high floods of Jan. 1851, Co. Leitrim	5,000	7·8	52,125	10·43	6675·

These results show how difficult it is to draw any inference from discharge and area of catchment alone, as the discharge, per minute per acre, must vary with the contour and elevation of the district in the same course; and with the climate, also, in different countries. We have ourselves observed the maximum discharges to vary up to 6 cubit feet per minute per acre, the lesser maximums being due to broad flat districts, and the greater maximums to higher and steeper districts, near the sources. In the Proceedings of the Institution of Civil Engineers, Ireland, vol. iv., from which we have collected and arranged some of the foregoing information, it is stated, p. 96, that the ratio of the discharge to the rain-fall, on a catchment on the Glyde, of 79,433 acres, for three months, ending March 13th, 1851, was 1·49 to 1 up to January 13th; 1·39 to 1 up to February 13th; and 3·86 to 1 up to March 13th, making a general average of 1·59 to 1; the whole rain-fall for the three months being only 5·89 inches, while the discharge was 9·35 inches! We fancy there is a mistake here. The whole catchment of the Glyde is 176,813 acres, and there is no data to show the discharge previous to or after the rain-fall from which to calculate the difference due to it *per se* for the three months; *nor is the place or method of gauging stated.* The supply from springs and the actual discharge before and after rain-fall must be correctly gauged before the proportion passing into the main channel in a given time, can be properly estimated; the results just stated clearly contradict themselves. The following anomalous results from p. 47 of the same work are also worthy of note. In five different districts the discharge is gauged, or estimated, greater than the fall, as shown in the following table. It is not stated, however, if the depths passed off, estimated over the catchments, include the flow before the commencement of the rain. If so the results are so far useless; and if they do not include it, there must be an error somewhere. Indeed, in the Robe we have evidence that not more than 58 per cent. passed from the catchment to the river, from Mr. Betagh's valuable paper, the results of which are arranged below. Also, in July, 1850, it is shown that in the Lannagh district only ·53 inch in depth passed off the catchment from a fall of 1·83 inches, or about one-third of the depth. The method of determining this was unobjectionable. Where such discrepancies as above exhibited exist, it is important that the method of gauging, and the whole calculation, should be shown, in order that other engineers should be able to judge of their accuracy; otherwise the results should be rejected, no matter under whose authority they may be published. But during the operations of the Arterial Drainage

---

* *Vide* Proceedings of the Institution of Civil Engineers, Ireland, vol. v. pp. 165 and 166, and Paper read by Thomas J. Mulvany, 11th February, 1851, vol. iv. p. 21.

READ 11TH MARCH, 1851.

District.	River.	Catchment in Acres.	DECEMBER 1850.		JANUARY 1851.	
			Total fall of rain by gauge in inches.	Total depth of discharge off catchment in inches.	Total fall of rain by Castlebar gauge n inches.	Total depth of discharge off catchment in inches.
Saleen . .	Saleen .	2,625	3·55	6·26	6·33	9·20
Lannagh .	Castlebar.	20,640		5·46		8·55
Balla . .	Manulla .	33,500		5·46		8·18
Mask and Robe	Robe .	70,000		,,		7·39
Dalla . .	Dalla .	3,200	4·00	6·527	,,	,,
	Owenmore	32,000		5·705	,,	,,

Commission in Ireland, from 1845 up to the year 1853, science was at a discount.

The following information has been collected and arranged by us from a paper by Mr. Betagh, in the Proceedings of the Institution of Civil Engineers, Ireland, vol. iv. In January 1851, 3·41 inches of rain fell in seven days, producing the maximum discharge of 85,836 cubic feet; while in December 1852, 3·17 inches, also falling in seven days, produced 115,656 for the maximum. At the beginning of the first fall

TABLES showing in detail, for the years 1851 *and* 1852, *the Monthly Fall of Rain and the corresponding Discharge of the River Robe, at Ballinrobe, County Mayo; the catchment basin being* 70,000 *acres, or* 110 *square miles; the lower end* 100 *feet, the upper end* 336 *feet; and the average height of the surface about* 180 *feet above the level of the sea. The average fall of the river, not including the rapids, is from one to two feet per mile; the catchment is about* 20 *miles long, about one-tenth of the area bog or low marsh, and nine-tenths clayey and gravelly. The river is about* 33 *miles long.*

RIVER ROBE OBSERVATIONS IN 1851.

MONTHS.	Rain-fall each month in inches.	Discharge each month of rain-fall in inches.	Discharge in cubic feet per minute, from a catchment of 70,000 acres, for each month.			Discharge in cubic feet per minute, per acre, for each month.		
			Maximum.	Minimum.	Average.	Maximum.	Minimum.	Average.
January .	9·2	7·4	85,836	20,133	43,373	1·158	·287	·620
February .	6·8	4·7	72,448	18,420	30,410	1·034	·263	·434
March . .	4·4	3·6	49,137	10,860	20,945	·702	·155	·300
April .	3·4	2·5	24,200	5,760	14,355	·345	·082	·205
May . .	1·0	0·8	5,820	4,125	5,001	·083	·059	·071
June . .	3·8	0·8	7,040	1,114	4,230	·100	·016	·060
July . .	3·8	0·5	4,920	1,500	2,558	·070	·021	·036
August .	2·4	0·9	17,055	1,240	4,866	·243	·017	·069
September	1·9	0·5	4,746	1,200	2,854	·067	·017	·040
October .	5·0	1·6	23,980	6,940	12,588	·342	·099	·179
November	1·3	1·2	12,852	6,000	7,827	·183	·085	·111
December .	2·6	2·5	44,715	6,210	14,373	·638	·088	·205
Total .	45·6	27·	352,749	83,502	163,380	4·965	1·189	2·33

RIVER ROBE OBSERVATIONS IN 1852.

Continued from last page.

MONTHS.	Rain-fall, each month, in inches.	Discharge, each month, of rain-fall, in inches.	Discharge in cubic feet per minute, from a catchment of 70,000 acres, for each month.			Discharge in cubic feet per minute, per acre, for each month.		
			Maximum.	Minimum.	Average.	Maximum.	Minimum.	Average.
January .	7·5	5·2	41,600	12,852	28,730	·594	·183	·410
February .	4·8	4·3	56,400	8,190	25,296	·805	·117	·361
March .	1·0	0·7	9,600	2,737	6,702	·137	·039	·095
April. .	1·1	0·5	3,931	1,468	2,477	·056	·020	·035
May . .	1·9	0·4	3,931	1,050	1,861	·056	·015	·026
June. .	6·6	1·2	22,764	1,400	6,547	·325	·020	·093
July .	2·5	1·0	15,439	3,172	6,057	·220	·045	·087
August .	4·5	0·6	3,856	2,236	3,070	·055	·032	·043
September	1·8	0·5	3,427	2,642	2,874	·048	·037	·041
October .	3·9	1·0	32,040	1,114	5,932	·457	·016	·084
November	5·5	5·2	45,360	17,000	30,742	·648	·242	·439
December.	12·0	9·5	115,656	23,232	54,846	1·657	·331	·783
Total .	53·1	30·1	354,004	77,093	175,134	5·058	1·097	2·497

there was flowing 26,640 feet, leaving the effects of the seven days' rain 85,836 − 26,640 = 59,196 cubic feet, while in the second year the quantity flowing at first was 75,360 cubic feet, leaving the effects of the seven days' rain-fall equal to 115,656−75,360=40,296 cubic feet. The effect of the previous state of the weather on the catchment must always modify, to a considerable extent, the discharge from a given rain-fall, and this has more to do with the results than the effect of arterial drainage itself, unless so far as one is a result of the other. Taking the mean of 1851 and 1852, it appears that the evaporation and absorption in the Ballinrobe catchment were to the rain-fall as 41·6 to 98·7, or about 42 per cent. This is certainly, from the nature of the catchment, less than the average through Ireland, which cannot be less than 60 per cent. In high, steep districts, fully three-fourths, or 75 per cent., of the rain-fall can be collected, and at times, when the catchment is saturated, nearly the whole; even in some few limited cases, when springs or hidden supplies are re-tapped, a larger discharge may take place than that due to the catchment and rain-fall; but these do not affect the general question.

The effects of absorption and evaporation are very variable; sometimes 58 or 60 per cent. of the annual fall is carried off in this way from ordinary flat tillage soils, and other estimates are much higher; much, however, depends on the soil, subsoil, inclination, stratification, geological formation, and season. The evaporation from water surfaces exceeds the annual fall in these countries by about one-third; and that from flat, marsh, and callow lands exceeds the evaporation from ordinary tillage, porous, and high lands. When the flat lands along the banks of rivers extend considerably on both sides, an extra fall is necessary into the main channel, along the normal drains, otherwise such lands must suffer from excessive evaporation as well as floods. Evaporation and absorption also vary with the climate, but in this country we may safely assume that one-third of the whole rain-fall passes on to the sea.

The absorption and evaporation must not, however, be taken as proportionate to the rain-fall. From 14 to 16 inches from land (and about 33 inches from water) may be taken in this country as the allowance to be made; equivalent to an average of about 15 inches. The evaporation from the surface of reservoirs is said to be about 4 feet in India. But it is probably greater.

In a paper in the Journal of the Royal Agricultural Society of England, vol. v. part 1, 1844, Mr. Josiah Parkes shows, that 42½ per cent. of the whole annual rain of England filters through the soil, and 57½ per cent. evaporated, being the mean results of eight years' observations, from 1836 to 1843, both included. The mean evaporation and filtration for each month during this period are shown and arranged by us in the following table :—

MONTHS.	Total falling.	Evaporated.		Remaining.		Deposited in tons and cubic feet per acre.	
	Inches.	Inches.	Per cent.	Inches.	Per cent.	Cubic feet.	Tons.
January . .	1·847	·540	29·3	1·307	70·7	4,744	132
February . .	1·971	·424	21·6	1·547	78·4	5,616	156
March . .	1·617	·540	33·4	1·077	66·6	3,910	109
April . . .	1·456	1·150	79·0	0·306	21·0	1,111	39
May . .	1·856	1·748	94·2	0·108	5·8	392	11
June . . .	2·213	2·174	98·3	0·039	1·7	142	4
July . .	2·287	2·245	98·2	0.024	1·8	87	2·4
August . .	2·427	2·391	98·6	0·036	1·4	131	3·6
September .	2·639	2·270	86·1	0·369	13·9	1,339	37
October . .	2·823	1·423	50·5	1·400	49·5	5,082	141
November .	3·837	0·579	15·1	3·258	84·9	11,826	328
December . .	1·641	0·164	00·0	1·477	100·0	6,552	182
Yearly averages	26·614	15·320	57·6	11·294	42·4	40,932	1145

The maximum quantity, 32·10 inches, fell in 1841, and the minimum in 1837, 21·10 inches. The maximum and minimum quantities respectively which fell in January were 3·95 and ·31 inches; in February 2·85 and 1·02 inches; in March 3·65 and 0·34 inches; in, April 2·57 and ·34 inches; in May 5·00 and ·70 inches;

in June 3·31 and 1·33 inches; in July 4·36 and 1·30 inches; in August 3·65 and 0·95 inches; in September 4·50 and 0·63 inches; in October 4·82 and 1·41 inches; in November 5·77 and 2·05 inches; and in December 3·02 and ·40 inches. The greatest quantities fall in September, October, and November, and the least in February, March, and April. The general mean fall for England is said to be 31¼ inches, and near London 25 inches.

The amount of rain varies, not only at different places and different elevations, but also at different elevations in the same place. The following table shows the amount of rain collected in each month in 1855 at Greenwich Observatory, at different elevations:—

Month in 1855.	Osler's anemometer gauge, inches.	On the roof of the library.	Cylinder partly sunk in the ground.
January	0·2	1·0	1·5
February	0·2	1·4	1·0
March	0·5	1·3	2·0
April	0·1	0·1	0·1
May	0·5	1·5	1·8
June	0·5	0·7	0·9
July	3·1	4·8	5·3
August	0·6	0·8	1·4
September . . .	0·8	1·1	2·0
October	2·6	4·5	5·2
November . . .	0·5	1·1	1·5
December . . .	0·4	0·9	1·1
Totals . .	10·0	19·2	23·8

The cylinder gauge was placed 155 feet above the level of the sea; the gauge on the roof of the library 22 feet over the cylinder gauge, and Osler's anemometer gauge 28 feet higher than the gauge on the roof of the library. In the valleys in the lake districts of Westmoreland and Cumberland, the annual fall varies occasionally from 50 to 100 inches, and the maximum fall is said to obtain at about 2000 feet above the level of the sea on high catchments.

At Ballinrobe, a gauge placed on the church tower, 60 feet above the ground, indicated 42 per cent. less rain than one on the ground; and another experiment with a change of gauges, gave 68 per cent. less at the greater elevation!

At Kinfauns Castle, Scotland, a gauge 600 feet high on a hill, gave 41½ inches, while one at the base, 580 feet lower, gave only 25½ inches. In Keswick, the fall is 65½ inches, and in Carlisle only 30 inches. At Kendal the fall is 60 inches; at Manchester 33 inches; at Lancaster 45 inches; at Liverpool 34 inches.

From the 23rd of February to the 6th of June, 1860, the rain at Dublin was 8 inches. At the Leefin Mountain, which is 2000 feet high, the rain was 13·1 inches. From the 23rd of February to the 9th of July, the rain at Dublin was 10·674 inches; and at the same time, on the Leefin Mountains (over Ballysmutten), 18·1 inches; that is, an increase of nearly 80 per cent. in that time. From the 23rd February to the 21st August, inclusive, the rain-fall at Dublin was 17 inches; at Blessington 21 inches; at Ballysmutten, on the site of a proposed reservoir, 27 inches. This showed an increase over Dublin of 10 inches. It would appear that from 50 to nearly 80 per cent. more rain fell at Ballysmutten than at Dublin. It would however have been more correct to compare the rain-fall at Kingstown or Bray with that on the adjacent mountains than the rain-fall of Dublin.

Experiments were made at York in 1832, 1833, and 1834, for the British Association, with three gauges —the first placed on a large grass plot in the grounds of the Yorkshire Museum; the second at a higher elevation, 43 feet 8 inches, on the roof of the Museum; and the third on a pole 9 feet above the battlements of the great tower of the Minster, at an elevation over the gauge on the ground of 212 feet 10½ inches. The quantities received were as follows:—

	Depth for three years.	Average depth for one year.
First gauge . . .	64·430 inches . .	21·477 inches
Second gauge . . .	52·169 ,, . .	17·389 ,,
Third gauge. . .	38·972 ,, . .	12·991 ,,

Professor Phillips gives the following formula for calculating the difference between the ratios of rain falling on the ground and at any height h in the same place—t° the temperature of the season, and c a co-efficient dependent upon it; then the difference d is

$$d = c\,h\,\frac{t^\circ}{110}.$$

The mean height at which rain begins to be formed by this formula is 1,747 feet over the ground; and at 356 feet high, the depth which falls is one-half of what falls on the ground.*

A discussion of the mean temperature in connexion with the fall of rain, was completed at Greenwich for the years 1852, 1853, and 1854; and at Oxford for the years 1855, 1856, and 1857. The result shows an average of 160·3 rainy days at Greenwich for each year, and 146·6 at Oxford. The difference of the mean temperatures of the day of rain and the day before is less than that of the day of rain and the day after.

---

* *Vide* Civil Engineer and Architect's Journal for 1860, p. 167.

	Mean temperature, day before rain.	Mean temperature, day of rain.	Mean temperature, day after rain.
Greenwich observations	49·25°	49·27°	48·98°
Oxford do.	49·50	49·63	49·44

Dividing the winds into two groups, northerly and southerly, the Oxford observations give the direction for 218·5 days' fair weather. The wind was northerly for 131·5 days, and southerly for 87 days. For the remaining 146·5 rainy days, the wind was northerly for 64·5 days, and southerly for 82 days.

SEWERAGE.

"The future population of the suburbs of London is calculated at 30,000 inhabitants per square mile. According to the following data, some of the densest portions of our large towns have a population of 220 persons to an acre. The population on the north side of the Thames is about 75 persons per acre, and on the south side 28 persons per acre. Taking the average density of population in our twenty-one principal towns, there appear to be 5045 inhabitants to the square mile; but, from the following table, extracted from Dr. Duncan's report on Liverpool, it will be seen that if we select five of our most populous cities, the average in these is much greater, while in others, it is equally certain that the crowding is far less than the general standard to which we have referred :—

Towns.	Inhabitants to a Square Mile. Total Area.	 Builded Area.
Leeds	20,892	87,256
London	27,423	50,000
Birmingham	33,669	40,000
Manchester	83,224	100,000
Liverpool	100,899	138,224

Dr. Duncan, however, states that there is a district in Liverpool containing 12,000 inhabitants crowded together on a surface of only 105,000 square yards, which gives a ratio of 460,000 inhabitants to the geographical square mile. In the East and West London Unions, Mr. Farr has estimated that there are nearly 243,000 inhabitants to a geographical square mile ; but, great as this overcrowding is, the maximum density of Liverpool is nearly double that of the metropolis." *

Great Towns.—The Registrar-General estimates the population of the metropolis in the middle of the year 1870 at 3,214,707, being 41·2 persons to an acre. This is London with the suburbs, from Hampstead to Streatham, and from Woolwich to Hammersmith. He estimates the population of Liverpool in the middle of the year 1870 at 517,567, or 101·3 persons to an acre; Manchester, 374,993, or 83·6 per acre ; and Salford, 121,580, or 23·5 per acre ; Birmingham, 369,604, or 47·2 per acre ; Leeds, 259,527, or 12 per acre ; Sheffield, 247,378, or 10·8 per acre ; Bristol, 171,382, or 36·6 per acre ; Bradford, 143·197, or 21·7 per acre ; Newcastle-upon-Tyne, 133·367, or 25 per acre ; Hull,

---

* Illustrated News, September 8th, 1855.

130,869, or 36·7 per acre; Portsmouth, 122,084, or 12·8 per acre; Leicester, 97,427, or 30·4 per acre; Sunderland, 94,257, or 19·6 per acre; Nottingham, 88,888, or 44·5 per acre; Norwich, 81,087, or 10·9 per acre; Wolverhampton, 72,990, or 21·5 per acre. The area taken is the municipal boundary in all cases except London. The population of Edinburgh is estimated at 178,970, or 40·4 per acre; of Glasgow, 468,189, or 92·5 per acre; of Dublin, with some suburbs, 321,540, or 33 per acre. The population of these twenty towns of the United Kingdom is thus estimated at 7,209,603. The population of Paris is estimated at 1,889,842; of Vienna, 605,200; of Berlin, 702,437.

The amount of sewage is calculated at about FIVE CUBIC FEET PER PERSON, including the supply from manufactories, breweries, distilleries, &c. So high as SEVEN FEET PER HEAD has been recommended as data to calculate from by Captain Galton and Messrs. Simpson and Blackwell, in their Report on the Main Drainage, and it has been found that about half of the estimated quantity of sewage would be passed off in six or eight hours.

In calculating the size of sewers, however, the rainfall must be provided for, in addition to the sewage matter from houses and public establishments. Mr. Bazalgette calculated this for the London sewerage at $\frac{1}{4}$th of an inch fall in 24 hours in the urban districts, and $\frac{1}{8}$th of an inch for the suburban districts. Captain Galton and Messrs. Simpson and Blackwell assumed $\frac{2}{5}$ths of an inch fall during eight hours' maximum flow. This would be 1,452 feet per acre.

Assuming the highest data, we shall have to provide sewers to discharge in eight hours

1,452 cubic feet of rain water per acre,
3½ cubic feet of sewage nearly per person.

Assuming a population of 80 persons per acre, then these figures would become

1,452 cubic feet for rain,
280 cubic feet for sewage, { in eight hours, *or about 3½ cubic feet per minute, per acre,*

which shows that the sewage is not more than ⅕th of the rain water; and that, in calculations for the size of sewers, the surface water is the most important element to be considered. If we had assumed a larger fall of rain, the difference between sewage and rain would be greater. On the 20th June, 1857, the day after heavy rain, the referees on the Metropolitan Drainage question found the Norfolk-street sewer to discharge 3 feet; the Essex-street sewer 5¼ feet; the Northumberland-street sewer 3¾ feet; and the Savoy-street sewer 20½ feet per minute per acre; but the last result has been controverted.

It appears that the daily amount of sewage varies from 4·8 cubic feet per head in the more thickly inhabited portions of London, occupied by a larger portion of the poorer classes, to 8 cubic feet per head in the western districts, where the value of water is more appreciated, and the cost less a matter of consideration; and the average of the whole metropolitan districts appears to be 5·8 cubic feet per head per diem. If the day be divided into three periods of eight hours each, the amount of the maximum flow is between nine A.M. and five P.M. and 49 per cent. of the whole; whilst only 18 per cent. flows during the eight hours of minimum flow, which occur between eleven P.M. and seven A.M.* The advantage of storm flows in flushing is shown by the heavy rain which occurred on the 20th of June, causing a flow in the Savoy-street sewer which was equivalent to 20 times the ordinary flow at the time. This was six times the maximum flow, and *although the sewer had been scoured, to a considerable extent, by a heavy fall of rain on the previous night, the sample contained more than double the amount of total impurity contained in specimens of ordinary sewage.*

In a town district, such as that drained by the Savoy and Northumberland-street sewers, the quantity running off into sewers, within six hours after the fall, varies from 10 to 60 per cent. of the quantity fallen. Of the rain during the storm of the 20th June, 1857, nearly one inch-and-a-quarter in an hour, 65 per cent. ran off within 15 hours of the fall, viz.:—

46 per cent. in 45 minutes after the rain ceased,
14 ,, in the next 6¾ hours,
5 ,, in the next 7½ hours.

In a suburban locality, such as the Counter Creek sewer drain, the quantity reaching the sewers would vary from 0 to 30 or 40 per cent. in 24 hours after the rain.†

In the Holborn and Finsbury divisions Mr. Roe calculated that an 18-inch cylindrical pipe, laid at an inclination of 1 in 80, is sufficient for 20 acres of house sewage, while a 5-inch pipe, laid at an inclination of 1 in 20, is necessary for 1 acre, and a 3-inch pipe, laid also at 1 in 20, for ¼ acre. A pipe 30″ in diameter, laid with an inclination of 1 in 200, would discharge 1700 cubic feet per minute, and perfectly drain 200 acres of urban land covered with houses to the extent of 4000 or upwards, and each house having a water supply of 150 gallons per diem. In each of these cases, however, the discharge must depend on the head and length of the pipe as well as the inclination at which it is laid. Assuming the inclination of those pipes to correspond with the hydraulic inclination, we have calculated their discharging powers with water to be respectively 807, 72, 20, and 1700 cubic feet per minute, the areas to be drained being 20, 1, ¼, and 200 acres. *In all calculations of this kind it is necessary to ascertain not only the maximum rain-fall per hour, but also the proportions discharged per hour, according to the season and district, into the main channel, as well as the junctions or places of arrival.* In urban districts, 1500, 2100, and sometimes 3600 cubic feet per hour per acre, have to be discharged after extraordinary rain-falls. These may be taken as maximum results. The gaugings of the Westminster sewers in summer give 53 cubic feet per hour for the urban, and 17 cubic feet for the suburban, according to Mr. Hawkins.

---

* Metropolitan Main Drainage Report, pp. 15, 17.

† Ibid., pp. 75, 76.

In urban districts, however, a much larger quantity of water is conveyed more rapidly, *cæteris paribus*, to the mains, than in suburban districts and catchment basins generally, in which the maximum discharge per acre per hour, even in the steeper and higher districts, seldom exceeds 700 cubic feet, and varies from about 20 cubic feet for the larger and flatter districts upwards. This arises from the impervious nature of the surfaces it falls upon in towns, and the lesser waste in passing to the drains, as well as a large portion of the supply being often artificial. From 70 to 90 cubic feet * per acre per hour, is generally taken for the maximum discharge from the average number of catchment basins; this is nearly equal to a supply of one-fiftieth part of an inch in depth from the whole area.

SECTION XIII.

WATER SUPPLY FOR TOWNS.—STRENGTH OF PIPES.—SEWERAGE ESTIMATES AND COST. — THOROUGH-DRAINAGE.—ARTERIAL DRAINAGE.

SUPPLY.—QUALITY.

The supply of water to towns has become latterly a subject of considerable importance. Three points have to be considered;—FIRSTLY, a sufficient and constant supply at high pressure, when it can be obtained within a reasonable expenditure; SECONDLY, the quality; and, THIRDLY, the cost. The advantages in towns of high pressure are now apparent to all in overcoming fire; fronts of houses and pavements may also be cleaned, and streets watered if the supply be abundant. The highest apartments can be supplied, and even mechanical power can be obtained for many purposes, as grinding coffee, at a reasonable cost. Mr. Glynn says,† "In many parts of London water is

* Some interesting observations on rain-fall and flood discharges are given in the Transactions of the Institution of Civil Engineers, Ireland, for 1851, pp. 19–33, and pp. 44–52.

† Power of Water.—WEALE.

supplied at 4*d.* for 1000 gallons, at a pressure of 150 feet: a gallon of water weighs 10 lbs., so that 1000 gallons of water falling 150 feet, are equal to 1,500,000 lbs. falling one foot; and if 1500 gallons of water be used in one hour, they are equal to 37,500 lbs. falling one foot in one minute, or somewhat more than a horse's power, which is 33,000; therefore, it may be assumed, that the cost of a horse's power for an hour in such cases, is only 6*d.*"

The number of gallons of water required for the supply of each person, including all collateral uses, has been differently estimated, and varies in almost every town, and even in the same city—London, for instance, when supplied by different companies and under different systems. 44 gallons per head, per diem, were supplied by the several companies of London in 1853, while evidence has been given to show that the actual average consumption for all purposes did not exceed 10 gallons per head, per diem; the remainder having been wasted under an imperfect system of distribution. It is asserted that when the supply is 25 gallons per head, per diem, that 5 gallons of it are used for purposes requiring filtration, 10 gallons for purposes not requiring filtration, and 10 gallons wasted, or two-fifths of the supply. As there must be a considerable loss under even the best system of supply, we may assume, with the Board of Health, that a *minimum supply of* 75 gallons per house, per diem, or 15 gallons per person, per diem, is necessary.

The following is an abstract of the average number of gallons of water furnished per diem, by different water companies in London, during the year 1853, to each house, including manufactories and public establishments as houses:—

	Gallons.	
	Per house.	Per person.
New River Company	193	38·3-5
East London Water Works	187	37·2-5
West Middlesex Water Works	204	40·4-5
Grand Junction Water Works	319 336	63·4-5 67·1-5
Southwark and Vauxhall Companies' Houses	175	35
Ditto average houses, manufactories, public establishments	209	41·4-5
Chelsea Water Works	227	45·2-5
Hampstead Water Works	111	22·1-5
Kent Water Works	270	54
	2231	446·1-5
Mean values	223·1-10	44·3-5

These quantities have been calculated from the parliamentary returns made in 1854; and if there be any truth in the calculations and returns of the quantities actually consumed per person—said to be 10 gallons—we get the proportion, as 10 is to 34 so is the quantity consumed to the quantity wasted. But, even assuming the quantity consumed to be 20 gallons per head, what an immense loss is here exhibited from want of a suitable system of check and distribution.

For large towns it is safe to provide for many purposes, besides present personal or house wants; and it is safer, *where it can be done without much extra cost,* to provide for a supply of 40 gallons to each inhabitant, even if this quantity should not be used or raised.

For high pressure, the supply required will generally vary from 15 to 40 gallons, or from 2·5 to 6·5 cubic feet to each inhabitant, or for an average of about 28 gallons, including the supply to stables, offices, manufactories, and breweries.

The storage in reservoirs should be for about 120 or 160 days' supply, including the quantity necessary for mills and riparian occupiers lower down. This latter is taken very often at about half the former; so that two-thirds of the storage may be available for the town, and one-third for mills and riparian lands. The actual relation, however, depends on local circumstances.

The quality of water for drinking, washing, or cooking, is also an important element in selecting a source of supply. Hardness is measured by the number of grains of chalk or carbonate of lime to a gallon of water, each called a degree. The average hardness of spring water is about 26°, that is, 26 grains of carbonate of lime to one gallon of water. Rivers and brooks have an average hardness of 13°, and water derived from surface drainage 5°; hence the great advantage of the latter kinds of water in washing. The average hardness of the London pipe waters is from 10° to 16°. The following extracts from a report and analyses furnished to me, in 1855, by Professor Sullivan, of the Museum of Irish Industry, Dublin, will show what is generally required on this head:—

"On the annexed page you will find the numerical results of my analyses of the four samples of water which you left with me for examination. From the table you will perceive that the water of the Mattock River appears to be the purest, so far as the nature and the amount of the foreign substances held dissolved in it is concerned. The water of the Boyne comes next in quality to that of the Mattock River, the pump water being in every sense the worst, so far as amount of ingredients can be taken as a test of the quality of a water; in this respect, indeed, it resembles the water of the deep wells of London and elsewhere.

"As the ordinary mode in which the quality of a water, for drinking and for culinary and like purposes, is judged of is, by the comparative amount of organic matter, the total amount of dissolved matter, and its hardness, according to the 'soap test,' I shall give in the following table the numbers representing each of these qualities:—

TABLE showing the number of grains of Organic Matter, and the number of grains of Solid Matter, in an Imperial Gallon of

Water from	Number of Grains of Organic Matter, per Imperial Gal.	Number of Grains of Solid Matter, per Imperial Gal.	Degree of Hardness according to the Soap Test.
No. 1. Tullyescar . .	8·975 grs.	31·175	15 8-10ths.
,, 2. River Mattock .	2· (about)	15·360	9 1-10th.
,, 3. River Boyne .	3·250	22·700	14 9-10ths.
,, 4. Burn's Pump .	7·100	76·850	34 4-10ths.

"In order to render this table more instructive, it may be well to subjoin a few of the results obtained from the analyses of the waters of other localities.

TABLE showing the number of grains of Solid Matter contained in one gallon of the following Water:

Thames at Greenwich	27·9 grains.
,, London	28·0 ,,
,, Westminster	24·4 ,,
,, Twickenham	22·4 ,,
,, Teddington	17·4 ,,

New River (London)	19·2 grains.
Lea ,, ,,	23·7 ,,
Trafalgar Square Fountain, Deep Well	68·9 ,,
Well in St. Giles's, Holborn	105·0 ,,
Artesian Well at Grenelle (Paris)	9·86 ,,

"The following are some of the results obtained from an examination of the waters in the neighbourhood of Dublin, or which have been proposed as a source of supply :*—

Locality from whence Water was obtained.	Total Number of Grains per Imperial Gallon.	Total Number of Grains of Organic Matter.	Degree of Hardness according to the Soap Test.
			degs.
Royal Canal (12th Lock)	21·0	2·80	14·0
Grand Canal (7th Lock)	16·300	2·30	10 3-4ths.
River Liffey, at Kippure	3·522	1·90	0 2-10ths.
,, Phoulaphouca	5·125	1·50	0 2-10ths.
Lough Dan, Co. Wicklow	2·800	1·225	0 8-10ths.
River Dodder, at City Weir	8·350	1·625	1 8-10ths.
Lough Owel	10·225	1·550	6 7-10ths.

* Dr. Apjohn gave the following analyses :—

	Total matter dissolved.	Organic matter.	Hardness.
Grand Canal—mean of seven analyses	20·78	·95	15·9
Royal Canal—mean of five analyses	20·76	1·64	14·1
Liffey—mean of eleven analyses	8·62	1·77	6·1

Analysis of the deposition on pipes from the Portobello basin :—

Water	2·20
Organic Matter	9·71
Sand	10·20
Peroxide of Iron and Alumina	3·50
Carbonate of Lime	74·20
Carbonate of Magnesia	·19
	100·

Professor Apjohn gave the following analyses of waters furnished to the city of Dublin in 1860. It shows how necessary it is to distinguish the time of taking specimens for analysis, and the previous state of the weather as affecting the foreign matters in the water. The specimens were collected on the 5th and 19th of May, 1860. The quantity operated upon in each instance was an imperial gallon, or 277·273 cubic inches:—

CITY WATER COURSE, DODDER.

	5th May.	19th May.	
Carbonate of lime	4·056	7·308	Specific gravity of specimen (5th May) 1·00011. Specific gravity of specimen (19th May) 1·00014.
Carbonate of magnesia		0·700	
Sulphate of lime and chlorides of sodium and magnesium	2·269	2·171	
Silex	0·166	0·526	
Organic matter	1·811	1·101	
	8·302	11·806	

PORTOBELLO BASIN.

	5th May.	19th May.	
Carbonate of lime	7·687	11·660	Specific gravity of specimen (5th May) 1·00023. Specific gravity of specimen (19th May) 1·00031.
Carbonate of magnesia		0·764	
Sulphate of lime and chlorides of sodium and magnesium	4·058	3·751	
Silex	0·073	0·194	
Organic matter	3·308	2·289	
	15·126	18·658	

It will be observed that the quantities of saline and other ingredients found in specimens of the same water collected at the two separate periods above mentioned are materially different; those obtained at the later date (May 19) containing the larger portion of foreign matters. The extent of this variation is very considerable, and it appears to Dr. Apjohn to have been the consequence of a very considerable fall of rain, which took place in the interval between the periods at which the specimens were taken up for analysis.

When the means of the preceding analyses are taken, we obtain the following results :—

	City Water Course.	Portobello Basin.
Mean amount of saline matter	8·598	14·094
,, ,, organic matter	1·456	2·798

The quality of a water for drinking purposes depends in a great degree upon the condition in which the organic matter is found, much more than upon its quantity. This is, however, a question outside of the domain of chemistry, and can only be solved by the aid of the microscope.

As a general rule, the water of clear flowing rivers, even though it may contain a large amount of solid matter, and even of organic matter, will be found wholesomer than well water, especially in towns.

The whole of the lime and magnesia existing as carbonates is precipitated by boiling, the water being thus proportionably rendered less hard; lime and magnesia existing as sulphates or chlorides, on the other hand, are not precipitated. This difference is of great consequence in culinary operations, as where boiled water is used, the carbonates of lime and magnesia are not injurious, and if no sulphates or chlorides be present, the water may be soft after boiling. The same observation applies to water used for washing clothes when boiled. And lastly, sulphate of lime forms one of the worst elements of fur or deposit upon steam boilers.

Tabular Results of the Special Analyses of Four Samples of Water from the neighbourhood of Drogheda.

Nature of dissolved matter.	No. 1. Tullyescar.	No. 2. Mattock River.	No. 3. Boyne River.	No. 4. Burn's pump water.	Observations.
Carbonate of lime . .	9·350	7·302	11·648	21·475	Inclusive of a very small quantity of phosphate of lime and iron not separated from the lime.
Carbonate of magnesia	0·429	0·510	0·888	0·585	
Sulphate of lime . .	9·043	2·514	4·459	4·568	
Chloride of magnesium	0·743	1.258	1·685	8·445	
Chloride of calcium .	..	..	..	9·524	
Chloride of sodium .	..	0·991	..	..	
Magnesia existing as crenate, &c., in the water	0·464	..	..	..	
Lime do. do.	..	..	..	0·548	
Silica do. do.	0·627	..	0·322	2·212	
Potash and soda existing in water, as nitrates, crenates, and other organic salts. .	1·544	2·785	0·448	22·393	
Organic matter . .	8·975	..	3·250	7·100	
Total number of grains per Imperial gallon .	31·175	15·360	22·700	76·850	

The saving in soap effected by a reduction of 10 degrees in hardness, is found to be over 50 per cent.

Some of the metropolitan waters analysed by Dr. Robert Dundas Thomson, F.R.S., were found, in May, 1860, much more impure than others, the samples of which had been taken at the beginning of the month, before the impurities conveyed by the rains had contaminated them. The supply afforded by large and small rivers, as in London, in this table, contrasts most unfavourably with that afforded by the drainage of mountain ridges, as at Glasgow and Manchester. The specimens of water from the two latter cities were taken by the instructions of Mr. Bateman, F.R.S., the engineer, from the main pipes during the month. It should be the object of the London Companies to avoid pumping the water in its most impure state, and to store it when in the condition of the greatest purity.

	Total Impurity per gallon.	Organic Impurity per gallon.
	Grs., or °.	Grs., or °.
Distilled water	0·0	0·0
Loch Katrine water, new supply to Glasgow	3·16	0·96
Manchester water supply	4·32	0·64
THAMES COMPANIES:—Chelsea	17·84	1·48
Southwark	17·08	1·64
Grand Junction	20·72	2·00
West Middlesex	20·08	2·08
Lambeth	20·80	2·40
OTHER COMPANIES:—New River	18·52	1·56
East London	23·64	3·20
Kent	21·68	2·96

The table is read thus:—Loch Katrine water contains in the gallon 3·16 degrees, or grains, of foreign matter in solution, of which ·96 degrees, or grains, are of vegetable or animal origin.

SOURCES AND GATHERING GROUNDS.

The sources from which a water supply for towns may be derived are lakes, rivers, and streams, springs, wells, and gathering grounds. Of the latter it may be said that, however ably put forward under the auspices of the Board of Health, it is far safer to resort to good river waters than trust to what has been termed, with some satirical truth, "new-fangled schemes of pot-piped gathering grounds." Springs and wells afford, at best, but a partial supply unless for villages or manufactories; and we must almost always trust to lakes, rivers, or

TABLE showing the Quantities of Gathering Ground and Reservoir Room to supply a given population with 15, 30, *and* 40 *gallons of water per head per diem. The reservoir room is calculated to hold* 12 *inches in depth of rain-fall per mile as a guide for lesser depths. For* 4 *inches the results are to be divided by* 3 *; and for* 6 *inches by* 2.

Population can be supplied at 15 gallons per head per diem.	Population can be supplied at 30 gallons per head per diem.	Population can be supplied at 40 gallons per head per diem.	Cubic feet per minute required for the supply. To find the net size and inclination of a pipe to convey it, see table, pp. 28 & 29.	Millions of gallons per diem required for the supply.	Gathering ground in square miles required for the supply for a depth of 12 inches collected, equal to a supply of 53 cubic feet per minute per mile.	Reservoir room required in millions of cubic feet per square mile of catchment, to hold a supply of 12 inches of rainfall.
2,500	1,250	937	4·179	·0375	·0789	2·196
5,000	2,500	1,875	8·358	·075	·1577	4·393
7,500	3,750	2,812	12·536	·1125	·2366	6·589
10,000	5,000	3,750	16·715	·15	·3154	8·786
12,500	6,250	4,687	20·894	·1875	·3942	10·982
15,000	7,500	5,625	25·072	·225	·4731	13·179
17,500	8,750	6,562	29·251	·2625	·5519	15·375
20,000	10,000	7,500	33·430	·300	·6308	17·572
25,000	12,500	9,375	41·788	·375	·7885	21·965
30,000	15,000	11,250	50·145	·45	·9462	26·358
35,000	17,500	13,125	58·5	·525	1·1039	30·75
40,000	20,000	15,000	66·9	·6	1·2616	35·144
45,000	22,500	16,875	75·217	·675	1·4193	39·537
50,000	25,000	18,750	83·57	·75	1·577	43·93
55,000	27,500	20,625	91·932	·825	1·734	48·32
60,000	30,000	22,500	100·29	·9	1·8924	52·716
65,000	32,500	24,375	108·65	·975	2·0501	57·109
70,000	35,000	26,250	117·	1·05	2·2078	61·502
75,000	37,500	28,125	125·36	1·125	2·3655	65·895
80,000	40,000	30,000	133·72	1·2	2·5232	70·288
85,000	42,500	31,875	142·1	1·275	2·6809	74·681
90,000	45,000	33,750	150·435	1·35	2·8386	79·074
95,000	47,500	35,625	158·8	1·425	2·970	83·467
100,000	50,000	37,500	167·15	1·5	3·154	87·86
105,000	52,500	39,375	175·5	1·57	3·311	92·25
110,000	55,000	41,250	183·86	1·65	3·469	96·64
115,000	57,500	43,125	192·22	1·72	3·62	101·10
120,000	60,000	45,000	200·58	1·8	3·785	105·43

streams, with reservoirs, for stowage, for a sufficient supply for large towns. The Croton aqueduct, conveying water with an average of three degrees of hardness, to New York, is perhaps the noblest work for water supply of modern times. The length of the aqueduct is about 44 miles, with a channel inclination of about 15 inches per mile. The receiving reservoir is about two miles higher up the channel than the distributing reservoir, which latter is 115 feet over the level of the sea, and commands the highest buildings of the city. In the driest weather the supply is equal to 28,000,000 gallons.* The cost of the work, including the purchase of land and water rights, was 8,575,000 dollars, or £8 per lineal foot nearly. The cost of distributing pipes was 1,800,000 dollars. We have had also the Loch Katrine and Glasgow aqueduct, a noble work, constructed after this model by Mr. Bateman, notwithstanding the previous supply of that city, or a portion of it, the Gorbals, from gathering grounds at a high level. The Vartry supply for Dublin, carried out under the same engineer and Mr. P. Neville, the city surveyor, is derived by embanking and storing the waters of the River Vartry. It is, however, sometimes necessary to make use of such grounds, particularly when flanking or lying above glens where an embankment may be easily thrown across, and the supply stored for use, which would otherwise pass quickly off. The table, page 353, gives the areas of reservoirs and gathering grounds according to a collection of one foot in depth from the catchment; it can be easily modified when the storage or required supply exceeds or falls short of this depth. *One acre of gathering ground with a collection of twelve inches of rain-fall from it annually will give a daily supply of five cubic feet per head to twenty-four inhabitants; or of four cubic feet, or twenty-five gallons, to thirty inhabitants.*

---

* Schramke's Croton Aqueduct, New York.

The next table will be of use in showing the actual quantities which have been collected, or could have been collected, for storage. Homersham, Hughes, and Beardmore's books have been consulted in arranging it.

The various methods employed for purification may be classed under three heads: mechanical, by filtering or straining; chemical, or antiseptic media, such as peat and animal charcoal, and precipitation by the use of lime water; and the natural precipitation of impurities when the water is at rest, as well as the purification which takes place from oxidation and neutralization on thorough exposure by the ozone of the atmosphere. This latter plan has, however, been tried, and signally failed. Filter beds may be constructed to have a surface area of one square yard for every 800 gallons filtered in twenty-four hours. For executed works the proportions vary from 1 in 460 to 1 in 1140.

COST.

With reference to cost, the following tables, arranged by us from various sources, will afford information from works executed.

The estimated cost of the water supply for Dublin from the Vartry was £300,000 for 12,000,000 gallons

TABLE showing information with reference to size of Reservoirs, Catchment Areas, &c., collected and arranged from various authorities. The first, fifth, and sixth columns contain information with reference to reservoirs and the collecting areas; the second, third, and fourth, show for different districts the whole rainfall, and the portions or per centage flowing off and available.

Names of Drainage Areas and names of Reservoirs.	Drainage area in square miles.	Rainfall in inches per annum.	Depth of rain in inches per annum flowing off the surface.	Per centage of rainfall which flows off the surface.	Reservoir room per square mile in millions of cubic feet of water.	Contents of Reservoir in millions of cubic feet.
Ashton	·59	40·0	15·5	39	21·0	12
Albany Works, U.S. .	·29	..	..	..	1·1	32
Ballinrobe, Ireland .	11·0	49·3	28·5	58	..	..
Belmont (moorland), mean of four years .	2·81	54·5	39·6	72	26·8	75
Bolton	·80	..	..	..	25·6	20
Bute (low country) . .	..	45·4	23·9	53	..	..
Bateman's evidence on the drainage area of Longdendale:—						
First half of 1845, very dry	..	21·2	13·5	64	..	..
Second half of 1845 . .	..	38·6	27·25	71	..	..
First half of 1846 . .	..	22·5	17·5	78	..	..
Oct., Nov., and Dec., 1846.	..	10·2	8·67	85	..	..
Bann Reservoir (moorland)	..	72·	48·0	66	..	..
Drainage areas on south side of Longridge Fell, near Preston, May, 1852, to April, 1853 . . .	..	..	15·5	29	..	..
	..	54·	18·0	33	..	..
	..	..	22·0	43	..	..
Dilworth Reservoir of Preston Works, Lancashire . . .	·092	..	..	..	54·0	5
Glencorse	6·00	37·0	22·3	60	7·66	46
Greenock	7·88	60·0	41·0	68	38·	300
Homersham's estimate of 24,000 cubic feet of Reservoir to each acre of drainage . . .	·1	..	..	..	15·36	15·36
Longdendale. . . .	23·8	..	..	..	12·3	292
Proposed Reservoir for Wolverhampton Works	22½·	..	..	..	·7	16
Rivington Pike . .	16·25	55·5	24·25	44	29·6	481
Sheffield	1·42	...	..	..	36·5	52
Turton and Entwistle .	3·18	46·2	41·0	89	31·43	100

daily. It is said to have cost £1 17*s.* 6*d.* a head, Glasgow £3, Manchester £2 18*s.*, and Birmingham £3. The annual cost of filtering 1,000,000 gallons daily, capitalized, is put down at £1,250 by Sir John Hawkshaw in his report on the Dublin supply. This would be £62 10*s.* yearly. It varies from £40 to £120 under different circumstances. The works of construction and the first cost of the filters may be taken at about £2,000 for each 1,000,000 gallons to be filtered daily.

The supply to the suburbs of Dublin is given at a charge of about 3½ pence for each 1,000 gallons.*

The actual cost of all works for house service varies very much in different towns, and with the quantities supplied, from a general average of 1*d.* per house per week, to 2*d.*, and from an annual rate of 9*d.* in the pound to 1*s.* 6*d.*, and higher. The cost of raising and supplying 1,000 gallons from a height of 135 feet in Nottingham is said to be 3*d.*, and the charge for house service to vary from 5*s.* to 60*s.* annually. In Rugby, the average cost per house is 19*s.* per year, 4½*d.* per week, or an annual charge of 3*s.* 3*d.* per year, or ¾*d.* per week per head of the population, and for a bare supply of 13 gallons. In Croydon, for a supply of only 14 gallons per head, the cost of works varied from 1½*d.* to 2½*d.* per house per week. The parliamentary returns, showing the number of houses supplied, and cost of supply, by different water companies of London, in 1834, give the following results :—

---

* In December, 1874, the quantity sent into the city and accounted for is stated to have been 7,226,000 gallons, and the waste 6,631,000 gallons daily ! District waste water meters are here essential.

COMPANIES.	Number of Houses.	Daily average Supply in Gallons.	Height of Supply over Thames.	Amount of charge per Company.		
				£	*s.*	*d.*
New River . .	73,212	241	145	1	6	6
Chelsea . .	13,891	168	135	1	13	3
West Middlesex	16,000	185	155	2	16	10
Grand Junction .	11,140	350	152	2	8	6
East London. .	46,421	120	107	1	2	9
South London .	12,046	100	80	0	15	0
Lambeth . .	16,682	124	185	0	17	0
Southwark . .	7,100	156	60	1	1	3

Cost of house apparatus for private supply from street mains, as averaged by the Board of Health, for first-rate houses, is £3 13*s.* 2*d.*; second-rate houses, £2 18*s.* 6*d.*; third-rate, £2 3*s.* 3*d.*; fourth-rate and cottages, 17*s.* 5*d.*; average cost for houses and cottages, £2 8*s.* 1*d.*

The actual cost of private works—to take water from mains for the supply of cottages—is shown in the following table:—

Work executed in	Name of Place.	Mean Expense of Private Works for each Cottage.			Annual Value of each Cottage.		
		£	*s.*	*d.*	£	*s.*	*d.*
Jan. 1852	Rugby, mean of 6 Cottages	1	12	11	5	10	0
Mar. 1852	Croydon . . 10 ,,	2	0	0	4	0	0
,, 1852	Barnard Castle 11 ,,	1	18	1⅓	3	2	6
Aug. 1852	Tottenham . 6 ,,	2	11	10½	10	0	0
Mean values for each Cottage . . .		2	0	9	5	13	1½

The water rate charged by the Local Board at Tottenham is given as follows:—

	In the Special District Rate Assessment.		Water Rate per week.	Water Rate per annum.
	Above	And not exceeding		
	£ s. d.	£ s. d.	£ s. d.	£ s. d.
On Premises assessed.	,,	10 0 0	,,	0 2 6
,,	10 0 0	15 0 0	,,	0 3 9
,,	15 0 0	20 0 0	,,	0 5 0
,,	20 0 0	25 0 0	,,	0 6 3
,,	25 0 0	30 0 0	,,	0 8 0
,,	30 0 0	40 0 0	,,	0 11 0
,,	40 0 0	50 0 0	,,	0 14 0

and 3*s*. for every additional rate of £10.

PUBLIC WORKS OF WATER SUPPLY, PRESTON.

Yards.	Cost of Pipes.	£	*s.*	*d.*
44	of 2-in. iron pipes, including valves, fire-plugs, outlet pipes, and all appurtenances, at 1*s.* 7*d.*	3	9	8
1,496	of 3-in. ditto, at 3*s.* 4*d.*	249	6	8
321	of 4-in. ditto, at 4*s.* 9*d.*	76	4	9
625	of 5-in. ditto, at 6*s.*	187	10	0
30	of 9-in. ditto, at 9*s.* 6*d.*	14	5	0
2,516		£530	16	1

Water Supply and its Cost for some Cities and Towns, from a Paper read to the British Association at Leeds, in 1858, *by Dr. Strang, of Glasgow. Vide* Builder, *for* 1858, *p.* 653.

TOWNS.	Population within bounds of Supply.	Daily Supply.	Daily Supply for each inhabitant.	Cost of undertaking.	Daily Supply for every £1 expended.	Prospective Supply daily in addition.
		Gallons	Gallons	£	Gallons	Gallons
London . .	2,667,917	81,025,842	30·3	7,102,823	11·4	..
Paris . .	1,100,000	26,350,000	24·	800,000	33·	20,000,000
Hamburgh .	160,000	5,000,000	31·25	170,000	29·50	..
New York .	713,000	28,000,000	39·27	1,800,000	15·5	..
Manchester .	500,000	11,000,000	22·	1,300,000	8·5	14,000,000
Liverpool .	500,000	11,000,000	22·	1,640,000	7·	..
Leeds . . .	153,000	1,850,000	12·	283,871	7·	..
Edinburgh .	215,000	4,800,000	22·3	456,000	10·5	2,000,000
Aberdeen .	65,000	1,200,000	18·4	50,000	24·	..
Dundee . .	96,000	1,750,000	18·2	139,000	12·5	..
Greenock .	40,000	2,112,500	52·8	90,000	23·4	..
Paisley . .	48,000	1,021,452	21·	60,000	17·	..
Glasgow .	420,000	16,710,000	39·8	651,199	26·	20,000,000

TABLE SHOWING THE AVERAGE CHARGE PER ANNUM FOR HOUSE SUPPLY IN LONDON;

ARRANGED FROM THE PARLIAMENTARY RETURNS OF 1855.

No.	Companies.	Houses containing 2 Rooms.	Houses containing 3 Rooms.	Houses containing 4 Rooms.	Houses containing 6 Rooms.	Houses containing 8 Rooms.	Houses containing 10 Rooms.	Houses containing 12 Rooms.	Houses containing 16 Rooms.	1st Rate Houses.	2nd Class Houses.	3rd Class Houses.	4th Class Houses.	5th Class Houses.	Horses.	Stables.	Water Closets.	Per centage paid on Capital expended.
		s. d.	*s. d.*	£ *s. d.*	£ *s. d.*	£ *s. d.*	£ *s. d.*	£ *s. d.*	£ *s. d.*	£ *s. d.*	£ *s. d.*	£ *s. d.*	£ *s. d.*	£ *s. d.*	*s. d.*	*s. d.*	*s. d.*	£ *s. d.*
1	New River . .	10 0	15 0	1 0 0	1 4 0	1 10 0	1 18 0	..	..	..	..	..	..	..	3 6	..	..	4 7 6
2	East London .	..	..	1 0 0	1 12 0	..	3 3 0	3 18 0	4 13 0	4 0 0	3 0 0	2 10 0	2 0 0	..	5 0	..	..	5 7 0
3	{ Southwark and Vauxhall . . }	..	..	0 14 0	1 11 0	..	2 10 0	3 3 0	5 1 0	3 12 0	2 6 8	1 5 0	0 12 6	0 6 11	..	..	..	4 17 0
4	West Middlesex	..	..	..	..	..	..	..	..	4 10 0	3 10 0	2 10 0	1 15 0	0 18 6	..	..	..	6 0 0
5	Lambeth . .	7 6	11 0	0 15 0	1 2 6	10 ℘C.	on ann.	value on Houses	above 6 Rooms	..	..	..	..	..	..	7 0	10 0	2 7 6
6	Chelsea . . .	13 0	17 0	1 2 0	..	..	..	..	..	..	..	..	..	..	..	10 0	15 0	1 7 6
7	Grand Junction	..	..	4 15 0	1 5 0	..	2 2 0	2 15 0	4 4 0	..	..	..	..	..	..	10 0	..	6 2 6
8	Kent . . .	..	..	..	..	..	..	..	..	2 0 0	1 13 0	1 1 0	0 16 0	..	..	10 0	..	5 0 0
9	Hampstead . .	10 0	15 0	1 0 0	1 10 0	2 0 0	..	..	..	..	..	..	..	..	..	..	..	1 13 0
	{ Average Annual Charge . . }	10 1½	14 6	0 18 0	1 7 5	1 15 0	2 8 3	3 5 4	4 12 8	3 10 6	2 12 5	1 16 6	1 5 10½	0 12 8½	4 3	9 3	12 6	4 2 6

The cost of pumping varies with circumstances; we believe that pumping engines cannot be put down at less than from £60 to £100 per horse power, dependent on the size of the engine, although the Board of Health adopted a standard of £50 per horse power. For the town of Drogheda we estimated for two engines at £75 per horse power. The following information respecting the cost of the Waterworks, Cork, was kindly furnished to the author by Sir John Benson, the engineer, who designed and carried out the works.

CORK WATER WORKS.

		£	s.	d.
Steam engine 100-horse power.	Direct acting Cornish Engine with three cylindrical flue boilers, including engine and boiler house, setting boilers, chimneys, &c., &c., per horse power	55	0	0
Two 50-horse power turbines.	Two turbines completed with four 11 in. ram pumps on each, including buildings, cisterns, sluices, gates, screens, per horse power	44	0	0
Reservoirs—				
One of 3,500,000 gallons.	One reservoir on a level of 186 ft. over weir	4,900	0	0
One of 563,000 gallons.	One reservoir on a level of 360 ft. over weir			
Cost per head.	The inhabitants in 1851, 86,000	0	15	3½
	The inhabitants in 1861, 100,000	0	13	0
Valuation standard per pound on the valuation.	City valuation, £112,000	0	11	7
Yearly cost per five inhabitants.	Distribution per house of every five persons	0	5	0
Water supplied.	Quantity supplied, including manufactories, to one person per day	30 gallons.		

The total estimated cost of engines, including pumps, engine houses, wells, &c., for raising the London sewage, is £70 per horse power, and the annual cost £20 per horse power.*

---

* Main Drainage Report, 1857, p. 29.

When coals are 10*s.* per ton, the cost of an engine exceeding 100-horse power, single acting Cornish, working night and day, will be £10 per horse power; when coals are 15*s.* per ton, the cost would be £13 per horse power; when coals are 20*s.* per ton, the cost would be £16 per horse power; when coals are 25*s.* per ton, the cost would be £19 per horse power. These estimates have been given by Mr. Hughes, and include every expense of coals, wages, oil, tallow, materials for packing, cleaning, &c., but none for interest of capital or depreciation of machinery.*

At Ely the cost of pumping is stated by a writer in the *Builder* to be as follows:—

To pump one million gallons 140 feet high, the old engine consumes:—

	£	*s.*	*d.*
Four tons of coal, at 16*s.* per ton	3	4	0
Oil, tallow, and packing	0	12	0
Wages	0	9	0
Total cost of pumping one million gallons	4	5	0

which gives 1*d.* per 1,000 gallons pumped 140 feet high (not a very high price).

The new engine requires:—

	£	*s.*	*d.*
Five and a half tons of coal at 16*s.*	4	8	0
Oil tallow, and packing	1	10	0
Wages	1	2	0
Total cost of pumping one million gallons 140 feet high	7	0	0

which is 65 per cent. more money than the old engine requires.

While another writer in the same periodical states, that the cost of pumping 1,000,000 gallons with the old engine was £4 18*s.* 8¼*d.*, and with the new engine,

---

* Main Drainage Report, 1857, p. 447.

ESTIMATED COST OF ENGINES FOR PUMPING AND OTHER INFORMATION.

Description of Works, and Engineers' estimates, with short description of engines.	Millions of Gallons daily.	Height to be raised in feet.	Millions of Gallons to be raised 1 foot in 24 hours.	Estimated cost in £s.	Work to be done in units of 33,000 lbs. raised 1 foot high per minute, or horse power	Cost for each unit of 33,000 lbs. raised 1 foot high per minute, or horse power, in £s.
Daglish's estimate for Liverpool Works, exclusive of buildings—one engine and pumps	2	300	600	8470	126·26	67·1
Ditto Two engines and pumps..................	2	300	600	9850	126·26	78·0
Ditto For one engine and pump........	1	75	75	3400	15·78	215·4
Homersham, for Watford supply	8	174	1392	35,357	292·92	120·7
Forester & Co.'s estimate for Liverpool engines, one or two, exclusive of buildings	2	300	600	9700	126·26	76·8
Ditto ditto ditto	1	75	75	3200	15·78	202·8
Hurry and Bateman's estimate, Wolverhampton Water Works..................................	1½ 4½	314 264	471 1188	35,400	349 11	101·4
Harvey and West's for ditto, exclusive of buildings, 84 inch cylinder engine	1½	306	459	11,000	96·59	113·9
Ditto ditto 64 inch cylinder engine	1½	180	270	7000	56·82	123·2
Ditto ditto 58 inch cylinder engine........	1½	140	210	6500	44·19	147·1
Messrs. Hawthorne for ditto, double power expansive condensing beam engines, exclusive of engines	1½	314	471	6055	99·11	61·1
Ditto	1½	163	244½	2520	51·45	48·9
Messrs. Hawthorne's high-pressure double acting horizontal engines, to be raised in 6 hours	1½	22	132	1170	27·78	42·1
Ditto to be raised in 9 hours..................	1½	22	108	780	22·73	34·3
Ditto to be raised in 12 hours	1½	22	66	570	13·89	41·0
Mr. Hocking's estimate for Wolverhampton, single acting Cornish	1½	500	750	20,800	157·83	131·8
Ditto exclusive of buildings	1½	500	750	13,200	157·83	83·6
Ditto to be raised in 12 hours, by two 60 inch, two 45 inch, and one 36 inch cylinder	1½	500	1500	32,600	315·65	103·3
Ditto exclusive of buildings	1½	500	1500	21,800	315·65	69·0
Mr. Quick's estimate, for Grand Junction Works, one 64, or two 45 inch cylinders, single acting Cornish..	5	46	230	7000	48·40	144·6
Ditto for Southwark and Vauxhall	8	40	320	7000	67·34	103·9
Ditto for West Middlesex	5	46	230	10,000	48·40	206·6
Sandys, Vivian & Co., two engines for Liverpool, exclusive of buildings...........................	2	300	600	8000	126·26	63·4
Ditto one engine, exclusive of buildings	1	75	75	1800	15·78	114·1
Seaward & Capel's, for Wolverhampton, exclusive of buildings	1½	566	849	22,000 to 25,000	178·65	123·2 139·9
West's estimate, 65 inch cylinder, exclusive of buildings	1½	314	471	6000	99·11	60·5
Ditto 50 inch cylinder, including buildings	1½	163	244½	7100	51·45	138·0
Ditto exclusive of buildings....................	1½	163	244½	5000	51·45	97·2
Ditto raised in 6 hours, by a 35 inch cylinder engine	1½	22	132	4600	27·78	165·5
Ditto exclusive of buildings....................	1½	22	132	3100	27·78	111·6
Ditto including duplicate engines.............	1½	566	849	38,200	158·66	240·8
Ditto 65 inch cylinder engine, including buildings (?)	1½	314	471	8400	99·11	84·8
AMERICAN ESTIMATES AND WORKS.						
Mr. McAlpine's, for Brooklyn, double acting, expansive, high-pressure, condensing engine, 72 inch cylinder engine, to be raised in 12 hours, exclusive of buildings	5	190	1950	18,000	399·83	45·0
Ditto non-condensing 30 inch cylinder engine, raised in 24 hours, exclusive of buildings......	5	190	1035	3000	199·92	15·0
Ditto in 12 hours, exclusive of buildings	10	190	3800	39,000	799·66	48·7
Ditto exclusive of buildings....................	20	190	7600	53,000	1599·33	33·1
Ditto ditto	30	190	11,400	85,000	2398·99	35·4
Ditto Albany Water Works, 58 inch condensing beam engine, and one duplicate not condensing, working for day only	2 1	156 238	624 476	13,320	231·48	57·5
Ditto Chicago Works, as in last, including buildings, 46 inch cylinder condensing engine and duplicate, as in last	3	107	642	11,258	135·10	83·3
Ditto 48 inch cylinder, &c..................	3	116	696	11,642	146·46	79·5

£4 10*s*. 7*d*. In the preceding table, arranged from information in Mr. Hughes' book,* the estimated cost of pumping engines for various works, English and American, is given.

In Example 28, pages 25 to 27, we have pointed out the method of calculating the increase of horse power required in raising water through pipes from friction, and also the great increase of this extra head if the velocity increases; the increase being nearly as the square of the velocity. In addition to this, an allowance of horse power must be made for bends, curves, junctions, and other obstructions, for the effects of which see Section XI. The more slowly the water is pumped, the less will the loss be from these causes through the same pipe. It is therefore, so far, advisable to give as large a diameter to the pipes supplying a reservoir from a pumping engine as other aspects of the question, cost, and engine power, will admit.

A report by the Water Committee of Plymouth, printed in a local newspaper of 13th October, 1864, contains much useful information respecting the water supply of the following places at that time:—

Bristol (population 140,000).—This city is supplied by a company drawing its water from the Mendip-hills; the pipes being too small and quantity deficient, it is not continuous, nor is the quality good. The scale of rates ranges from 5 per cent. on low rentals to 3 per cent. on rentals of £100, and 2½ per cent. on rentals of £200 and upwards. For trade purposes the water rate is 6*d.* per 1,000 gallons the minimum, to 1*s.* 6*d.* per 1,000 gallons the maximum. Water closets, stables, baths, &c., charged extra to domestic supply and reduced rates—no overflow or waste pipe permitted to communicate with any cistern or bath unless supplied by meter. No cistern for closet containing more than two gallons allowed without extra payment. The company undertakes plumbers' work on moderate terms.

---

* Weale, London.

GLOUCESTER (population 30,000).—The Local Board of Health hold the water works in trust for this city. For domestic purposes the rates are moderate, houses under £10 paying 8*s.* 8*d.* per annum; from £10 to £60, 5 per cent.; £60 to £70, 3¼ per cent.; £70 to £80, 3½ per cent.; above £80, 3¾ per cent. Meter rates were only determined on in December, 1862; range from 6*d.* per 1,000 gallons on the million and upwards, to 1*s.* per thousand gallons under 100,000 gallons. Service box cisterns are not enforced, although generally adopted by the better class of consumers.

DERBY (population 45,000).—A private company supplies this city. The supply is constant at high pressure, and is of good quality. All house apparatus and fittings are in accordance with the regulations of the company, and subject to the inspection of their officers. Service box cisterns to closets are strictly enforced. No overflow or waste pipes are permitted to cisterns without meters, and rain water carefully excluded therefrom. The scale of rates for domestic use and by meter is moderate. The Local Board of Health contracts for the supply of baths and washhouses, at the rate of 3*d.* per 1,000 gallons. Water for streets, when not drawn from the river, is charged at the lowest meter rate. The consumption is rather above 19 gallons per head per day, including numerous wells and large manufactories.

NOTTINGHAM (district population, 76,000). — We cannot say too much in commendation of the superior arrangement and management of the water works of this town, for whilst the greatest economy is used, an abundant amount of contentment is manifested by the consumers: the consumption averaging 17 galls. per head per day. A constant high pressure supply has been maintained by the company for the last 20 years. Overflow or waste pipes to cisterns are prohibited, but warning pipes in exceptional cases fitted with the approval of inspectors. All water closet cisterns are fitted with service boxes, and in no case is rain water allowed to flow into any such cisterns. No supply is allowed to be laid on or apparatus fixed by any other than an authorised plumber or the workmen of the company. The rates vary according to the level supplied: consumers by meter pay—

		Price per 1,000 gallons.		
		Lower.	Middle.	Higher.
		d.	*d.*	*d.*
Not exceeding	50,000 galls.	6	9	12
,, ,,	400,000 galls.	4½	6¾	9
Exceeding	1,600,000 galls.	3	4½	6

Tanks with meters affixed are placed in suitable parts of the town for supplying carts for street watering, a cart being filled in three minutes, the Local Board paying the lowest meter rate. The company have adopted call-books for complaints, repairs, &c., plumber's book for instruction, and issue cards for securing attention to the work required to be done.

NORWICH (population 75,000).—The water works of this city furnish another example of good management. The average consumption for private and public purposes is 14½ gallons only. A beneficial change is due entirely to the zealous care in distribution, seconded in a praiseworthy manner by a discriminating public. The company does a large portion of the plumbing and water fittings, the best feeling exists between the various tradesmen and the officers of the company. The service box cisterns here provided for closets are very simple, effective, and not liable to get out of order; iron rods take the place of wire. The cost is also so small as to be within reach of the poorer classes. Patterns of those and cast-iron street or courtyard stand pipes will be submitted to the inspection of the Plymouth Water Committee. The same regulations are in force here as at Nottingham as to domestic supply, &c. The rates are moderate, commencing at 4*s*. 4*d*. per annum for tenements under £5 per annum; above £5 and under £100, 5 per cent.; above £100, 4½ per cent. This is exclusive of water closets, which, with stables, gardens, &c., are charged extra. Meter rate—Not exceeding 200,000 gallons per annum, 1*s*. per 1,000 gallons; not exceeding 1,000,000 gallons per annum, 10*d*. per 1,000 gallons; exceeding 1,000,000 gallons per annum, 8*d*. per 1,000 gallons; Local Board for street watering, if amounting to 7,000,000 gallons per annum, 7*d*. per 1,000 gallons. It might be noticed that the Town Council of this city guaranteed the Water Company 5 per cent. on their outlay, after which the Council was to receive a moiety of the profits. The civic authorities of Edinburgh have adopted the arrangements carried out in this city, and we heartily recommend their application where, necessary, for Plymouth.

LEICESTER (population 69,000). — The supply of water to this town is in the hands of a private company, who have expended the sum of £90,000. The Local Board of Health holds 680 shares, and is entitled to half the profits after a dividend of 5 per cent. shall have been paid to the shareholders. The water is at high pressure and continuous. Storing cisterns for house purposes are not much used, but if adopted must be without waste pipes, or inlet of rain water. Water closets in all cases are fitted with flushing service boxes, or must be self-acting apparatus. The scale of rates ranges from 5 per cent. on low rentals to 3½ per cent. up to £100, and 3 per cent. above £200. Water rates range from *5d.* per 1,000 gallons the minimum to 10*d.* per 1,000 the maximum. The Local Board of Health is charged 2½*d.* per 1,000 for street watering, having meters attached to the mains. The company's regulations to prevent waste of water are strictly enforced.

GREAT YARMOUTH (population 30,000).—This town has the benefit of a well-governed water company. There is a constant supply at high pressure. The scale of rates is 6 per cent. on the rental up to £100 per annum, with 5 per cent. on the excess in addition —this including one water closet only—5*s.* per annum is charged for all others in addition, and 10*s.* for baths.

The meter rate is also high, ranging from 2*s.* per 1,000 gallons to 1*s.* 3*d.* The Local Board has an especial rate for watering the streets. Water-closets are in all cases fitted with double-valve service boxes; but few cisterns for domestic supply are in use (not being needed), but in these overflow and waste pipes are prohibited. The manager of the company will, however, permit detective or warning pipes where necessary. The company is willing to execute all plumbers' work if required, but all such work must be done under inspection. We have during our tour of inspection kept in view the object for which it was proposed, namely, that of ascertaining the means by which a constant supply of water may be secured to the inhabitants of this borough, and to suggest the practical application of such means. The attainment of so desirable an object can be secured very speedily by the submission of water consumers to the absolute control of appointed officers over all fittings, and the adjustment of apparatus of every kind. Having witnessed the beneficial effect of such wholesome regulations as have been referred to in the preceding remarks, we cannot but anticipate similar advantages by their adoption.

In Plymouth.—We desire therefore to recommend the general application of the rules and regulations as adopted in Nottingham and Norwich, which embody among them the following provisions—1st, that all applications for the supply of water, notice of insufficient supply, and other complaints, be recorded in a call book; that visiting cards be issued authorising the attendance of the workmen necessary, and a complete registry being kept of all such work performed; 2nd, that wherever practicable, cheap water closets be recommended in lieu of open privies; 3rd, that all water closets be fitted with full and complete apparatus for flushing by means of service cisterns, or such other description of closets as shall be approved, and that valves be worked by rods instead of wires or chains; 4th, that cisterns be without overflow or waste pipes, but, with the approval of officers, detective or warning pipes be substituted; 5th, that high pressure taps be introduced with new fittings, and all drawing and ball taps to be of the approved kinds: in open court yards and exposed places, taps to be protected with iron casing, and be made to open with keys supplied to the ratepayers only; 6th, that wire gauze screens, in the absence of filtration, be placed where desired by your surveyor. We also further recommend that as soon as practicable the condition of all the fittings and premises at present supplied with water be duly registered, with a view to an early repair and correction where necessary. It is not necessary, in adopting the preceding recommendations, that the supply of water shall in any degree be stinted, but *the exercise of a moderate amount of economy would enable a constant supply to be given*, and would doubtless tend to the development of many branches of manufacture in the town in which water forms an essential. The domiciliary visits for the inspection of premises would be in no way offensive, but would be of the same nature as those now made for inspection of gas apparatus, and we have the evidence of numerous householders in towns we have visited that the greatest respect and civility are manifested. It is not our aim or desire to abolish cisterns now in use, *but the* CONSTANT SUPPLY SYSTEM *will render so large an expenditure unnecessary in future*, cisterns for flushing closets being then only necessary.

THICKNESS OF PIPES FOR WATER WORKS.

It is evident that the thickness of a pipe should be at least sufficient to bear the pressure of the atmosphere, and therefore the whole pressure in a pipe is best expressed by a determinate number of pressures, each equal to that of a column of water 33 feet high. If n be the number of such pressures, or the number of units each equal to 33 feet high, d the diameter of the pipe in inches, and t the thickness, also in inches, we shall have for

(A.)	1.	Iron pipes, plate	$t = \cdot 0009\, n\, d + \cdot 13$
	2.	Iron pipes cast horizontally . .	$t = \cdot 0024\, n\, d + \cdot 33$
	3.	Iron pipes cast vertically . .	$t = \cdot 0016\, n\, d + \cdot 32$
	4.	Copper pipes, plate	$t = \cdot 0015\, n\, d + \cdot 16$
	5.	Lead pipes	$t = \cdot 0024\, n\, d + \cdot 19$
	6.	Zinc pipes	$t = \cdot 0051\, n\, d + \cdot 16$
	7.	Artificial stone	$t = \cdot 0054\, n\, d + 1\cdot 60$

For cast-iron pipes the engineer of the Paris water works, M. Dupuis, adopted in his practice a formula which is equivalent to

(B.) $t = \cdot 0016\, nd + \cdot 32 + \cdot 013\, d$

in the foregoing measures. This formula may also be expressed as follows:—

(C.) $t = (\cdot 0016\, n + \cdot 013)\, d + \cdot 32.$

If d be 12 inches, and $n = 9$, corresponding to a pressure of 297 feet, we shall find from the last equation, $t = (\cdot 0144 + \cdot 013) \times 12 + \cdot 32 = \cdot 3336 + \cdot 32$

= ·6536 inch. All pipes should however be proved with ten atmospheres, or 330 feet, and in practically applying the above formulæ in equation (A), for finding the thickness of pipes, the value of n should always have 10 added to it. Hence, applying formula (A), No. 3, to our example, we get $t = {\cdot}0016 \times 19 \times 12 + {\cdot}32 = {\cdot}6848$ inch, which is the same practically as found from equation (C).

SEWERAGE COST.

As for water-works, the minimum rain-fall of a district should be calculated upon; so the maximum fall must be considered for sewerage and drainage works. We have already shown, page 340, that for a population of 80 persons per statute acre, and a discharge of two-fifths of an inch in eight hours, sewers should be calculated to discharge about 3½ cubic feet per minute, the rain supply being about seven times the house supply, or sewage, including house water supply. Instances are quoted in which the discharge, after a heavy rain-fall, amounted to 20½ cubic feet per minute per acre, as in the Savoy-street sewer, which of course was principally surface water, as the sewage of 80 persons at 7 cubic feet per person, one-half of which, if discharged in eight hours, would only be $\frac{80 \times 7}{8 \times 2} = 35$ cubic feet per hour, or $\frac{35}{60} = {\cdot}59$ feet nearly per minute, which is only about the thirty-third part of 20½ feet. In other words, the storm waters were thirty-three times the amount of house sewage. It would be waste to provide drainage for so much surface water considered in itself, where it can be passed off from the surface channels. But sewage is not water, and it is essential, in the greater number of cases, that sewers should be flushed occasionally. It is absurd to calculate the size of sewers, as if the sewage matter were thoroughly diluted or passed off like water. In fact, the sewage in part lies at the bottom of the sewer, or is deposited there in nine cases out of ten, while the house supply of water passes on and escapes over it, removing only diluted and detached portions. It is, therefore, of importance, where artificial flushing and cleansing out are not provided, that storm waters should occasionally pass through and flush a system of sewers, particularly the main or arterial lines. An engineer must be guided, in calculating the dimensions, &c., of main sewers, by the circumstances of each case. The inclinations to be obtained, the form of the bottom or invert, the rain-fall, the amount of sewage which will not affect the size to any considerable extent, the material and the cost consistent with permanency.

The discharging power of a water channel is more than doubled by increasing its dimensions by one-third; and it is increased in the proportion of 5·7 to 1 by doubling the dimensions. By giving four times the fall, the same channel will only double the discharge. Now a pipe 2 feet in diameter with a fall of 1 in 200, would discharge fully 1000 cubic feet of water flowing full with a velocity of 5·4 feet per second: at 3½ cubic feet per minute per acre, for a population of 80 to the acre, the thoroughly diluted sewage of 280 acres would be passed off by one such pipe; that is, the sewage from 20,400 persons, on 280 acres, and also two-fifths of an inch of rain falling for eight hours, can be conveyed off by a 2 feet pipe, with a fall of 1 in 200. But as this rain supply is about seven times the house supply, passing 2½ feet per person off in eight hours, made up of fæces and used-up water supply, it is apparent that such a pipe would convey about eight times the sewage alone of the district, if flowing as water; and, under any circumstances, would be abundantly large for the duty, even when assuming the whole quantity to pass in at the upper end. For a fall of 1 in 800, two such pipes would be required, or one pipe 32 inches in diameter; for a fall of 1 in 3,200, four 2 feet pipes would be required, or one pipe 3 feet 6 inches.

House drains should not be less than 6 inches in diameter, and should have facilities for being cleaned, either by using half-flange joints, or by having a moveable upper segment. The inclination for these drains should be uniform, but the amount is not so important as some appear to think, if proper provision be made for cleaning. Where flushing is used, cast-iron pipes are the best, but they are also the most expensive. House drains of brick with a V tile bottom covered with flags or bricks are perhaps the best, as the capacity can be considerably augmented by adding to the height of the sides, and they can be at all times easily opened and cleaned. If inclinations from 1 in 50 to 1 in 20 can be had, so much the better. The following items as to cost have been selected from the "Builder":—

COST OF SEWERS, NEWPORT, MONMOUTHSHIRE.

Total lengths.	Average depths.		Sizes of sewers.					Thicknesses.	Cost per foot lineal.	
feet.	ft.	in.	ft.	in.		ft.	in.	in.	*s.*	*d.*
1,322	15	6	4	6	by	3	6	9	11	8
2,217	13	0	4	6	by	3	0	9	10	1½
6,110	12	0	3	0	by	2	2	9	7	7¼
12,354	11	8	3	0	by	2	2	6	5	3¾
1,953	9	3	2	6	by	1	10	6	4	7
9,663	10	0	2	6	by	1	10	4½	3	8¼
690	10	2	2	3	by	1	9	4½	3	5¼
3,264	8	6	1	2 diameter				4½	2	4¾

COST OF SEWERS AND PIPES IN PRESTON.

The following extract from a published summary of public works executed during the year ending April 30th, 1859, contains some useful information:—

Yards.		£	s.	d.	£	s.	d.
60	of Brick Sewers, 2 ft. 6in. diameter at 7*s.*	21	0	0			
538	3ft. by 2ft., at 17*s.* 6*d.*	470	15	0			
294	3ft. 6in. by 2ft. 4in., at 28*s.*	412	12	0			
372	3ft. 9in. by 2ft. 6in., at 28*s.*	520	16	0			
250	4ft. 3in. by 2ft. 10in., at 41*s.* 9*d.*	521	17	6			
56	4ft. 6in. by 3ft., at 75*s.* 7*d.*	211	12	8			
66	4ft. 6in. diameter, at 40*s.* 9*d.*	134	9	6			
					2,293	2	8
1,636							
42	of Cast-iron Sewer, 2ft. diameter, at 36*s.*				75	12	0
22	of Earthenware Pipe Sewer, 6in. diameter, at 4*s.*	4	8	0			
1,129	9in. diameter, at 7*s.* 5*d.*	418	13	5			
565	12in. diameter, at 8*s.* 9*d.*	247	3	9			
88	15in. diameter, at 11*s.* 3*d.*	49	10	0			
98	18in. diameter, at 13*s.*	63	14	0			
145	21in. diameter, at 18*s.* 6*d.*	134	2	6			
					917	11	8
2,089							
	Total, including superintendence, also man-holes, street gullies, and all appurtenances				£3,286	6	4

TABLE showing the prices of Tubular Drains as made by the Board of Health in 1852, *fifty per cent. being added for profit, &c.; and the sale prices in the market.*

Diameter in inches.	Lengths.	Red earthen-ware pipes made by the Board.			Red pipes at Sale prices.			Stoneware glazed at Sale prices.			Assumed gain.					
											On red ware pipes.			Over glazed stoneware pipes.		
		£	*s.*	*d.*	£	*s.*	*d.*	£	*s.*	*d.*	£	*s.*	*d.*	£	*s.*	*d.*
5	For 1,000 feet	6	15	0	20	16	8	25	0	0	14	1	8	18	5	0
6	For 1,000 feet	9	14	0	25	0	0	29	3	4	15	6	0	19	9	4
9	For 1,000 feet	15	1	6	37	10	0	50	0	0	22	8	6	34	18	6

Did the Board of Health here add the cost of their own establishment and staff to the cost of production? The manufacturer and salesman must surely live, at least the Author thinks so.

The following estimates were made for laying pipes at Tottenham, not including their cost:—

Diameter of pipe in inches.	Depth 6 feet.	Depth 8 feet.	Depth 10 feet.
6	8½*d.*	11*d.*	12*d.*
9	9½*d.*	14½*d.*	15½*d.*
12	11½*d.*	15¼*d.*	19¼*d.*

The cost of laying alone at St. Thomas's, Exeter, was—

6	inch pipes	5*d.*	per foot lineal	3 to 4 feet deep.
9	,,	5*d.*	,,	3 to 4 feet deep.
12	,,	8*d.*	,,	5 feet deep.
15	,,	9*d.*	,,	5 feet deep.
18	,,	11*d.*	,,	5 feet deep.

2*d.* per foot lineal for relaying pitching; 4*d.* for macadamised roads; and 6*d.* for pavements.

Relative Cost of Earthenware Pipe Drainage on executed works previous to 1852. From Minutes of Information collected by the General Board of Health, pp. 179 *and* 180, *ordered to be printed for the use of Local Boards and their officers. [We have altered the cost to the next penny where half-pence and farthings are stated in the original. The cost of junctions is not shown, but it is stated to amount to one-eighth in two of the towns, and one-tenth in two others.]*

Diameter of pipes.	CROYDON. Pipes laid 9 feet deep.			BARNARD CASTLE. Pipes laid 9 feet deep.			SOUTHAMPTON. Pipes laid 9 feet deep.			HITCHIN. Pipes laid 8 feet deep.			ORMSKIRK. Pipes laid 8 feet deep.		
	Cost of pipes per yard delivered.	Cost of laying pipes per yard run.	Total cost of pipes laid per yard.	Cost of pipes per yard delivered.	Cost of laying pipes per yard run.	Total cost of pipes laid per yard.	Cost of pipes per yard delivered.	Cost of laying pipes per yard run.	Total cost of pipes laid per yard.	Cost of pipes per yard delivered.	Cost of laying pipes per yard run.	Total cost of pipes laid per yard.	Cost of pipes per yard delivered.	Cost of laying pipes per yard run.	Total cost of pipes laid per yard.
inches	s. d.	s. d.	s. d.	s. d.	s. d.	s. d.	s d.	s. d.	s d.	s. d.	s. d.	s. d.	s d.	s. d.	s. d.
21	16 0	3 3	19 3	..	..	..	..	..	..	..	..	..	..	..	..
20	..	..	..	..	..	..	..	..	..	8 9	5 0	13 9	7 0	1 8	8 8
18	8 0	3 3	11 3	..	..	..	..	..	..	7 3	2 9	10 0	6 0	1 8	7 8
15	6 0	3 2	9 2	5 0	1 10	6 10	6 0	3 7	9 7	5 5	2 6	7 11	4 3	1 7	5 10
12	3 6	3 1	6 7	3 2	1 10	5 0	3 6	3 5	6 11	3 3	2 4	5 7	2 6	1 7	4 1
9	2 2	2 9	4 11	2 2	1 8	3 10	2 3	3 0	5 3	2 0	2 3	4 3	1 5	1 6	2 11
6	1 3	2 6	3 9	1 3	1 8	2 11	1 4	2 8	4 0	1 3	2 2	3 5	0 11	1 6	2 5
4	0 10	2 2	3 0	0 11	1 8	2 7	..	..	..	0 11	2 2	3 1	0 8	1 6	2 2

Diameter of pipes.	RUGBY. Pipes laid 8 feet deep.			SANDGATE. Pipes laid 6 feet deep.			TOTTENHAM. Pipes laid 5 to 13 feet deep.			ST. THOMAS'S, EXETER. Pipes laid 3 to 7½ feet deep.			OTTERY ST. MARY. Pipes laid 6 feet deep.		
	s d.	s. d.	s. d.	s. d.	s d.	s. d.	s. d.	s d.	s. d.	s d.	s. d.	s. d.	s. d.	s. d.	s. d.
24	..	..	..	..	..	..	..	..	..	..	3 6	..	..	..	..
21	..	..	..	..	..	..	..	..	..	10 0	3 3	13 3	..	..	..
20	8 5	2 9	11 2	..	..	..	..	..	..	..	..	..	..	..	..
18	7 2	2 9	9 11	..	..	..	7 4	2 5	9 9	8 0	2 9	10 9	..	..	..
15	5 5	2 6	7 11	..	..	..	5 5	1 8	7 1	6 0	2 3	8 3	6 6	2 5	8 11
12	3 3	2 6	5 9	3 11	1 9	5 8	3 5	1 5	4 10	3 8	2 0	5 8	3 3	2 2	5 5
9	2 0	2 0	4 0	2 5	1 9	4 2	2 1	1 0	3 1	2 3	1 3	3 6	2 0	1 4	3 4
7½	..	..	..	..	..	..	1 8	0 11	2 7	..	..	..	..	..	..
6	1 2	2 0	3 2	1 5	1 6	2 11	1 2	0 9	1 11	1 4	1 3	2 7	1 2	1 2	2 4
4	0 11	1 9	2 8	1 1	1 6	2 7	..	..	..	1 0	1 0	2 0	0 11	1 1	2 0
3	0 9	1 9	2 6	0 11	1 6	2 5	..	..	..	0 10	1 0	1 10	..	..	..
2	..	..	..	..	..	..	..	..	..	0 8	1 0	1 8	..	..	..

ESTIMATE FOR SEWERS AT BRIGHTON.

DESCRIPTION OF SEWERS.	Length in yards.		Price per yard.		Amount.	
Brick Sewers :—						
Diameter.			£	s.	£	s.
6 ft.	4,850		3	10	16,975	0
4 ft. 6 in.	350		2	10	875	0
4 ft. 6 in. by 3 ft.	4,000		2	8	9,600	0
3 ft. 9 in. by 2 ft. 6 in.	1,890		2	2	3,969	0
3 ft. by 2 ft.	2,820		1	16	5,076	0
2 ft. 3 in. by 1 ft. 6 in.	8,580		0	18	7,722	0
Total brick sewers		22,490				
Earthenware Pipe Sewers:			s.	d.		
15 inches diameter	9,466		13	6	6,389	11
12 ,, ,,	44,430		10	0	22,215	0
Total earthenware pipe sewers		53,896				
Cast-iron Pipe Sewers :—			£	s.		
3 in. diameter	750		7	0	5,250	0
1 ft. 6 in. ,,	1,260		3	0	3,780	0
Total cast-iron pipe sewers		2,010				
Total length of sewers Or 44 miles 956 yards		78,396				
		Number				
Man holes and ventilating shafts		600	20	0	12,000	0
Lamp-holes		600	4	0	2,400	0
Gullies		3,000	3	10	10,500	0
Outlet works, overflows, and extra work on steep gradients, &c.					5,000	0
Contingencies, including repairs, &c., of existing sewers, 10 per cent.					11,178	9
Total					£122,930	0

The author has constructed a large quantity of main sewers, from 18 inches to 2 feet and 2 feet 6 inches wide, and 4 feet 6 inches high; the side walls built with rubble masonry, 9-inch segment inverts laid with

4½-inch courses in cement; the top sometimes flagged, when flags of sufficient length could be procured, and sometimes arched with rough rubble arches. The invert was laid on, well bedded, well rammed, rubble to prevent subsidence, and preserve the bottom inclination uniform. The cost, at an average depth of about 9 feet, was 9*s*. per running foot, the side walls being about 18 inches thick. Upright side walls, where rubble is cheap, have many advantages in giving a considerable increase of capacity for a small outlay. The tenement and house drains were of earthenware pipes. Cast-iron gully grates and traps, weighing 3 cwt., cost 30*s*. each; the grate fastened by a wrought-iron chain.

The following regulations were laid down for Cambridge and Carlisle:—

STIPULATIONS FOR CAMBRIDGE DRAINAGE.

"Water from the rear of premises should not be conveyed to the front under the basement floor.

"Rain-water from the roofs should not be conveyed into the basement, but conducted into the sewer by shallow drains.

"Cast-iron pipes may be used for basement drains in some instances.

"The scullery sink should be kept as high as possible, and approached by a step. A flat trap should be fixed between the sink and sewer.

"There should be no water-closet on the basement floor; if it cannot be arranged elsewhere, the soil-pipe should have a flap trap, or similar contrivance, to prevent the influx of sewage water."

FOR CARLISLE DRAINAGE.

"STIPULATION 1.—If water-closets are to be generally used, the description of such to be sanctioned by the Board, the same to be fixed to the satisfaction of the Surveyor.

"2.—All down-spouts to be connected with the sewers where it may be proper to connect the same; in all cases where they are not connected with the sewer they are to be connected with the channel.

"3.—All stench traps to be similar to samples furnished by the Surveyor, or others approved by him, and properly fixed to his satisfaction.

"4.—All sewers to water-closets not to be less than six inches diameter.

"5.—All sewers to yards, stables, kitchens, and sculleries, not to be less than four inches diameter.

"6.—In every case the whole of the fall to be made available from the junction with the main sewer to the end of the private drain, that is to say, only one inclination to be used from the junction with the public sewer to the end of the private drain; and all branches from the private drain to sinks, water-closets, &c., to have one inclination from the junction of such drain. None of the above instructions to be departed from without the express sanction of the Surveyor.

"7.—In no case must a private drain be put in with a less fall than one in fifty, without the sanction of the Surveyor.

"8—No pipes, water-closets, stench traps, gullies, kitchen sinks, bends, junction or tapering pipes, to be used without being approved by the Surveyor.

"9.—All ash pits and dung depôts to be raised to the level of the adjoining ground, to be properly paved and drained as the Surveyor may direct.

"10.—All buildings, outhouses, &c., to be properly spouted, and the water conveyed into the sewers where approved of by the Surveyor."

THOROUGH LAND DRAINAGE.

The following instructions and general specifications, have been prepared by the Commissioners of Public Works in Ireland, for the use of the district inspectors, and persons reporting on thorough-drainage. The drains are made in general parallel, and to suit the fall of the ground. The depths must alter in order that the bottoms should have an uninterrupted fall, and may vary from 2 feet to 4 feet 6 inches in practice, averaging, say about 3 feet 6 inches, but dependent on circumstances. The portions printed in italics are from specifications prepared by officers of the Board, and are varied according to each particular case :—

GENERAL OBSERVATIONS.

"No drainage works should be undertaken until it has been clearly ascertained that the surface level of the maximum floods in the main drain can be discharged at a level that will admit of the submain drains venting the waters from the lowest point of the lands proposed to be thorough-drained, at a level sufficiently below the surface of such land, that the highest floods shall not prevent the free discharge of such submain.

"When sufficient out-fall can be obtained, no open main drain should be of a less depth than five feet, and in all cases a greater depth is desirable, in order to insure a permanent and efficient drainage, and at the same time to prevent cattle, &c., from crossing.

"As it has been found by practical experiments on different varieties of soils, that deep drains, say from four to five feet deep, are more effective than shallow ones : they should always be estimated for, when the open main drains admit of their being cut to that depth, or when, by a moderate outlay per acre, the main drains can be cut to a sufficient depth ; the distance between the parallel drains must necessarily vary with the texture of the soil,—forty feet may be taken as a general rule.

OPEN MAIN DRAINS.

"Main drains should have gradients of such inclination, and be sunk to a depth that will admit of the above stipulations, as to the discharge of the submain drains being carried out. They should have such width at bottom and side slopes as may be necessary ; and be free of sharp angles, projecting stones, and other impediments to the quick discharge of the waters.

"The spoil or material raised in sinking and improving the drains, where not available for filling up useless holes or drains, should be removed to a proper distance from the edge of the main drains, and dressed off in a workmanlike manner.

"The abutments and piers of such bridges as have sufficient breadth of water-way, should, if necessary, be carefully under-pinned ; and those bridges which are insufficient to discharge floods, should be taken down and rebuilt of suitable dimensions.

COVERED MAIN DRAINS.

"Whenever, from the nature of the lands, the extent of the district under drainage, and the quantity of water to be voided, it may be necessary to form covered main drains to receive the water discharged from the submains, their dimensions must be proportional to the amount of water to be voided, well flagged or paved at bottom, the sides built of stone or brick, and covered with a flag or arch at top.

SUBMAINS.

"The submains to be of such depth and width at top and bottom as may be necessary. The fall in each to be as great as the above-described main drainage of the district will allow, and not to be allowed to run beyond a suitable length without discharging itself into a covered or open main drain.

THE MINOR DRAINS

"To be of such depth, width at top and bottom, and at such distance apart, as will secure the perfect drainage of the land, to be run in a straight direction parallel to each other, directly up and down the declivity, unless where the declivity happens to be very steep, and then to be carried across the fall at such an angle as to secure a free discharge for the water. The fall in each minor drain to be as great as the main drainage and submain drainage, previously described, will admit.

"In filling in the stones, great care should be taken that the bottom of the drain be clean, and that no clay or dirt be put in with them; a sod, grass side down, or a few inches of tough clay, to be placed on the surface of the stones, and trodden firmly. The drain should then be filled up with the stuff previously shovelled out, observing to keep the active soil for the top. The putting in of the stones to be commenced at the highest part or head of the drain.

"In using draining pipes or other tiles, care should be taken that they be laid firmly on the bottom for their entire length, so as to prevent them being deranged by the filling of the drain, and that the points be fitted as closely together as possible.

"In cases of unfavourable ground, caused by running sand or otherwise, whereby the level of the conduit might be deranged, collared pipe tiles offer considerable advantages in the way of remedy.

"When gripes may be necessary on the sides of farm roads, they should be on the field side of the fences."

SPECIFICATION FOR MAIN DRAINAGE.

OPEN MAIN DRAINS.

"The deepening and improving of the main drain, No. ——, is to be commenced at the point —— on the accompanying map, and from thence a gradient carried up to the point ——, having an inclination of at least —— feet per statute mile, and sunk to the depth of —— feet. It shall be —— feet wide at bottom, and the side slopes shall average —— at least, unless in rock cutting, when the side slopes may be diminished to six inches to one foot; all sharp angles, projecting stones, and other impediments to the free discharge of the water, must be carefully removed. The spoil or material raised in sinking and improving the drain, when not immediately used for top-dressing the adjoining lands, or for filling useless holes or drains, is to be removed to a distance of —— feet from the edge of the main drain, and dressed off in a workmanlike manner.

"The bridge marked at the point —— on the accompanying map to be ——.

"The whole to be executed in a proper and workmanlike manner, and the works to be maintained in good order for so long as any interest shall be payable for the money advanced on account of its execution."

SPECIFICATION FOR THOROUGH-DRAINAGE (WITH TILES).

COVERED MAIN DRAINS.

"These shall be cut *fifty-four* inches deep, *thirty-six* inches wide at top, *twenty-four* inches wide at bottom; the materials used in them shall be *double row of three-inch pipe tiles.*

"The side walls shall be —— inches in height, —— inches thick, and well —— at bottom. They shall be covered with a flag not less than —— in thickness.

SUBMAINS.

"These shall be cut *fifty* inches deep, *thirty* inches wide at top, *eighteen* inches wide at bottom. They shall be carried along the low side of the fields, or portions of land to be drained, at a distance from the fence of *fifteen* feet, and through natural hollows where necessary. No submain to be allowed to run beyond a length of *two hundred* yards without discharging itself into a covered or open main drain.

MINOR DRAINS.

"These shall be cut *forty-eight* inches deep, *sixteen* inches wide at top, *five* inches wide at bottom, and at a distance of *forty* feet apart. They shall be run in a straight direction, parallel to each other, directly up and down the declivity (when possible). No minor drain to be allowed to run beyond a length of *two hundred* yards without discharging itself into a submain.

FILLING IN.

"All the drains (or a large number of them) having been opened and cut in a workmanlike manner, and it being ascertained that no water is standing in any of them, the filling in may be commenced.

MINOR DRAINS.

"Into each minor drain shall be put *pipe* tiles *twelve* inches in length, *one-and-a-half* inch in the ope, for *one hundred* yards, commencing from the upper end of the drain, and *pipe* tiles *twelve* inches in length, *one-and-three-quarter* inch in the ope, in continuation from thence to the submains.

SUBMAINS.

"Into each submain shall be put *pipe* tiles *twelve* inches in length, *two* inches in the ope, for *one hundred* yards, commencing from the upper end of the drain, and *pipe* tiles *twelve* inches in length, *three* inches in the ope, in continuation to the end or point where they discharge themselves.

GENERAL RULES.

"All tiles to be of good sound material, and well burned. The tiles shall be laid firmly on the bottoms of the drains for their entire length; the joints fitted as closely as possible, they shall be carefully covered with a *thin grassy sod or screen.* The stuff previously taken out of the drains shall then be returned, observing to keep the active soil uppermost.

"The mouths of the covered main or submain drains shall be built about with solid masonry set in mortar, carried up with the same slope as the sides of the open main drain, into which they discharge themselves.

"Before laying the tiles, great care must be taken that the bottom of the drains be clean. The putting in of the tiles to be commenced at the highest point or head of the drains.

"In case of an entire field being thorough-drained, a drain shall be cut at the top of it, parallel to the fence, and running at a distance from it equal to one-half of the distance between each of the minor drains, into one or more of which (as may be necessary) it shall discharge itself. The remainder of the minor drains to be discontinued at a distance from this drain equal to one-half the entire distance between each of the minor drains; this drain to be of the same dimensions, and filled with the same materials, and in like manner, as the above described.

"No open drain shall run into a closed one.

"In passing through unfavourable ground, caused by running sand or otherwise, whereby the level of the conduit might be deranged, and where pipe tiles are the materials used for forming the conduit, collars must be used, so as to connect the ends of the tiles, and they must be fitted as closely as possible.

"Soles must, in all cases, be used when laying single D tiles, and they must be so laid that the ends of the tiles shall rest equally on them; when inverted D tiles are used, they shall also be connected from end to end by placing one-half of the upper tiles on one-half of the adjoining tiles below them.

"The whole to be executed in a proper and workmanlike manner; and the work to be maintained in like good order as when approved of at its completion, for so long as any interest shall be payable for the money advanced on account of its execution." [*Collars for up to 4-inch pipes can be had at the Florence Court Tilery.*]

SPECIFICATION FOR THOROUGH-DRAINAGE (WITH BROKEN STONES).

COVERED MAIN DRAINS.

"These shall be cut *forty-two* inches deep, *thirty* inches wide at top, *twenty-four* inches wide at bottom; the materials used in them shall be ——.

"The side walls in them shall be twelve inches in height, six inches thick, and well —— at bottom. They shall be covered with ——.

SUBMAINS.

"These shall be cut *forty-two* inches deep, *eighteen* inches wide at top, *fourteen* inches wide at bottom. They shall be carried along the low side of the fields, or portions of land to be drained, at a distance from the fences of *thirteen* feet, and through natural hollows, where necessary. No submain to be allowed to run beyond the length of *one hundred and fifty* yards, without discharging itself into a covered or open main drain.

MINOR DRAINS.

"These shall be cut *thirty-six* inches deep, *fifteen* inches wide at top, *four* inches wide at bottom, and at a distance of *twenty-six* feet apart. They shall be run in a straight direction, parallel to each other, directly up and down the declivity (when possible). No minor drain to be allowed to run beyond the length of *two hundred* yards without discharging itself into a submain.

FILLING IN.

"All the drains (or a large number of them) having been opened and cut in a workmanlike manner, and it being ascertained that no water is standing in any of them, the filling in may be commenced.

MINOR DRAINS.

"Into each minor drain shall be put *ten* inches of broken stones in depth, the stones having been broken to a size not exceeding two-and-a-half inches in diameter. Great care should be taken that the bottom of the drain be clean, and that no clay or dirt be put in along with the stones ; a sod (or clay, as may be convenient) *three* inches thick shall be placed carefully on top, and the whole trampled upon or rammed hard. The drain shall then be filled up with the stuff previously shovelled out, observing to keep the active soil for covering the top.

The putting in of the stones shall invariably be commenced at the highest part or head of the drain.

FILLING IN SUBMAINS.

"In each submain a conduit shall be formed of *six* inches in height, *four* inches wide, and the filling in completed as above described.

GENERAL RULES.

"The mouths of the covered main or submain drains shall be built about with solid masonry set in mortar, carried up with the same slope as the sides of the open main drain into which they discharge themselves.

"Before filling in the stones, great care must be taken that the bottom of the drains be clean, and that no clay or dirt be put in along with them. The putting in of the stones to be commenced at the highest part or head of the drains.

"In case of an entire field being thorough-drained, a drain shall be cut at the top of it, parallel to the fence, and running at a distance from it equal to one-half the distance between each of the minor drains, into one or more of which (as may be necessary) it shall discharge itself. The remainder of the minor drains to be discontinued at a distance from this drain, equal to one-half the entire distance between each of the minor drains; this drain to be of the same dimensions, to be filled with the same material, and in like manner, as the above described.

"No open drain shall run into a closed one.

"The whole to be executed in a proper and workmanlike manner, and the work to be maintained in like good order as when approved of at its completion, for so long as any interest shall be payable for the money advanced on account of its execution."

One of the officers of the Commissioners of Public Works, Ireland, the Inspector of Drainage for Roscommon, a gentleman residing in that county, wrote to us as follows, with reference to tile and broken-stone drains on the carboniferous formation:—

"With respect to tile drainage, my experience ha not been very extensive, as the proprietors of the district, with scarcely any exception, give a decided preference to broken stones; but from what I have seen, I am very much inclined to prefer good well-burnt pipes to any other draining material, provided that collars be used, but not otherwise. As to the best diameters, I have found the 1¼″ collared pipes of the Clonbrock Tile Works (now closed) very satisfactory; but when the length of minor drains exceeded 100 yards, I should like an increase to 1½ or 1¾. For submains (say 150 or 180 yards long) I have recommended pipes of 2 inches, 2½, and 3 inches in succession, all of which were to be had with collars: if 4-inch pipes were to be had with collars, I should have recommended longer submains. The larger-sized pipes are not provided with collars in our present tileries, *and on this account I generally put a note on the margin of the printed form, suggesting that a stone duct of the ordinarg size of submain, say 6 inches in height and 4 inches wide, be substituted for the tile filling.*

"I decidedly prefer an open duct to broken-stone filling; and in nine-tenths of my own drainage I have made the minor drains on the same plan as the submain, with an open stone conduit; the only difference being, that the minor drains are a few inches shallower, with a smaller duct. The increase of expense is a mere trifle, and when the substratum (as very frequently occurs here) is a fine calcareous gravel, containing 40 to 60 per cent. of carbonate of lime, the additional spoil is a very cheap fertilizer for the land.

"With respect to depths and distances apart, the two most commonly used in my specification are 3½ feet deep, 33 feet apart,—and 4 feet deep, 42 feet apart. These arrangements will not suit all cases, and I vary accordingly. Thus, in one case of exceedingly retentive land of peculiar texture, 4-feet drains, 27 feet apart, produced the required result, while in another, 3½-feet drains, 66 feet apart, effected all that was required. In the latter case there was a mixed soil, which might be described as 'half wet;' yet the water lingered sufficiently long to make the land unsound for sheep, and greatly to injure the crops in quality as well as quantity."

Mr. Josiah Parkes says, in 1843:—"Experiment and experience have rapidly induced the adoption of a system of parallel drains, considerably deeper, and less frequent, than those commonly advocated by professed drainers, or in general use. I gave several instances of this practice in Kent in the Report of last year, 1843, already alluded to, and it is rapidly extending. Mr. Hammond stated to you that he drained 'stiff clays 2 feet deep, and 24 feet between the drains, at £3 4*s.* 3*d.* per acre, and porous soils 3 feet deep, 33½ feet asunder, at £2 5*s.* 2*d.* per acre.' I now find him continuing his drainage at 4 feet deep, wherever he can obtain the outfall, from a conviction founded on the experience of a cautious progressive practice as to the depth and distance, that depth consists with economy of outlay as well as with superior effect. He has found 4-feet drains to be efficient, at 50 feet asunder, in soils of varied texture —not uniform clays—and executes them at a cost of about £2 5*s.* per acre, being 18*s.* 4*d.* for 871 pipes, and £1 6*s.* 6*d.* for 53 rods of digging. Communications have been recently made to me by several respectable Kentish farmers, of the satisfactory performance of drains deeply laid in the Weald clays, at distances ranging from 30 to 40 feet, but I have not had the opportunity of personally inspecting these drainages.

"The following little table shows the actual and respective cost of the above three cases of under-draining, calculated on the effects really produced, that is, on the masses of earth effectively relieved of their surplus water at an equal expense. I conceive this to be the true expression of the work done, as a mere statement of the cost of drainage per acre of surface conveys but an imperfect, indeed a very erroneous, idea of the substantive and useful expenditure on any particular system. This will be apparent on reference to the two last columns of the table, which give the cost in cubic yards and square yards of soil drained for one penny, at the above-mentioned prices, depths, and distances.

Depth of drains, in feet.	Distance between the drains, in feet.	Mass of soil drained per acre, in cubic yards.	Mass of soil drained for one penny, in cubic yards.	Surface of soil drained for one penny, in square yards.
2	24	3226½	4·1	6·27
3	33½	4840	8·93	8·93
4	50	6453	12·00	8·96

"I may here observe, that Mr. Hammond, when draining tenacious clays, chooses the month of

February for the work, when he lays his pipes (just covering them with clay to prevent crumbs from getting in), and leaves the trenches open through March, if it be drying weather, by which means he finds the cracking of the soil much accelerated, and the complete action of the drains advanced a full season. The process of cracking may, doubtless, be hastened both by a choice of the period of the year in which drains are made, and by such a management of the surface as to expose it to the full force of atmospheric evaporation."

With reference to drains, we have known a case in the Queen's County in which inch pipes had to be taken up, and pipes of 2½-inch bore substituted. The drains were 40 feet apart, and 4 feet deep, and the pipes had collars. The minor drains should discharge into submains at convenient distances, say 100 yards, on flat grounds. Small pipes will choke unless the velocity in them be sufficient to carry off deposits, and the diameters should vary according to the inclinations of the ground, and distance apart of the drains.

Mr. Mechi, in 1844, laid down the following rules:—

"1st.—That it is not the size or form of the drains that regulate perfect drainage; but the *depth at which they are placed.* The depth also governs the distances at which the drains should be cut according to the quality of the soil.

"2nd.—The pipes of 1-inch bore, without stones, are amply sufficient, placed at 4 feet deep and 30 feet wide in dense soils, and the same depth and 50 feet wide in mixed soils.

"3rd.—The deep drains receive more water than shallow ones, and consequently lay dry a greater extent of ground.

"4th.—The deep drains *begin and end running sooner than shallow ones,* and carry off more water in a given time.

"5th.—That where shallow drains are made and deep ones cut below them, the shallow ones no longer act, all the water passing to the deeper drains.

"6th.—That when round stones are used as well as pipes, the latter should always be placed at the bottom, as I find, practically, water flows more quickly through pipes than amongst stones.

"Before persons begin draining, I would recommend their perusing attentively the facts developed by Mr. Parkes, at pages 39 and 40, and my remarks at page 36 of Letters on Agricultural Improvements.

"Pipes made to socket into each other (by Ford's Patent Socketing Machine) are best adapted to loose or mixed soils."

Pipes laid, however, too near the surface, are frequently choked with the roots of plants. The principal advantage of submains alongside open mains is, that the mouths of the minor drains should not be choked from vegetation, and that the water from them, flowing into and taken up by this submain, may be discharged by a few apertures only, and thereby keep themselves open, or as much so as the nature of the case will admit. The following tables show the cost per statute acre, in Ireland, of thorough-drainage, which must vary with circumstances, locality, and the value of labour.

The average cost per statute acre for Sir Richard O'Donnell's Gold Medal was £3 5*s.* 7*d.*, and £4 13*s.* 10*d.*

TABLE showing a Return of the number of Acres thorough-drained in the years 1843 *and* 1844, *by the different Competitors for Sir Richard O'Donnell's Gold Medal, together with the Average Prices per Perch, and Cost per Acre respectively.* (*Given in Reports to the Royal Agricultural Improvement Society of Ireland.*)

Competitors.	Number of statute acres.	Number of statute perches.	Average price per statute perch.	Average price per statute acre	Cost to the tenants.	Rate of wages per day and by task work.
	A. R. P.	P.				
Marquess of Waterford	501 2 15	66,900	9*d.* in 1842, reduced to 5⅓*d.* in 1845.	£5 in 1842, reduced to £3 2*s.* 8*d.* in 1845.	5 per cent. charged to the rent.	..
Viscount Templetown .	564 0 25	54,351	4¼*d.*	£1 14 4	£500 0 0	..
Sir R. O'Donnell, Bart.	551 0 0	53,478	4*d.*	1 12 4	408 6 0	8*d.* a day.
The Earl of Caledon .	321 0 19	47,183	4*d.*	2 9 5	396 9 9	10*d.* to 1*s.* a day.
J. L. W. Naper, Esq. .	254 1 29	34,433	9¾*d.*	5 8 3	700 7 4	7*d.* a day in winter, and 10*d.* in summer.
Lord Blayney .	201 2 30	34,634	5*d.*	3 12 0	427 18 0	10*d.* to 1*s.* a day.

TABLE showing a Return of the number of Acres thorough-drained by Proprietors, for the Society's Gold Medal, and the Average Prices per Perch and per Acre respectively.

	A. R. P.	P.		£ *s.* *d.*		
The Earl of Erne . .	110 0 37	..	..	3 12 3	..	..
Lord Dufferin and Clanboye . . .	203 1 0	29,478	10¾*d.*	6 11 9	..	1*s.* per day.
Messrs. Andrews . .	117 1 4	16,614	9¾*d.*	5 15 0	..	13*d.* per day.
Dr. O'Neill . . .	115 0 12	..	..	2 16 3	..	7*d.* to 8*d.* a day.

for the Society's Gold Medal; average of both, £4 per statute acre nearly.

The average number of acres annually improved in Ireland, about the year 1860, was about 5530, at an average cost per acre of £4 17*s*.

In Ireland, thorough-drainage is almost generally carried out by loan, under the Commissioners of Public Works, and there is no branch of the public service which has given more satisfaction to owners of property. The works are, we believe, always executed within the estimates, *and the owner having the expenditure in his own hands, can satisfy himself of its proper application.* How different from the Arterial Drainage, when the Board executed the works themselves, a system now so happily changed! No loans are made unless where immediately, or prospectively, a return of 6½ per cent. is estimated on the expenditure, a rent-charge for this amount being made for 22 years.

ARTERIAL DRAINAGE.

The effect of thorough-drainage on the arterial channels of a district, is to discharge the rain-fall into the main channels in a shorter time than before, particularly during wet seasons. This frequently causes floods to rise higher as well as more rapidly. During dry seasons the supply is less, and so far, when it is limited, an injury is done to the adjacent districts requiring it for use. The effect of obstructions in the main channel is to impound the upland water, sometimes made available for water power or navigation purposes, but in general to the injury of the drainage of adjacent lands, and the regimen of the

TABLE showing Estimates of the Quantities and General Cost for the Thorough-Drainage of a Statute Acre of Land, with broken Stones or Tiles, with the distances apart for different class soils.

Description of land, section of parallel drain, and cost.	Distance between the parallel drains.	Lineal perches of drains per statute acre.	Cubic yards per acre.		Expense per statute acre.			Number of feet of tiles per statute acre.
			Of clay excav.	Of broken stones.				
	Feet	Perches	Cu. yds.	Cu. yds.	£	s.	d.	No.
Hard subsoil stiff and sandy clay drains, from 12 to 20 feet apart, at 8*d.* per lineal perch.	12	220	277	38·5	7	6	8	3,630
	13	203	255⅔	35·5	6	15	4	3,351
	14	188½	237	33·0	6	5	8	3,111
	15	176	221	30·8	5	17	4	2,904
	16	165	207	28·9	5	10	0	2,722
	17	155⅓	195½	27·2	5	3	6½	2,562
	18	146⅓	184	25·9	4	17	6⅛	2,420
	19	139	175	24·3	4	12	0	2,293
	20	132	166⅔	23·1	4	8	0	2,178
Freestone bottom drains, from 20 to 30 feet apart, at 9*d.* per lineal perch.	21	125⅛	182½	29·3	4	14	6	2,074
	22	120	174⅔	28·1	4	10	0	1,971
	23	115	167	26·9	4	6	3	1,894
	24	110	159⅓	25·7	4	2	6	1,815
	25	105½	153½	24·7	3	19	1½	1,742
	26	101½	147⅓	23·7	3	16	1½	1,675
	27	97¾	142	22·9	3	13	4	1,613
	28	94	136½	22·0	3	10	6	1,556
	29	91	132⅓	21·3	3	8	3	1,502
	30	88	127¾	20·6	3	6	0	1,452
Beds of gravel, sand, and rocky stratification, from 30 to 100 feet apart, at 10*d.* per lineal perch.	31	85	151½	24·7	3	10	10	1,405
	32	82½	147	24·0	3	8	9	1,361
	33	80	142⅔	23·2	3	6	8	1,320
	34	77⅔	138½	22·6	3	4	8½	1,280
	35	75⅓	134	21·9	3	2	9¼	1,245
	36	73⅓	130⅓	21·3	3	1	1¼	1,210
	37	71	126½	20·6	2	19	2	1,177
	38	69	123	20·0	2	17	6	1,146
	39	67⅓	119⅔	19·6	2	16	1¼	1,117
	40	65⅔	116½	19·1	2	14	8½	1,089
	100	27	47	8·0	1	2	6	436

Estimates and Expenditure for some Works in Ireland, arranged from the Parliamentary Report and Papers of June 16, 1853.

No.	Name of District.	Original Estimate, exclusive of Works proposed to be omitted and of County Works, but including Interest up to the time the Works were stopped for Second Assents.	Total Cost of the Works, exclusive of Interest on Borrowed Money and of County Works.	Area of Land to be Drained and Improved.	Average Cost per Acre, exclusive of Interest on Borrowed Money and of County Works.	Original Estimate, Average Cost per Acre, exclusive of Works proposed to be omitted and of County Works, but including Interest to the time the Works were stopped for Second Assents.
		£ s. d.	£ s. d.	Acres.	£ s. d.	£ s. d.
1	Kilbeggan . .	8,850 15 0	24,055 5 9	5,970	4 0 7	1 9 8
2	Brosna (Ferbane)	40,085 1 1	71,966 9 1	15,723	4 11 6	2 11 0
3	Dunkellin . .	16,943 16 11	29,602 14 0	7,921	3 14 9	2 2 9
4	Cappagh . .	8,133 3 8	13,415 6 6	3,621	3 14 1	2 4 11
5	Lavally . .	9,763 1 10	12,740 4 7	2,167	5 17 7	4 10 1
6	Fergus . . .	37,359 3 5	44,568 0 3	8,419	5 5 10	4 8 9
7	Shanagolden .	7,393 12 2	10,299 10 9	1,000	10 6 0	7 7 10
8	Inny . . .	29,191 4 5	44,471 4 4	14,770	3 0 2	1 19 6
9	Rinn and Black River . .	17,205 17 9	21,998 10 7	5,691	3 17 4	3 0 5
10	Lough Gara and Mantua . .	10,719 11 8	18,380 2 6	4,374	4 4 0	2 9 0
11	Kill . . .	2,145 19 11	2,035 5 6	1,025	1 19 8	2 1 10
	Total .	186,916 5 0	293,532 13 10	70,681	4 3 0	2 12 10

TABLE of Expenditure and Cost of Works on Eleven Arterial Drainage Works in Ireland.

NAME OF WORK.	Quantity of Earthwork Gravel.	COST.			Quantity of Rockwork.	COST.			Cost of Un-watering.			Cost of Superintendence, not including Engineers' Establishments.			Cost of Plant, Tools, and Materials not used in the Works.			Analysis of Cost per Cube Yard in Pence.						
																		Gravel.	Rock.	Unwatering.	Implements.	Superintendence.	Total.	
																							Gravel.	Rock.
	Cu. yds.	£	s.	d.	Cu. yds.	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.	Pence	Pence	Pence	Pence	Pence	Pence	Pence
Kilcolgan .	80,992	1,984	9	11	22,572	868	7	4½	144	1	11	500	2	10	374	18	8	5·88	9·23	0·30	0·87	1·15	8·20	11·55
Craughwell .	51,491	600	18	10	2,006	100	0	0	8	6	2	123	0	0	88	0	0	2·80	12·00	0·04	0·39	0·55	3·78	12·98
Strongfort .	17,475	285	12	8	400	12	0	0	13	17	11	55	0	0	38	0	0	4·00	7·20	0·18	0·51	0·73	5·42	8·62
Ballymore .	75,810	1,771	16	11	9,350	509	1	0	7	5	10	360	0	0	255	0	0	5·61	13·00	·02	0·72	1·01	7·36	14·75
Raford . .	64,100	1,151	10	2	800	60	9	0	47	18	3	215	0	0	150	0	0	4·31	18·00	0·18	0·56	0·80	5·85	19·54
Ballykeeran .	17,033	285	9	9½	60	9	10	0	17	5	6	55	0	0	38	0	0	4·00	38·00	0·24	0·53	0·77	5·54	39·54
Dunsandle .	35,724	669	15	9½	3,526	114	5	0	35	12	2	130	0	0	89	0	0	4·50	7·77	0·21	0·54	0 79	6 04	9·31
St. Clerans .	40,842	745	8	7	850	44	0	0	29	9	5½	147	15	0	101	0	0	4·38	12·42	0·17	0·58	0·85	5·98	14·02
Loughrea .	89,296	1,512	12	8	2,700	180	18	4	12	19	0	297	10	0	205	0	0	4·06	16·08	0·03	0·53	0·77	5·39	17·41
Monksfield .	61,122	594	2	8	320	33	14	0	9	18	7	157	15	0	110	0	0	2·33	25·25	0·04	0·42	0·61	3·40	26·32
Lackafinna .	48,790	864	6	9	522	53	14	0	15	19	1	108	0	0	77	0	0	4·25	24·69	0·08	0·37	0·52	5·22	26·66
Carra . .	74,850	1,186	5	6½	7,900	444	0	0	25	8	2	198	0	0	136	0	0	3·80	13·48	0·07	0·39	0·57	4·83	14·51
Raruddy . .	28,854	388	9	8½	3,060	113	12	0	4	3	7	79	0	0	56	10	0	3·22	8·90	0·03	0·42	0·59	4·36	9·94
	686,179	12,041	0	0	54,066	2,543	10	9	372	5	7½	2,426	2	10	1,718	8	8	4·21	11·29	0·12	0·55	0·78	5·66	12·74

river, particularly in flat districts. The arterial drainage in Ireland has effected a vast amount of good, but up to 1853 the estimates appear to have been usually doubled; those for eleven of these works being £186,916, and the expenditure £293,532. The average cost per acre, on the land improved by these projects, varied from £1 19*s.* 8*d.* to £10 6*s.*, the average of the eleven districts being £4 3*s.*, which is about the average for thorough-drainage.*

The following table affords valuable information of the cost of arterial drainage works in Ireland: it is extracted from the Report of the Commissioners of Inquiry presented to the House of Commons, June 16th, 1853.

The abstract of 84 arterial drainage awards, made by the Commissioners of Public Works in Ireland, in 1854, gives for different years, 1849 to 1854—

Number of districts drained.	Total combined catchment acreage of districts.	Area of flooded lands.	Average cost of arterial drainage per acre improved by drainage. £	*s.*	*d.*
12 districts	90,332	9,453	3	14	2
27 ,,	95,582	11,579	3	16	7
19 ,,	237,466	13,707	4	17	3
16 ,,	374,427	29,452	3	13	4
2 ,,	49,840	3,275	5	0	0
8 ,,	266,420	21,033	3	9	4
84 ,,	1,114,067	88,501	3	17	7

The last line gives the general average, and shows that in these 84 districts, about 1 acre in 13 is the average of flooded lands to the catchment area, or 8 per cent. nearly. The original and revised estimates for these works were considerably exceeded in almost every case, and the landowners having justly resisted any payments *exceeding their original* assents, they succeeded. See REPORT OF THE COMMISSIONERS OF INQUIRY INTO THE WORKS OF ARTERIAL DRAINAGE IN ELEVEN DISTRICTS IN IRELAND, presented to the House of Commons, June 16th, 1853.

---

* See Parliamentary Report, by Sir Richard Griffith, Sir W. Cubitt, and Jas. M. Rendel, June 16th, 1853, pp. 14, 15.

SECTION XIV.

WATER AND HORSE POWER.—FRICTION BRAKE, OR DYNAMOMETER.—CALCULATION OF THE EFFECTIVE POWER OF WATER WHEELS.—OVERSHOT, UNDERSHOT, AND BREAST VERTICAL WHEELS.—HORIZONTAL WHEELS AND TURBINES.—HYDRAULIC RAM.—WATER ENGINE.

Taking the representative of a horse's power at 33,000 foot-pounds, or 33,000 lbs. raised one foot high in one minute, the theoretical horse-power of an overfall is expressed by the fall in feet, multiplied by the discharge in cubic feet per minute, the product multiplied by $62\frac{1}{3}$ (the weight in lbs., nearly, of a cubic foot of water), and divided by 33,000. The following table (page 400) gives the weight in air of a cubic foot of pure water at different temperatures, Fahrenheit's thermometer.

WEIGHT OF A CUBIC FOOT OF WATER.

The weight of 36 cubic feet of water is one ton, nearly.

Temperature, in degrees.	Weight of a cubic foot of water. Pounds Avoirdupois.	Temperature, in degrees.	Weight of a cubic foot of water. Pounds Avoirdupois.	Temperature, in degrees.	Weight of a cubic foot of water. Pounds Avoirdupois.
32	62·375	51	62·365	69	62·278
33	62·377	52	62·363	70	62·272
34	62·378	53	62·359	71	62·264
35	62·379	54	62·356	72	62·257
36	62·380	55	62·352	73	62·249
37	62·381	56	62·349	74	62·242
38	62·381	57	62·345	75	62·234
39	62·382	58	62·340	76	62·225
40	62·382	59	62·336	77	62·217
41	62·381	60	62·331	78	62·208
42	62·381	61	62·326	79	62·199
43	62·380	62	62·321	80	62·190
44	62·379	63	62·316	81	62·181
45	62·378	64	62·310	82	62·172
46	62·376	65	62·304	83	62·162
47	62·375	66	62·298	84	62·152
48	62·373	67	62·292	85	62·142
49	62·371	68	62·285	86	62·132
50	62·368			87	62·122

16,500 foot-pounds, or one half of the above, is much nearer the average power of a horse, working for 10 hours only, as the work is ordinarily done through the country; 33,000 lbs. raised one foot per minute is equivalent to 884 tons, nearly, raised one foot in an hour, and 14·73 tons in a minute. Therefore, a river discharging 884 tons, over a fall one foot high in an hour, or 884 tons, over a fall 24 feet high in 24 hours, has also a horse-power. The drainage of 10 square miles, with an average collection of 12 inches in depth of rain annually, will give an unceasing one-horse power for each foot of fall in a receiving channel; or five square miles will give the same result, if the collection amounts to 24 inches in depth. The collection of 10 square miles, one foot deep, yearly, is nearly equal to the delivery of 530 cubic feet per minute, for the same period; or to one-horse power theoretical, working day and night. See pp. 322 & 323.

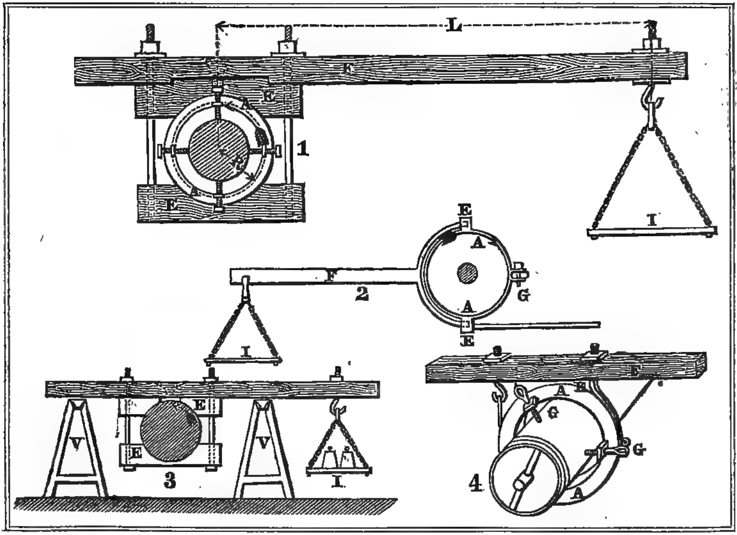
The effective power of a fall depends on the nature, proportions, and construction of the wheel or machine, and also upon the manner in which the theoretical power is applied. When the velocity of a stream acting on a wheel only is known, the theoretical head, h, due to it is found in feet from the formula $h = \cdot 0155\ v_2$, v being the velocity in feet per second.

In order to gauge the quantity of water applied to a wheel, and thereby determine with accuracy its effective power, the water used must be passed through an orifice, or over a notch or a weir, the coefficients for which has been previously ascertained from experiment. Greater accuracy can be obtained from gaugings through thin plates, or planks having the downstream arrises chamfered off, than with any other form of orifice or notch; and when it can be effected, the channel above should be sufficiently enlarged to prevent the effects of an approaching current. We have already in the body of this work dwelt in detail on the various formulæ required for gauging under different circumstances. The accuracy of results, showing the effective powers of wheels, depends in the first place, on the accuracy of the gaugings or estimates of the quantity of water used, and next on the fall employed. The loss of head in passing through the penstock sluice, or orifice, and until the water acts on the wheel, requires to be estimated also, and deducted from the fall or head, to find the effective fall, and the effective power of a wheel.

FRICTION BRAKE, OR DYNAMOMETER.

The power applied to a revolving shaft through a water wheel of any construction, is the weight of water multiplied by the fall. It is evident that the portion of this power available to turn a shaft and machinery, or the effective power, must depend on the construction of the wheel, as portion of the theoretical power is lost mechanically, in applying it; in changes of direction, friction, eddies, and discharging currents. The greater the effective power conveyed

FIG. 44.
FRICTION BRAKES, OR DYNAMOMETERS.



to a shaft, the greater becomes the power of the wheel, or medium through which the original power is transmitted. The mechanical effect produced by a revolving shaft is best measured by a friction brake, the principle of which is as follows. In diagram 1, Fig. 44, let the friction pulley AA be firmly fixed to the revolving shaft or axis of the wheel; E and E, two wooden clamps grasping the friction pulley by means of the screw bolts, delineated, which can be tightened on the axis, and also to the arm F, by means of suitable nuts. The more tightly the bolts are screwed, the greater will be the friction between the friction pulley AA, and the clamping pieces EE. If, while the axis and friction pulley AA, are revolving in the direction indicated by the arrow, a weight be applied in the scale at I, so that the arm F shall not be carried round, but remain fixed; it is clear that the work done by the revolving shaft in one revolution, will be measured by the circumference of the friction pulley, multiplied by the friction due to the pressure on it, or by its equivalent, the weight in the scale I, multiplied by the circumference of a circle whose radius is L, or by $2\,L \times 3{\cdot}14159 \times w$, in which expression w is the weight in lbs. in the scale I. If n be the number of revolutions in a given time, say one minute, we shall therefore have the useful effect of the wheel on the shaft in foot-pounds per minute, equal to

$$2\,L \times 3{\cdot}14159 \times w \times n.$$

We have also the power of the water acting on the wheel, equal to

$$h \times D \times 62{\cdot}37,$$

in which h is the head and D the discharge in cubic feet per minute; therefore we shall have for the ratio of the effect to the power the expression

$$\frac{2\,\text{L} \times 3{\cdot}14159 \times w \times n}{h \times \text{D} \times 62{\cdot}37} = \frac{\text{L}\,w\,n}{9{\cdot}926\,\text{D}\,h}.$$

If the revolving shaft be horizontal, the weight of the arm F, acting at its centre of gravity and reduced to the length L, where the weight w is suspended, will have to be included in the weight w. If the weight w be suspended at the end of a connection of levers, or other mechanical powers, the length L will have to be determined accordingly. Diagram 2, Fig. 44, shows the Armstrong brake; Diagram 3, the common form; and Diagram 4, Egen's brake.

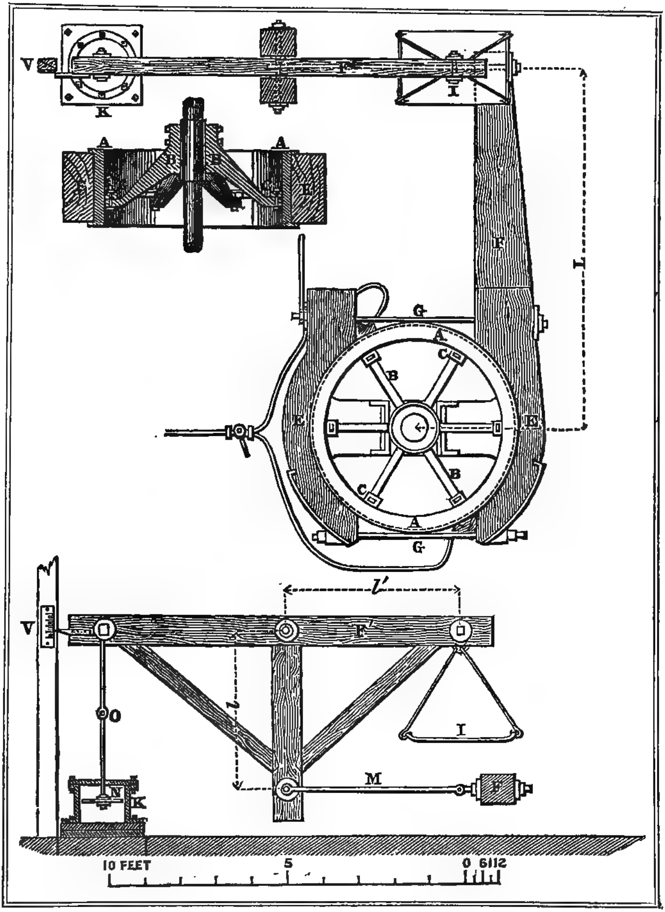
Fig. 45, is a general representation of the brake used by Francis, in the Lowell experiments. The length of the arm of the brake L, was 9·745 feet; the length of the vertical arm l of the bell crank 4·5 feet; and the length of the horizontal arm l' 5, feet. The following detailed description is by Francis:—

" *The Friction Pulley* A is of cast-iron, 5·5 feet in diameter, two feet wide on the face, and three inches thick. It is attached to the vertical shaft by the spider B, the hub of which occupies the place on the shaft intended for the bevel gear.

" The friction pulley has, cast on its interior circumference, six lugs, C C, corresponding to the six arms of the spider. The bolt holes in the ends of the arms are slightly elongated in the direction of the radius, for the purpose of allowing the friction pulley to expand a little as it becomes heated, without throwing much strain upon the spider. When the spider and friction pulley are at the same temperature, the ends of the arms are in contact with the friction

FIG. 45.

DYNAMOMETER FOR DETERMINING THE USEFUL EFFECT OF THE TREMONT TURBINE.



pulley. The friction pulley was made of great thickness for two reasons. When the pulley is heated, the arms cease to be in contact with the interior circumference of the pulley, consequently they would not prevent the pressure of the brake from altering the form of the pulley. This renders great stiffness necessary in the pulley itself. Again, it is found that a heavy friction pulley insures more regularity in the motion, operating, in fact, as a fly-wheel, in equalizing small irregularities.

"*The brakes* E *and* F are of maple wood; the two parts are drawn together by the wrought-iron bolts G G, which are two inches square.

"*The bell crank* F′, carries at one end the scale I, and at the other the piston of the hydraulic regulator K; this end carries also the pointer L, which indicates the level of the horizontal arm. The vertical arm is connected with the brake F, by the link M.

"*The hydraliuc regulator* K, shown in the figures, is a very important addition to the Prony dynamometer, first suggested to the author by Mr. Boyden, in 1844. Its office is to control and modify the violent shocks and irregularities, which usually occur in the action of this valuable instrument, and are the cause of some uncertainty in its indications.

"The hydraulic regulator used in these experiments, consisted of a cast-iron cylinder K, about 1·5 feet in diameter, with a bottom of plank, which was strongly bolted to the capping stone of the wheel pit, as represented in figure 1. In this cylinder, moves the piston N, formed of plate-iron 0·5 inches thick, which is connected with the horizontal arm of the bell crank by the piston rod O. The circumference of the piston is rounded off, and its diameter is about $\frac{1}{16}$ inch less than the diameter of the interior of the cylinder. The action of the hydraulic regulator is as follows. The cylinder should be nearly filled with water, or other heavy inelastic fluid. In case of any irregularity in the force of the wheel, or in the friction of the brake, the tendency will be, either to raise or lower the weight, in either case the weight cannot move, except with a corresponding movement of the piston. In consequence of the inelasticity of the fluid, the piston can move only by the displacement of a portion of the fluid, which must evidently pass between the edge of the piston and the cylinder; and the area of this space being very small, compared to the area of the piston, the motion of the latter must be slow, giving time to alter the tension of the brake screws before the piston has moved far. It is plain that this arrangement must arrest all violent shocks, but, however violent and irregular they may be, it is evident, that, if the mean force of them is greater in one direction than in the other, the piston must move in the direction of the preponderating force, the resistance to a slow movement being very slight. A small portion of the useful effect of the turbine must be expended in this instrument, probably less, however, than in the rude shocks the brake would be subject to without its use.

"For the purpose of ascertaining the velocity of the wheel, a counter was attached to the top of the vertical shaft, so arranged, that a bell was struck at the end of every fifty revolutions of the wheel.

"To lubricate the friction pulley, and at the same time to keep it cool, water was let on to its surface in four jets, two of which are shown. These jets were supplied from a large cistern, in the attic of the neighbouring cotton mill, kept full, during the working hours of the mill, by force pumps. The quantity of water discharged by the four jets was, by a mean of two trials, 0·0288 cubic foot per second.

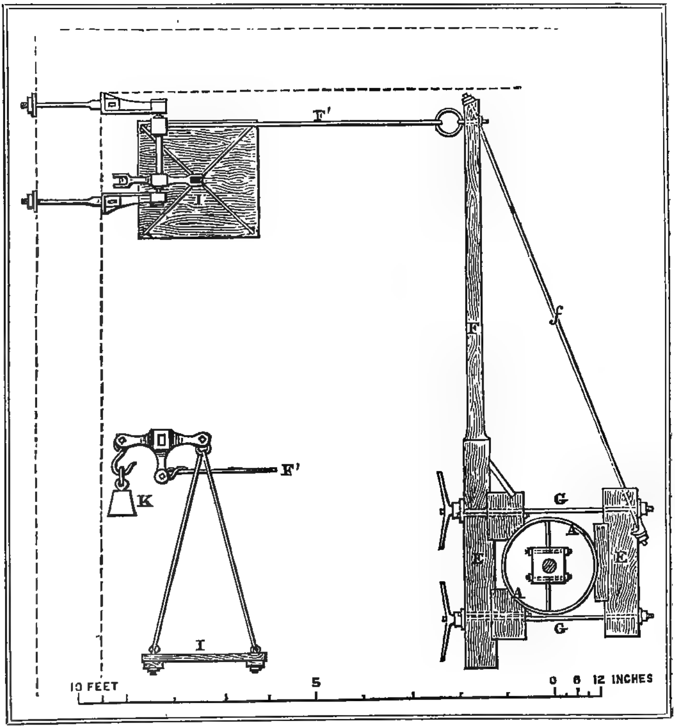
"In many of the experiments with heavy weights, and consequently slow velocities, oil was used to lubricate the brake, the water, during the experiment, being shut off. It is found, that, with a small quantity of oil, the friction between the brake and the pulley is much greater than when the usual quantity of water is applied; consequently, the requisite tension of the brake screws is much less with the oil, as a lubricator, than with water. This may not be the whole cause of the phenomenon, but, whatever it may be, the ease of regulating in slow velocities is incomparably greater with oil as a lubricator, than with water applied in a quantity sufficient to keep the pulley cool. The oil was allowed to flow on in two fine continuous streams; it did not, however, prevent the pulley from becoming heated sufficiently to decompose the oil, after running some time, which was distinctly indicated by the smoke and peculiar odour. When these indications became very apparent, the experiment was stopped, and water let on by the jets, until the pulley was cooled. As the pulley became heated, the brake screws required to be gradually slackened. Water, linseed oil, and resin oil, were each used for lubrication."

Fig. 46 is a representation of a brake used by Professor Thomson, at Crawford and Lindsay's mill, to determine the power of a turbine put up there, by Mr.

Gardner, of Armagh. One of the common causes of the swinging or vibratory jumps of the arms F, in Figs. 44, 45, and 46 is, that very often the friction pulley, or drum A, A, must be made in two parts, so as to be fixed

Fig. 46.

FRICTION BRAKE, USED AT CRAWFORD AND LINDSAY'S MILL, BY PROFESSOR JAMES THOMSON, TO DETERMINE THE POWER OF THE TURBINE.



Length of the brake, L, Fig. 45, adjusted . .	9·745 feet.
Effective length of vertical arm l	4·5 ,,
Effective length of horizontal arm l' . . .	5·0 ,,

to its place on the shaft. This fixing is liable to give an oval shape, and causes an irregular action with the clamps E, E. Oil gives greater regularity of motion than water, but without the use of the latter abundantly, the friction pulley would usually get too much heated. The following calculations from practical operations will point out pretty clearly the use of the brake, and the manner of determining the useful effect in the tables of experiments, by Francis and Thomson, pp. 383 and 392:—

The effective length of the brake was therefore $\frac{9{\cdot}745 \times 5}{4{\cdot}5} = 2 \times 10{\cdot}827778$ feet; and the circumference of a circle of this radius $= 10{\cdot}827778 \times 3{\cdot}14159 = 68{\cdot}0329$ feet.

In the first experiment on the Tremont turbine, page 429, the number of revolutions of the wheel per second was ·89374, and the weight in the scale 1443·34 lbs. The useful effect of the brake was therefore in foot-pounds per second $68{\cdot}0329 \times {\cdot}89374 \times 1443{\cdot}34 = 87680{\cdot}3$ lbs. raised one foot per second. The quantity of water which passed the gauge-weir in cubic feet per second was 139·4206, and the total fall acting on the wheel 12·864 feet; therefore, the total power of the water acting on the wheel was $12{\cdot}864 \times 139{\cdot}4206 \times 62{\cdot}375 = 111870$ feet-pounds per second, 62·375 being taken as the weight in lbs. of a cubic foot of water at 32° Fahrenheit. The ratio of the useful effect, at the given velocity of the wheel (viz. 450 revolutions in 503·5 seconds), to the power expended, is therefore $\frac{87680{\cdot}3}{111870} = {\cdot}784$, or about $78\frac{1}{2}$ per cent. The effect in the experiments generally appears to have been a maximum, when the velocity of the interior circumference of the wheel was about 66 per cent. of the velocity due to the fall; and this was about half of the maximum velocity, which was 1·333 times that due to the fall alone, when the turbine was doing no work.

In Thomson's brake for determining the useful effect of the vortex turbine, erected from his designs at Ballysillan, Ireland, L = 4 ft. 2 in., and the circumference of a circle that would be described by the arm 3·14159 × 8 feet 4 inches = 26·18 feet. In the first experiment, taken from the tabulated results, page 419, we get 26·18, the circumference multiplied by 46·31, the weight in lbs., and the product by 323·3, the number of revolutions per minute, equal to 391,967 foot-pounds, for the effect transmitted from the turbine or work done. We have also 354·4, the number of cubic feet of water passed to the wheel per minute, multiplied by 62·37, the weight of a cubic foot of water in lbs., multiplied by 23·73 feet, the available fall, equal to 524,526 foot-pounds: therefore $\frac{391,967}{524,526} = \cdot 747$ is the useful effect, that in the table being ·7481, which probably arose from taking a different weight per cubic foot for the water. Of course the difference is immaterial. The drum attached to the vortex wheel shaft for fixing the brake to, was in two parts, bolted together, and firmly enclosing the shaft. It was of cast-iron, 20 inches diameter, and 8 inches wide; the shaft to which it was attached was $2\frac{3}{8}$ inches diameter. The arm of the brake was $5'\cdot4'' \times 6'' \times 4''\frac{1}{2}$, of timber, and extending 1 foot 2 inches beyond the centre of the shaft and drum. The clamping pieces were about 2 feet 5 inches long externally.

For overshot wheels the ratio of the power to the effect may be taken as 3 to 2, and therefore the effective horse-power,—taking 33,000 foot-pounds per minute, or 530 cubic feet falling one foot, as a standard,—will be 49,500 lbs. of water, or 795 cubic feet, falling one foot in one minute. The maximum effect varies with the construction of the wheel. Smeaton found it ·76 times the theoretical power; Weisbach ·78 times for the wheel of a Stamp Mill at Frieberg, which was 23 feet high, 3 feet wide, carrying 48 buckets.* To find the effective horse-power, the theoretical horse-power must be here divided by the coefficient of effect ·76 or ·78, which will give 43,600 foot-pounds, or 43,300 foot-pounds per minute. The following experimental results from a model wheel are by Smeaton.

In this table the effective power of the water must be reckoned upon the whole descent, because it must be raised that height, in order to be in a condition of producing the same effect a second time.

The ratios between the *powers* so estimated, and the *effects* at the *maximum*, deduced from the several sets of experiments, are exhibited at one view, in column 9, of Table II.; and from hence it appears, that those ratios differ from that of 10 to 7·6 to that of 10 to 5·2, that is, nearly from 4 to 3 to 4 to 2. In those experiments where the heads of water and quantities expended are least, the proportion is nearly as 4 to 3, but where the heads and quantities are greatest, it approaches nearer to that of 4 to 2; and by a medium of the whole, the ratio is that of 3 to 2, nearly. We have seen before, in our observations upon the effects of undershot wheels, that the general ratio of the power to the effect, when greatest, was 3 to 1; *the effect, therefore, of over-*

---

* Some valuable experiments on the power of water wheels are given by Rennie, in Weale's Quarterly Papers on Engineering, vol. vi. They however require reduction. An effect of ·75 requires a flow of 707 cubic feet per minute, with a fall of one foot, for a horse power.

TABLE containing the Result of Sixteen Experiments, on a Model Overshot Wheel, by Smeaton.

No.	Whole descent, in inches.	Water expended in a minute, in lbs.	Turns at the maximum in a minute.	Weight raised at the maximum in lbs.	Power of the whole descent.	Power of the Wheel.	Effect.	Ratios of the whole effect to the power.	Ratios of the effect to the power or coefficient of effect.	Mean Ratios.
1	27	30	19	6½	810	720	556	·69	·77	·81
2	27	56⅔	16¼	14½	1530	1360	1060	·69	·78	
3	27	56⅔	20¾	12½	1530	1360	1167	·76	·84	
4	27	63⅓	20½	13½	1710	1524	1245	·73	·82	
5	27	76⅔	21½	15½	2070	1840	1500	·73	·82	
6	28½	73⅓	18¾	17½	2090	1764	1476	·70	·84	·82
7	28½	96⅔	20¼	20½	2755	2320	1868	·68	·80	
8	30	90	20	19½	2700	2160	1755	·65	·81	·82
9	30	96⅔	20¾	20½	2900	2320	1914	·66	·82	
10	30	113⅓	21	23½	3400	2720	2221	·65	·82	
11	33	56⅔	20¼	13½	1870	1360	1230	·66	·90	·85
12	33	106⅔	22¼	21½	3520	2560	2153	·61	·84	
13	33	146⅔	23	27½	4840	3520	2846	·59	·81	
14	35	65	19¾	16½	2275	1560	1466	·65	·95	·85
15	35	120	21½	25½	4200	2880	2467	·59	·86	
16	35	163½	25	26½	5728	3924	2981	·52	·76	

shot wheels, under the same circumstances of quantity and fall, is at a medium double to that of the undershot; and, as a consequence thereof, that non-elastic bodies, when acting by their impulse or collision, communicate only a part of their original power; the other part being spent in changing their figure, in consequence of the stroke.

The powers of water, computed from the height of the wheel only, compared with the effects as in column 10, appear to observe a more constant ratio: for, if we take the medium of each class, which is set down in column 11, we shall find the extremes to differ no more than from the ratio of 10 to 8·1 to that of 10 to 8·5; and as the second term of the ratio gradually increases from 8·1 to 8·5, by an increase of head from 3 inches to 11, the excess of 8·5 above 8·1 is to be imputed to the greater impulse of the water at the head of 11 inches, above that of 3 inches: so that if we reduce 8·1 to 8, on account of the impulse of the 3-inch head, *we shall have the ratio of the power, computed upon the height of the wheel only, to the effect at a maximum, as* 10 *to* 8 *or as* 5 *to* 4, *nearly;* and from the equality of the ratio between power and effect subsisting, where the constructions are similar, we must infer, *that the effects,* as well as the powers, *are respectively as the quantities of water and perpendicular heights, multiplied together.*

For breast wheels, the ratio of the theoretical power to the effective power must vary considerably, the mean value being about ·5 and, therefore, the effective horse-power would be 66,000 foot-pounds, or 1,060 cubic feet falling 1 foot in one minute. Morin gives an efficiency of from ·52 to ·7. Egen, with a wheel 23 feet in diameter, 4½ feet wide, having 69 ventilated buckets, very well constructed, found at best an efficiency of only ·52, under ordinary circumstances ·48, the mean amount being ·5. Very wide wheels give a larger effect, sometimes as high as ·7; but a

TABLE containing the Result of Twenty-seven Experiments, on a Model Undershot Wheel, by Smeaton.

	Height of water in the cistern in inches.	Turns of the wheel unloaded.	Virtual head deduced therefrom in inches.	Turns at the maximum.	Load at the equilibrium, in lbs. and ozs.	Load at the maximum, in lbs. and ozs.	Water expended in a minute.	POWER.	EFFECT.	Ratio of the Effect to the power or coefficient of effect.	Ratio of the velocity of the wheel and water.	Ratio of the load at the equilibrium, to the load at the maximum.
1	33	88	15·85	30·	13 10	10 9	275·	4358	1411	·324	·340	·775
2	30	86	15·0	30·	12 10	9 6	264·7	3970	1266	·320	·350	·740
3	27	82	13·7	28·	11 2	8 6	243·	3329	1044	·315	·340	·750
4	24	78	12·3	27·7	9 10	7 5	285·	2890	901·4	·312	·355	·753
5	21	75	11·4	25·9	8 10	6 5	214·	2439	735·7	·302	·345	·732
6	18	70	9·95	23·5	6 10	5 5	199·	1970	561·8	·285	·336	·802
7	15	65	8·54	23·4	5 2	4 4	178·5	1524	442·5	·290	·360	·830
8	12	60	7·29	22·	3 10	3 5	161·	1173	328	·280	·377	·910
9	9	52	5·47	19·	2 12	2 8	134·	733	213·7	·290	·365	·910
10	6	42	3·55	16·	1 12	1 10	114·	404·7	117·0	·282	·380	·930
11	24	84	14·2	30·75	13 10	10 14	342·	4890	1505	·307	·366	·790
12	21	81	13.5	29·	11 10	9 6	297·	4009	1223	·301	·362	·805
13	18	72	10·5	26·	9 10	8 7	285·	2993	975	·325	·360	·875
14	15	69	9·6	25·	7 10	6 14	277·	2659	774	·292	·362	·900
15	12	63	8·0	25·	5 10	4 14	234·	1872	549	·294	·397	·870
16	9	56	6·37	23·	4 0	3 13	201·	1280	390	·305	·410	·950
17	6	46	4·25	21·	2 8	2 4	167·5	712	212	·298	·455	·900
18	15	72	10·5	29·	11 10	9 6	357·	3748	1210	·323	·402	·805
19	12	66	8·75	26·75	8 10	7 6	330·	2887	878	·305	·405	·810
20	9	58	6·8	24·5	5 8	5 0	255·	1734	541	·301	·422	·910
21	6	48	4·7	23·5	3 2	3 0	228·	1064	317	·299	·490	·960
22	12	68	9·3	27·	9 2	8 6	359·	3338	1036	·302	·397	·917
23	9	58	6·8	26·25	6 2	5 13	332·	2257	686	·304	·452	·950
24	6	48	4·7	24·5	3 12	3 8	262·	1231	385	·313	·510	·935
25	9	60	7·29	27·3	6 12	6 6	355·	2588	785	·303	·455	·945
26	6	50	5·03	24·6	4 6	4 1	307·	1544	450	·292	·490	·930
27	6	50	5·03	26·	4 15	4 9	360·	1811	534	·295	·520	·925

great deal depends on the manner of bringing on the water and the construction of the wheel and buckets.

PONCELET'S WHEEL is seldom if at all used in these countries. The effective power is ·5, or 1,060 cubic feet of water falling one foot for the standard horse-power.

FOR UNDERSHOT WHEELS the mean effect may be taken at one-third, or ·33, or 100,000 foot-pounds, or 1,590 cubic feet falling one foot in one minute, for an effective horse's power; a maximum effect of ·5 is sometimes approached, and a minimum of ·26 or less. The following results, obtained from a model, are given by Smeaton. The *virtual* or *effective* head is here termed the theoretical head due to the velocity of the wheel, at the circumference, which was 75 inches girth.

Smeaton derived the following "maxims" from the foregoing experiments. Their truth, independent of any experiment, will be apparent:—

I.—That the virtual or effective head being the same, the effect will be nearly as the quantity of water expended.

II.—That the expense of water being the same, the effect will be nearly as the height of the virtual or effective head.

III.—That the quantity of water expended being the same, the effect is nearly as the square of the velocity.

IV.—The aperture being the same, the effect will be nearly as the cube of the velocity of the water.

FOR TURBINES OR HORIZONTAL WHEELS, a useful effect of two-thirds or ·67 may be assumed, or 49,500 foot-pounds in a minute for a horse-power, and the efficiency varies from ·5 to ·8, or less.* Poncelet's turbine gives an efficiency of ·5 to ·6. Floating wheels ·38, impact wheels from ·16 to ·4, and Barker's mill from ·16 to ·35. We believe that the efficiency of the turbine has been too often over-estimated, and that the great advantage of this wheel, as a medium of power, is derived from its capability of employment for all falls, whether large or small, without any considerable loss of effect. In Ireland, Mr. Gardner, of Armagh, was amongst the first, if not the first, to apply this wheel to practical purposes; and Professor Thomson has, in his vortex wheels, produced, we believe, the highest efficiencies which have yet been obtained in practice. In the experiments on the Ballysillan wheel higher efficiencies would probably have been attained with a supply pipe of larger diameter. It will be seen from the remarks, at pp. 164 to 167, and the tables, at pp. 146 and 199, that quite apart from bends, &c., a loss of mechanical power always results from the passage through orifices and pipes; and that it is necessary to take this loss into account, before the head acting on the wheel can be accurately used to determine its effective power. The table, p. 419, contains the experiments on the Ballysillan turbine.

---

* In our first edition we gave an efficiency of ·821, on the authority

The following remarks on the vortex turbine, read at the meeting of the British Association at Belfast, in 1852, are also by Professor Thomson:—

of a paper by Dr. Robinson, Armagh, in the Proceedings of the Royal Irish Academy, vol. iv., p. 214. On again glancing over this paper, we believe there are mistakes, which vitiate the results there given; first, in the formula for calculating the discharge over the weir, and next, in the formula for finding the effect of the brake. Francis gives an efficiency of ·88, p. 3, in his book.

"Numberless are the varieties, both of principle and of construction, in the mechanisms by which motive power may be obtained from falls of water. The chief modes of action of the water are, however, reducible to three, as follows:—First, the water may act directly by its weight on a part of the mechanism which descends while loaded with water, and ascends while free from load. The most prominent example of the application of this mode is afforded by the ordinary bucket water wheel. Secondly, the water may act by fluid pressure, and drive before it some yielding part of a vessel by which it is confined. This is the mode in which the water acts in the water pressure engine, analogous to the ordinary high-pressure steam-engine. Thirdly, the water, having been brought to its place of action subject to the pressure due to the height of fall, may be allowed to issue through small orifices with a high velocity, its inertia being one of the forces essentially involved in the communication of the power to the moving part of the mechanism. Throughout the general class of water wheels called turbines, which is of wide extent, the water acts according to some of the variations of which this third mode is susceptible. In our own country, and more especially on the Continent, turbines have attracted much attention, and many forms of them have been made known by published descriptions. The subject of the present communication is a new water wheel, which belongs to the same general class, and which has recently been invented and brought successfully into use by the author.

"In this machine the moving wheel is placed within

***TABULATED STATEMENT** of Experiments made on March 29th, 1854, by Professor James Thomson, C.E., University of Glasgow; on the Vortex Wheel at Ballysillan, to determine its efficiency. The Radius of the Friction Brake was 4 feet 2 inches. The corresponding Circumference is 26·18 feet. In this Table the quantity of Water passing over the Weir is calculated by means of MM. Poncelet and Lesbros' Coefficients for Notches.*

Number of Experiments.	Total weight on cord of Break in pounds.	Time of each Experiment.	Number of Revolutions of Shaft during Experiment.	Number of Revolutions of Shaft per minute.	Total height of fall in feet.	Height in feet of Water over Weir, length of Weir being 3 feet.	Number of cubic feet of Water per minute over Weir.	Number of cubic feet of Water per minute, supplied to the Wheel, 1·39 being deducted to wet the Break.	Work given out at Friction Break, in foot pounds per minute.	Horse Power given out by the Wheel.	Work due to the fall of water in foot-pounds per minute.	Efficiency of the Wheel or Work given out by the Wheel, the Work due to the Water being 100.	Remarks entered at the time of the experiments, and relating to the supposed accuracy of the experiments.
		min. sec.											
1	46·31	5 34	1800	323·3	23·73	·718	355·8	354·4	392000	11·88	523900	74·81	Satisfactorily accurate.
2	46·31	5 38	1800	319·5	23·71	·718	355·8	354·4	387400	11·73	523500	73·99	Satisfactorily accurate.
3	53·31	6 11	1800	291·	23·72	·718	355·8	354·4	406100	12·31	523700	77·55	Good.
4	60·31	7 9	1800	251·7	23·71	·718	355·8	354·4	397400	12·04	523500	75·91	Water supposed running over.
5	53·31	6 12	1800	290·3	23·71	·718	355·8	354·4	405100	12·27	523500	77·39	One of the best, no water lost.
6	39·31	5 26	1800	351·2	23·21	·718	355·8	354·4	340800	10·33	512400	66·51	Some water lost at water case, and water too low part of time.
7	46·31	6 9	1800	292·7	23·77	·665	318·1	316·7	354900	10·75	469000	75·66	Good.
8	46·31	6 13	1800	289·5	23·75	·665	318·1	316·7	351000	10·63	468600	74·90	Good, one of the best.
9	32·31	5 8	1800	350·6	23·75	·665	318·1	316·7	296600	8·99	468600	63·28	Very good.
10	36·31	5 24	1800	333·3	23·75	·665	318·1	316·7	316800	9·59	468600	67·61	Very good.
11	36·31	5 21	1800	336·4	23·75	·665	318·1	316·7	319800	9·69	468600	68·24	Excellent.
12	53·31	7 5	1800	254·1	23·75	·663	316·7	315·3	354600	10·74	466500	75.01	Arm not very steady.
13	53·31	7 14	1800	248·8	23·75	·663	316·7	315·3	347200	10·52	466500	74·43	Arm not very steady.

Abstract of the Data and Calculations according to which the Vortex Wheel was designed.

Total Height of Fall = 24 feet, Standard quantity of Water = 420 cubic feet per minute. Calculated speed 292·5 revolutions per minute. Effective Standard Power at an efficiency of 75 per cent = 16·25 Horse Power. Supply pipe, 21 inch bore. Diameter of wheel, 1 ft. 8¼ ins.

REMARKS.—It is to be observed, that as the experiments happened to be made in dry weather, the stream in none of them supplied the standard quantity of water for which the wheel was more particularly adapted. Even with the diminished quantity of water, the efficiencies experimentally found were very high. It is also to be observed, that in some of the experiments the resistance to the wheel was purposely made less than what was most suitable for the water supply. The wheel thus ran considerably too fast in those experiments, and consequently the efficiencies in them came to be not so high as in the others that were more properly arranged. Also, from information received after the time of the experiments, it appeared that during the experiments the Joint Rings of the Vortex were not screwed properly close to the rings of the Water Wheel; and that on their being afterwards screwed, the power of the wheel was sensibly increased. It is therefore probable that still higher efficiencies are attainable than those shown by the above experiments.

a chamber of a nearly circular form. The water is injected into the chamber tangentially at the circumference, and thus it receives a rapid motion of rotation. Retaining this motion it passes onwards towards the centre, where alone it is free to make its exit. The wheel, which is placed within the chamber, and which almost entirely fills it, is divided by thin partitions into a great number of radiating passages. Through these passages the water must flow on its course towards the centre; and in doing so it imparts its own rotatory motion to the wheel. The whirlpool of water acting within the wheel chamber, being one principal feature of this turbine, leads to the name *Vortex* as a suitable designation for the machine as a whole.

"The vortex admits of several modes of construction, but the two principal forms are the one adapted for high falls and the one for low falls. The former may be called the High-pressure Vortex, and the latter the Low-pressure Vortex. Examples of these two kinds are in operation at two mills near Belfast.

"The height of the fall for the first vortex is about 37 feet, and the standard or medium quantity of water, for which the dimensions of the various parts of the wheel and case are calculated, is 540 cubic feet per minute. With this fall and water-supply the estimated power is 28 horse-power, the efficiency being taken at 75 per cent. The proper speed of the wheel, calculated in accordance with its diameter and the velocity of the water entering its chamber, is 355 revolutions per minute. The diameter of the wheel is 22⅝ inches, and the extreme diameter of the case is 4 feet 8 inches.

"In the second vortex, the fall being taken at 7 feet, the calculated quantity of water admitted, at the standard opening of the guide-blades, is 2,460 cubic feet per minute. Then, the efficiency of the wheel being taken at 75 per cent., its power will be 24 horse-power. Also, the speed at which the wheel is calculated to revolve is 48 revolutions per minute.

"The two examples which have now been described of vortex water wheels, adapted for very distinct circumstances, will serve to indicate the principal features in the structural arrangements of these new machines in general. Respecting their principles of action some further explanations will next be given. In these machines the velocity of the circumference is made the same as the velocity of the entering water, and thus there is no impact between the water and the wheel; but, on the contrary, the water enters the radiating conduits of the wheel gently, that is to say, with scarcely any motion in relation to their mouths. In order to attain the equalization of these velocities, *it is necessary that the circumference of the wheel should move with the velocity which a heavy body would attain, in falling through a vertical space equal to half the vertical fall of the water, or in other words, with the velocity due to half the fall;* and that the orifices through which the water is injected into the wheel-chamber should be conjointly of such area that when all the water required is flowing through them, it also may have a velocity due to half the fall. Thus one-half only of the fall is employed in producing velocity in the water; and, therefore, the other half still remains acting on the water within the wheel-chamber at the circumference of the wheel, in the condition of fluid pressure. Now, with the velocity already assigned to the wheel, it is found that this fluid pressure is exactly that which is requisite to overcome the centrifugal force of the water in the wheel, and to bring the water to a state of rest at its exit; the mechanical work due to both halves of the fall being transferred to the wheel during the combined action of the moving water and the moving wheel. In the foregoing statements, the effects of fluid friction, and of some other modifying influences, are, for simplicity, left out of consideration; but in the practical application of the principle, the skill and judgment of the designer must be exercised in taking all such elements, as far as possible, into account. To aid in this, some practical rules, to which the author as yet closely adheres, were made out by him previously to the date of his patent. These are to be found in the specification of the patent, published in the Mechanics' Magazine for January 18 and January 25, 1851 (London).

"In respect to the numerous modifications of construction and arrangement which are admissible in the vortex, while the leading principles of action are retained, it may be sufficient here merely to advert,—first, to the use of straight instead of curved radiating passages in the wheel; secondly, to the employment, for simplicity, of invariable entrance orifices, or of fixed instead of moveable guide-blades; and lastly, to the placing of the wheel at any height, less than about thirty feet, above the water in the tail-race, combined with the employment of suction pipes descending from the central discharge orifices, and terminating in the water of the tail-race, so as to render available the part of the fall below the wheel.

"In relation to the action of turbines in general, the chief and most commonly recognized conditions, of which the accomplishment is to be aimed at, are that the water should flow through the whole machine with the least possible resistance, and that it should enter the moving wheel without shock, and be discharged from it with only a very inconsiderable velocity. The vortex is in a remarkable degree adapted for the fulfilment of these conditions. The water moving centripetally (instead of centrifugally, which is more usual in turbines), enters at the period of its greatest velocity (that is, just after passing the injection orifices) into the most rapidly moving part of the wheel, the circumference; and, at the period when it ought to be as far as possible deprived of velocity, it passes away by the central part of the wheel, the part which has the least motion. Thus, in each case, that of the entrance and that of the discharge, there is an accordance between the velocities of the moving mechanism and the proper velocities of the water.

"The principle of injection from without inwards, adopted in the vortex, affords another important advantage in comparison with turbines having the contrary motion of the water; as it allows ample room, in the space outside of the wheel, for large and well-formed injection channels, in which the water can be made very gradually and regularly to converge to the most contracted parts, where it is to have its greatest velocity. It is as a concomitant also of the same principle, that the very simple and advantageous mode of regulating the power of the wheel, by the moveable guide-blades already described, can be introduced. This mode, it is to be observed, while giving great variation to the areas of the entrance orifices, retains at all times very suitable forms for the converging water channels.

"Another adaptation in the vortex is to be remarked as being highly beneficial, that, namely, according to which, by the balancing of the contrary fluid pressures due to half the head of water and to the centrifugal force of the water in the wheel, combined with the pressure due to the ejection of the water backwards from the inner ends of the vanes of the wheel when they are curved, only one-half of the work due to the fall is spent in communicating *vis viva* to the water, to be afterwards taken from it during its passage through the wheel; the remainder of the work being communicated through the fluid pressure to the wheel, without any intermediate generation of *vis viva*. Thus the velocity of the water, where it moves fastest in the machine, is kept comparatively low; not exceeding that due to half the height of the fall, while in other turbines the water usually requires to act at much higher velocities. In many of them it attains at two successive times the velocity due to the whole fall. The much smaller amount of action, or agitation, with which the water in the vortex performs its work, causes a material saving of power by diminishing the loss necessarily occasioned by fluid friction.

"In the vortex, further, a very favourable influence on the regularity of the motion proceeds from the centrifugal force of the water, which, on any increase of the velocity of the wheel, increases, and so checks the water supply; and on any diminution of the velocity of the wheel, diminishes, and so admits the water more freely; thus counteracting, in a great degree, the irregularities of speed arising from variations in the work to be performed. When the work is subject to great variations, as for instance in saw-mills, in bleaching works, or in forges, great inconvenience often arises with the ordinary bucket water-wheels and with turbines which discharge at the circumference, from their running too quickly when any considerable diminution occurs in the resistance to their motion.

"The first vortex which was constructed on the large scale was made in Glasgow, to drive a new beetling-mill of Messrs. C. Hunter and Co., of Dunadry, in County Antrim. It was the only one in action at the time of the meeting of the British Association in Belfast; but the two which have been particularly described in the present article, and one for an unusually high fall, 100 feet, have since been completed and brought into operation. There are also several others in progress; of which it may be sufficient to particularize one of great dimensions and power, for a new flax-mill at Ballyshannon in the West of Ireland. It is calculated for working at 150 horse-power, on a fall of 14 feet, and it is to be impelled by the water of the River Erne. This great river has an ample reservoir in the Lough of the same name; so that the water of wet weather is long retained, and continues to supply the river abundantly even in the dryest weather. The lake has also the effect of causing the floods to be of long duration, and the vortex will consequently be, through a considerable part of the year, and for long periods at a time, deeply submerged under backwater. The water of the tail-race will frequently be seven feet above its ordinary summer level; but as the water of the head-race will also rise to such a height as to maintain a sufficient difference of levels, the action of the wheel will not be deranged or impeded by the floods. These circumstances have had a material influence in leading to the adoption in the present case of this new wheel in preference to the old breast or undershot wheels."

The next tables have been arranged by us from Mr. Francis' valuable experiments. They show the ratio of the effect to the power in two wheels, the first a centre-vent wheel, erected at the Boott Cotton Mills, and the second a turbine, erected at the Tremont Mills, Lowell, Massachusetts.

The maximum effect ·794 was obtained from the

Tremont turbine experiments, when the velocity of the interior circumference of the wheel was to that due to the whole fall as ·63 to 1; and an effect of 78 per cent. was obtained when these velocities were as ·51 to 1. In the Boott centre-vent wheel the maximum effect ·797 was obtained when the velocity of the exterior circumference was to that due to the fall as ·64 to 1; and a like effect was produced when this ratio was ·708 to 1. Indeed, between these ratios the useful effect was nearly the same; an effect of ·78 to ·79 was obtained for all such ratios between limits of ·59 and ·71 to 1, averaging a ratio of ·65 to 1. If a turbine have a variable fall, say from 2 to 1, and be of sufficient capacity to give the required power always, the dimensions should be determined from the lesser fall, and if correctly so determined, it will not have sufficient velocity for the greater fall. When the fall is greatest the quantity in the same place is generally least, giving thereby a lessened effect when most is required. For such cases two turbines may be used with advantage.

TABLE showing the Results of Experiments upon a Model of a Centre-vent Water Wheel, and also upon a Centre-vent Water Wheel at the Boott Cotton Mills, Lowell, Massachusetts, arranged from Mr. Francis' valuable Experiments. Diameter of Wheel to the outside of the Buckets, about 9·3 feet. Depths of External Guide Curves about ·75 foot. External height of Wheel about 1·5 feet. Number of Buckets, 40. Mean height of the orifices between the Guides, 1 foot. Diameter of Supply Pipe, 8 feet. The first Seven Experiments were made on a Model, the Exterior Diameter of the Wheel being $22\frac{5}{8}$ inches, Interior Diameter $19\frac{1}{2}$ inches, height between the Crowns $2\frac{13}{16}$ inches, and the number of Buckets 36. The Construction of the Wheel is shown in Mr. Francis' Book, and all necessary Details. The general principle of the Centre-vent Wheel of Francis' and Thomson's Vortex Wheel appears to be the same; the Guide Blades being fewer in the latter, and capable of adjustment.

Numbers of the Experiments.	Falls acting on the Wheels.	Depths on the Weirs in feet.	Cubic feet of Water acting on the Wheel per second.	Number of pounds avoirdupois. If raised one foot per second.	Revolutions of the Wheel per second.	Weight in the scale in pounds avoirdupois.	Ratios of the effect to the power calculated by means of Prony's Brake.	Velocity due to the fall acting on the Wheel, in feet per second.	Velocity of the outside circumference of the Wheel, in feet per second.
1	2	3	4	5	6	7	8	9	10
1	2·52	·365	2·15	337·7	1·14	16·	·679	12·73	6·83
2	2·46	·366	2·16	331·5	1·34	14·	·711	12·58	8·03
3	2·50	·367	2·17	338·0	1·54	12·5	·716	12·68	9·23
4	2·60	·372	2·21	358·2	1·70	12·	·705	12·93	10·03
5	2·60	·373	2·22	361·3	1·73	11·5	·692	12·95	10·36
6	2·60	·373	2·22	559·6	1·71	11·5	·689	12·93	10·26
7	2·60	·374	2·23	360·8	1·90	10·0	·648	12·93	11·15
8	14·60	1·296	67·53	61493·4	·59	575·6	·377	30·65	17·39
9	14·67	1·262	64·89	59364·4	·88	202·1	·203	30·72	25·77
10	14·57	1·282	66·43	60347·0	·72	407·2	·332	30·61	21·21
11	14·16	1·284	66·61	58821·6	·54	606·0	·382	30·18	15·97
12	14·20	1·290	67·03	59351·7	·50	666·3	·381	30·22	14·65
13	14·14	1·288	66·89	59002·2	·45	720·5	·373	30·16	13·18
14	14·24	1·294	67·37	59858·1	·25	931·9	·269	30·27	7·46
15	14·30	1·211	61·08	54486·4	1·01	..	..	30 33	29·75
16	14·29	1·514	85·	75732·8	1·01	334·1	·303	30·32	29·63
17	14·23	1·531	86·35	76608·0	·96	441·2	·376	30·25	28·16
18	14·20	1·539	87·08	77093·2	·93	501·8	·413	30·22	27·35
19	14·19	1·547	87·68	77607·2	·90	562 6	·444	30·21	26·43
20	14·19	1·554	88·28	78143·8	·86	656·6	·489	30·22	25·12
21	13·78	1·576	90·17	77480·4	·69	955·5	·582	29·77	20·37
22	13·61	1·594	91·70	77812·1	·60	1140·9	·596	29·58	17·54
23	13·94	1·418	77·11	67076·7	1·18	..	..	29·9	34·51
24	13·52	1·642	95·76	80736·3	1·05	263·0	·245	29·49	32·44
25	13·37	1·673	98·49	82145·2	·99	531·8	·437	29·33	29·14
26	13·37	1·695	100·42	83728·2	·90	786·8	·576	29·32	26·43
27	13·40	1·718	102·42	85571·4	·82	1001·5	·657	29·35	24·20
28	13·38	1·723	102·82	85800·0	·79	1107·4	·690	29·34	23·07
29	13·34	1·731	103·52	86138·0	·75	1205·0	·710	29·30	21·89
30	13·32	1·734	103·77	86218·8	·79	1259·2	·720	29·27	21·26
31	13·33	1·733	103·70	86229·3	·71	1297·3	·728	29·29	20·86
32	13·30	1·739	104·23	86483·7	·70	1329·8	·731	29·25	20·50
33	13·70	1·598	92·02	78648·0	1·25	..	..	29·70	36·78
34	13·40	1·832	112·52	94057·5	·71	1554·2	·797	29·36	20·81
35	13·43	1·837	112·99	94662·2	·70	1584·0	·796	29·39	20:51
36	13·33	1·832	112·56	93603·9	·68	1613·9	·797	29·28	19·93
37	13·38	1·837	113·00	94296·4	·67	1644·4	·797	29·33	19·71
38	13·39	1·838	113·07	94415·2	·66	1675·1	·796	29·34	19·36
39	13·38	1·839	113·16	94471·1	·65	1705·5	·796	29·34	19·03
40	13·36	1·838	113·09	94219·0	·64	1735·9	·797	29·31	18·67
41	13·38	1·844	113·67	94881·9	·62	1768·4	·791	29·34	18·32
42	13·40	1·851	114·29	95571·2	·61	1802·1	·787	29·36	18·00
43	13·32	1·848	113·97	94703·1	·59	1836·2	·781	29·27	17·38
44	13·54	1·809	110·45	93270·1	..	3155·3	..	29·51	..
45	13·57	1·807	110·32	93422·4	..	2793·3	..	29·55	..
46	13·60	1·688	99·79	84635·0	1·29	..	..	29·57	37·70

TABLE showing the Results of Experiments upon the Turbine at Tremont Mills, Lowell, Massachusetts, arranged from Mr. Francis' valuable Experiments. Diameter, measured to the Exterior Circumference of Crowns of the Wheel, 8·333 *feet. Height of Buckets from top of the Disc to the bottom of the Garniture,* ·97 *feet. Number of Buckets,* 44. *Width of the Buckets,* ·8 *foot nearly. Width of Guide Curves,* 2·3 *feet nearly. Number of Ditto,* 33. *A Double Weir with* 4 *end constructions, and* 16·98 *feet long, used for gauging the Water, the Crest being* 6·5 *feet above the floor of the Wheel Pit. The Falls show the difference of heads in the Forebay and Wheel Pit. For further details, see Francis' Lowell Hydraulic Experiments, pp.* 1 *to* 43. *The supply pipe is fully a quadrant, and varies from* 6 *to* 9 *feet in diameter.*

Numbers of the Experiments.	Falls acting on the Wheel.	Depths on the Weir in feet.	Cubic feet of Water acting on the Wheel per second.	Number of pounds avoirdupois, if raised one foot per second.	Number of revolutions of the Wheel per second.	Weight in the scale in pounds avoirdupois.	Ratio of the effect to the power, calculated by means of Prony's brake or dynamometer.	Velocity due to the fall acting on the Wheel, in feet per second.	Velocity of the interior circumference of the Wheel, in feet per second.
1	2	3	4	5	6	7	8	9	10
1	12·864	1·88	139·42	111870·0	·894	1443·34	·784	28·76	18·95
2	12·869	1·88	139·47	111951·2	·894	1443·34	·784	28·77	18·96
3	12·611	2·02	154·40	121444·2	1·532	411·48	·353	28·48	32·49
4	12·696	1·97	149·46	118363·5	1·382	638·36	·507	28·58	29·32
5	12·777	1·94	146·02	116373·2	1·245	854·87	·622	28·67	26·40
6	12·819	1·92	143·91	115067·3	1·125	1057·49	·703	28·71	23·86
7	12·856	1·91	142·52	114284·2	1·067	1156 27	·735	28 76	22·63
8	12·888	1·90	142·04	114187 1	1·024	1229·41	·750	28·79	21·71
9	12·896	1·90	141·28	113640·9	·970	1319·22	·766	28·80	20·57
10	12·883	1·89	140·08	112568·7	·923	1397·12	·779	28·79	19·57
11	12·899	1·88	139·90	112563·3	·902	1433·43	·781	28·80	19·13
12	12·905	1·88	139·01	111893·5	·892	1454·24	·789	28·81	18·92
13	12·899	1·88	139·03	111859·4	·885	1464·80	·788	28·80	18·77
14	12·902	1·87	138‘85	111740·3	·880	1474·37	·790	28·81	18·66
15	12·906	1·87	138·51	111504·9	·866	1498‘66	·792	28·81	18·37
16	12·915	1·87	138·27	111384·0	·836	1552·44	·794	28·82	17·73
17	12·934	1·87	138·23	111521·1	·813	1597·08	·792	28·84	17·25
18	12·939	1·86	137·71	111139·7	·742	1724·49	·783	28·85	15·74
19	12·940	1·84	135·14	109077·1	·646	1911·45	·770	28·85	13·69
20	12·963	1·84	135·34	109433·4	·647	1911·45	·769	28·88	13·72
21	12·977	1·83	133·75	108265·8	·532	2167·38	·725	28·89	11·29
22	12·948	1·82	133·43	107764·7	·454	2367·88	·679	28·86	9·63
23	12·954	1·87	138·62	112009·3	·832	1565·21	·791	28·86	17·64
24	12·932	1·87	138·50	111720·6	·817	1590·50	·791	28·84	17·32
25	12·951	1·87	138·37	111777·2	·789	1641·34	·788	28·86	16·74
26	12·758	1·92	143·33	114060·7	1·483	390·95	·346	28 65	31·45
27	12·909	1·86	137·75	110917·6	1·142	963·30	·675	28·82	24·21
28	12·950	1·86	137·00	110664·7	1·025	1150·77	·725	28·86	21·73
29	12·965	1·84	135·10	109252·2	·930	1293·63	·750	28·88	19·74
30	12·999	1·82	133·30	108082·5	·865	1396 11	·760	28·92	18·35
31	13·026	1·81	131·99	107246·3	·844	1494·68	·763	28·95	17·05
32	13·028	1·80	130·89	106366·6	·708	1656·98	·750	28.95	15·01
33	13·077	1·68	118·55	96699·5	1·351	316·03	·300	29·00	28·66
34	13·134	1·66	116·10	95112·0	1·219	519·69	·453	29·06	25·84
35	13·215	1·63	113·24	93346·0	1·004	832·26	·609	29·15	21·28
36	13·282	1·59	109·71	90893·3	·843	1033·30	·652	29·23	17·88
37	13·310	1·58	107·95	89620·7	·776	1115·02	·656	29·26	16·44
38	13·362	1·54	103·85	86555·6	·616	1277·98	·619	29·32	13·06
39	12·883	1·86	137·36	110380·8	·855	1482·56	·781	28·79	18·12
40	12·896	1·86	136·97	110176·6	·822	1544·87	·784	28·80	17·42
41	12·912	1·85	136·55	109973·0	·789	1604·85	·783	28·82	16·73
42	13·369	1·27	78·84	65746·8	1·146	118·59	·141	29·32	24·30
43	13·395	1·25	76·62	64018·1	·960	325·39	·332	29·35	20·36
44	13·435	1·22	74·05	62062·0	·772	519·86	·440	29·40	16·37
45	13·478	1·19	71·87	60424·6	·672	612·22	·463	29·44	14·25
46	13·513	1·17	70·01	59006·4	·560	704·44	·455	29·48	11·87
47	13·559	1·11	64·50	54554·9	·300	882·02	·330	29·53	6·36
48	13·985	·78	38·22	33340·8	·620	118·59	·150	29·99	13·14
49	14·001	·78	38·57	33683·5	·689	73·14	·102	30·01	14·61
50	14·020	·76	37·17	32508·0	·388	296·39	·240	30·03	8·22

TABLE for Turbines of different Diameters, modified from Francis, operating with different Falls; assuming the useful effect is seventy-five per cent. of the power expended, that the Velocity of the Interior Circumference is fifty-six per cent. of the Velocity due to the Fall, and that also the Height between the Crowns is one-tenth of the Outside Diameter.

Fall in feet.	Outside diameter 2 ft. Inside „ 1·56 „ Number of buckets 36.			Outside diameter 3 ft. Inside „ 2 38 „ Number of buckets 39.			Outside diameter 4 ft. Inside „ 3 24 „ Number of buckets 42.			Outside diameter 5 ft. Inside „ 4·11· „ Number of buckets 45.		
	Water discharged in cubic feet per second.	Horse-power.	Revolutions per minute.	Water discharged in cubic feet per second.	Horse-power.	Revolutions per minute.	Water discharged in cubic feet per second.	Horse-power.	Revolutions per minute.	Water discharged in cubic feet per second.	Horse-power.	Revolutions per minute.
5	4·5	1·9	123	10·1	4·3	80	17·88	7·6	59	27·9	11·9	47
6	4·9	2·5	135	11·0	5·6	88	19·6	10·0	65	30·6	15·6	51
7	5·3	3·1	146	11·9	7·1	95	21·17	12·6	70	33·1	19·7	55
8	5·7	3·8	156	12·7	8·7	102	22·63	15·4	75	35·3	24·0	59
9	6·0	4·6	165	13·5	10·3	108	24·00	18·4	79	37·5	28·7	63
10	6·3	5·4	174	14·2	12·1	114	25·30	21·5	84	39·5	33·6	66
11	6·6	6·2	183	14·9	13·9	119	26·53	24·8	88	41·5	38·8	69
12	6·9	7·1	191	15·6	15·9	125	27·71	28·3	92	43·3	44·2	72
13	7·2	8·0	199	16·2	17·9	130	28·84	31·9	95	45·1	49·8	75
14	7·5	8·9	206	16·8	20·0	135	29·93	35·6	99	46·8	55·7	78
15	7·7	9·9	213	17·4	22·2	139	30·98	39·5	103	48·4	61·7	81
16	8·0	10·9	220	18·0	24·5	144	32·00	43·5	106	50·0	68·0	83
17	8·2	11·9	227	18·5	26·8	148	32·99	47·7	109	51·5	74·5	86
18	8·5	13·0	234	19·1	29·2	153	33·94	51·9	112	53·0	81·1	88
19	8·7	14·1	240	19·6	31·7	157	34·87	56·3	115	54·5	88·0	91
20	8·9	15·2	247	20·1	34·2	161	35·78	60·8	118	55·9	95·0	93
21	9·2	16·4	253	20·6	36·8	165	36·66	65·4	121	57·3	102·2	96
22	9·4	17·5	259	21·1	39·5	169	37·52	70·2	124	58·6	109·6	98
23	9·6	18·7	264	21·6	42·2	172	38·37	75·0	127	59·9	117·2	100
24	9·8	20·0	270	22·0	45·0	176	39·19	79·9	130	61·2	124·9	102
25	10·0	21·2	276	22·5	47·8	180	40·00	85·0	132	62·5	132·8	104
26	10·2	22·5	281	22·9	50·7	183	40·79	90·1	135	63·7	140·9	106
27	10·4	23·8	286	23·4	53·7	187	41·57	95·4	138	65·0	149·1	108
28	10·6	25·2	292	23·8	56·7	190	42·33	100·7	140	66·1	157·4	110
29	10·8	26·5	297	24·2	59·7	194	43·08	106·2	143	67·3	165·9	112
30	10·9	27·9	302	24·6	62·8	197	43·82	111·7	145	68·5	174·6	114
31	11·1	29·3	307	25·0	66·0	200	44·54	117·4	147	69·6	183·4	116
32	11·3	30·8	312	25·5	69·2	203	45·25	123·1	150	70·7	192·3	118
33	11·5	32·2	317	25·8	72·5	207	45·96	128·9	152	71·8	201·4	120
34	11·7	33·7	321	26·2	75·8	210	46·65	134·8	154	72·9	210·6	122
35	11·8	35·2	326	26·6	79·2	213	47·33	140·8	157	73·9	220·0	123
36	12·0	36·7	331	27·0	82·6	216	48·00	146·9	159	75·0	229·5	125
37	12·2	38·3	335	27·4	86·1	219	48·66	153·0	161	76·0	239·1	127
38	12·3	39·8	340	27·7	89·6	922	49·32	159·3	163	77·0	248·9	129
39	12·5	41·4	344	28·1	93·2	225	49·96	165·6	165	78·1	258·8	130
40	12·6	43·0	349	28·5	96·8	227	50·60	172·0	167	79·1	268·8	132

TABLE of Turbines of different Diameters, modified from Francis, operating with different Falls; assuming the useful effect is seventy-five per cent. of the power expended, that the Velocity of the Interior Circumference is fifty-six per cent. of the Velocity due to the Fall. and that also the Height between the Crowns is one-tenth of the Outside Diameter.

Fall in feet.	Outside diameter 6 ft. Inside „ 5 „ Number of buckets 48.			Outside diameter 7 ft. Inside „ 5·90 „ Number of buckets 51.			Outside diameter 8 ft. Inside „ 6·81 „ Number of buckets 54.			Outside diameter 10 ft. Inside „ 8·67 „ Number of buckets 60.		
	Water discharged in cubic feet per second.	Horse-power.	Revolutions per minute.	Water discharged in cubic feet per second.	Horse-power.	Revolutions per minute.	Water discharged in cubic feet per second.	Horse-power.	Revolutions per minute.	Water discharged in cubic feet per second.	Horse-power.	Revolutions per minute.
5	40·2	17·1	38	54·8	23·3	32·5	71·5	30·4	28·1	111·8	47·5	22·1
6	44·1	22·5	42	60·0	30·6	35·6	78·4	40·0	30·8	122·5	62·5	24·2
7	47·6	28·3	45	64·8	38·6	38·4	84·7	50·4	33·3	132·3	78·7	26·2
8	50·9	34·6	48	69·3	47·1	41·1	90·5	61·5	35·6	141·4	96·2	28·0
9	54·0	41·3	51	73·5	56·2	43·6	96·0	73·4	37·8	150·0	114·7	29·7
10	56·9	48·4	54	77·5	65·9	46·0	101·2	86·0	39·8	158·1	134·4	31·3
11	59·7	55·8	57	81·3	76·0	48·2	106·1	99·2	41·7	165·8	155·0	32·8
12	62·4	63·6	59	84·9	86·6	50·3	110·8	113·1	43·6	173·2	176·7	34·3
13	64·9	71·7	62	88·3	97·6	52·4	115·4	127·5	45·4	180·3	199·2	35·7
14	67·3	80·1	64	91·7	109·1	54·4	119·7	142·5	47·1	187·1	222·6	37·0
15	69·7	88·9	66	94·9	121·0	56·3	123·9	158·0	48·7	193·6	246·9	38·3
16	72·0	97·9	69	98·0	133·3	58·1	128·0	174·1	50·3	200·0	272·0	39·6
17	74·2	107·2	71	101·0	146·0	59·9	131·9	190·6	51·9	206·2	297·9	40·8
18	76·4	116·8	73	103·9	159·0	61·7	135·8	207·7	53·4	212·1	324·6	42·0
19	78·5	126·7	75	106·8	172·5	63·3	139·5	225·3	54·9	217·9	352·0	43·1
20	80·5	136·8	77	109·6	186·3	65·0	143·1	243·3	56·3	223·6	380·1	44·3
21	82·5	147·2	79	112·3	200·4	66·6	146·6	261·7	57·7	229·1	409·0	45·4
22	84·4	157·9	80	114·9	214·9	68·2	150·1	280·7	59·0	234·5	438·5	46·4
23	86·3	168·8	82	117·5	229·7	69·7	153·5	300·0	60·4	239·8	468·8	47·5
24	88·2	179·9	84	120·0	244·8	71·2	156·8	319·8	61·7	244·9	499·7	48·5
25	90·0	191·2	86	122·5	260·3	72·7	160·0	340·0	62·9	250·0	531·2	49·5
26	91·8	202·8	87	124·9	276·1	74·1	163·2	360·6	64·2	254·9	563·4	50·5
27	93·5	214·6	89	127·3	292·2	75·5	166·3	381·6	65·4	259·8	596·3	51·4
28	95·2	226·7	91	129·6	308·5	76·9	169·3	403·0	66·6	264·6	629·7	52·4
29	96·9	238·9	92	131·9	325·2	78·3	172·3	424·8	67·8	269·3	663·7	53·3
30	98·6	251·4	94	134·2	342·2	79·6	175·3	446·9	68·9	273·9	698·3	54·2
31	100·2	264·1	95	136·4	359·4	80·9	178·2	469·5	70·1	278·4	733·5	55·1
32	101·8	277·0	97	138·6	377·0	82·2	181·0	492·4	71·2	282·8	769·3	56·0
33	103·4	290·0	98	140·7	394·8	83·5	183·8	515·6	72·3	287·2	805·7	56·9
34	105·0	303·3	100	142·9	412·9	84·7	186·6	539·2	73·4	291·5	842·6	57·7
35	106·5	316·8	101	144·9	431·2	86·0	189·3	563·2	74·5	295·8	880·0	58·5
36	108·0	330·5	103	147·0	449·8	87·2	192·0	587·5	75·5	300·0	918·0	59·4
37	109·5	344·3	104	149·0	468·7	88·4	194·6	612·2	76·6	304·1	956·5	60·2
38	111·0	358·4	106	151·0	487·8	89·6	197·3	637·1	77·6	308·2	995·5	61·0
39	112·4	372·6	107	153·0	507·2	90·8	199·8	662·5	78·6	312·2	1035·1	61·8
40	113·8	387·1	108	154·9	526·8	91·9	202·4	688·1	79·6	316·2	1075·2	62·6

THE HYDRAULIC RAM has been applied with advantage in raising water to a considerable height by the momentum of a larger quantity at a lower level. The shock of the valves, and vibration of the machine, require heavy and strong setting, and considerable strength in all the parts. This limits its application, and prevents its use for raising large quantities of water. The work done by the ram, in over one thousand experiments by Eytelwein, did not exceed in any of them 1480 lbs. raised one foot in one minute; and in France, the ram put up by the younger Montgolfier, said to be the largest constructed, raised only 7400 lbs. one foot high per minute, and had a useful effect, it is reported, of ·65. This ram was put up at Mello, near Clermont-sur-Oise. Its dimensions were—

Length of the body pipe or injection pipe . .	108 feet.
Diameter	4·3 inches.
Weight of body pipe.	3190 lbs.
Weight of head	440 lbs.
Contents of air-chamber	1½ gallons.

This ram worked under a head of 37 feet, discharging in use 31½ gallons each minute, and raising 3·85 gallons a height of 195 feet.

The largest ram employed by Eytelwein in his experiments had the following dimensions—

Length of the body pipe or injection pipe .	43 feet 9 inches.
Diameter of ditto	0 feet 2¼ inches.
Contents of air-chamber	1·94 gallons.
Area of tail or escape valve	3·74 square inches.

and his experiments led to the following practical formula by D'Aubuisson—

$$\frac{d\,h'}{\mathrm{D}\,h} = 1{\cdot}42 - {\cdot}28\sqrt{\frac{h'}{h}}\,:$$

in which D is the water used per minute in gallons, d the quantity raised in gallons, h the head used, and h' the lift of the quantity d. By a slight reduction we get

$$d\,h' = 1{\cdot}42\,\mathrm{D}\,(h - {\cdot}28\sqrt{h\,h'})$$

for the effect produced, which is reduced nearly one-sixth for practical application, giving the formula

$$d\,h' = 1{\cdot}2\,\mathrm{D}\,(h - {\cdot}2\sqrt{h\,h'})$$

for the work done.

EXPERIMENTAL RESULTS—HYDRAULIC RAM.

Number of strokes per minute.	Height in feet of				Ratio of heights.	Gallons of water per minute.		Ratio $\frac{d\,h'}{D h}$		Ratios $\frac{D}{d}$
	Fall h		Elevation h'		$\frac{h'}{h}$	Expended D	Raised d	Experiments.	Formula.	
	Ft.	In.	Ft.	In.						
66	10′	0″	26′	4″	2·63	10·65	3·39	·9	·97	2·92
54	10	2	32	4	3·18	13·97	3·83	·873	·92	3·67
50	9	11	38	8	3·9	12·01	2·622	·85	·87	4·58
52	8	0	32	4	4·	8·16	1·687	·847	·85	4·72
45	8	9	38	8	4·4	10·85	2·09	·845	·84	5·2
42	7	5	38	8	5·21	9·92	1·5	·787	·78	6·62
36	6	0	38	8	6·5	8·89	1·05	·754	·71	8·62
26	4	6½	32	4	7·2	5·23	·495	·672	·67	10·7
31	5	0	38	7	7·7	8·05	·704	·667	·65	11·54
23	4	1	38	8	9·47	11·11	·649	·548	·56	17·2
17	3	0	32	2	10·7	10·8	·479	·473	·51	22·6
15	3	3	38	8	11·9	12·34	·363	·352	·45	33·8
14	2	6	38	8	15·5	11·95	·22	·284	·32	54·6
10	1	11½	38	8	19·3	9·81	·088	·181	·18	106·6

Eytelwein recommends, that the length of the body-pipe should not be less than three-fourths of the height to which the water is to be raised; its diameter in inches equal ·58 $\sqrt{D}$; the diameter of the rising pipe ·3 $\sqrt{D}$; and the contents of the air-chamber equal to that of the rising pipe. If D be in cube feet, then diameter, in inches, of the body-pipe may be taken = 1·5 $\sqrt{D}$, and that of the rising pipe = ·75 $\sqrt{D}$.

The following table gives the result of experiments made by Montgolfier and his son:—

TABLE OF EXPERIMENTAL RESULTS—HYDRAULIC RAM.

Height.				Water per Minute.		$\frac{d\,h'}{D\,h}$	Mean Ratio $\frac{d\,h'}{D\,h}$
Fall h		Elevation h		Expended D	Delivered d		
Ft.	In.	Ft.	In.	Gallons.	Gallons.		
8′	6″	52′	8″	15	1·37	·57	...
37	2	195	0	31	3·85	·653	...
34	9	111	11	18·5	3·74	·651	·65
3	3	14	11	437	59·18	·629	...
22	10	196	10	2·86	0·22	·671	...

In several experiments made by the author in 1866, for the Directors of the Midland Great Western Railway, Ireland, on two rams at work at the Broadstone Terminus, Dublin, where the lifts varied from 22 to 24 feet, the ratio of the effect to the power varied from ·4 to ·84; the latter effect having been got with a fall of 8 feet, a lift of 22 feet, and 95 beats in a minute. An effect of ·395 was got with a fall of 14 feet, a lift of 24 feet, and 45 beats in a minute.

Latterly, the Messrs. Easton and Amos have patented improvements in this machine, and have raised water to a height of 330 feet. The injection pipe is laid by them at an inclination of about one in four for high falls, and varies down to one in eighteen for smaller falls. The quantities raised in their practice vary up to six gallons per minute.

Water-pressure engines give a useful effect, varying up to 70 per cent. for the best constructed. An immense amount of mechanical skill and invention has been brought to bear on their construction, and in Weisbach's book* a useful effect of 83 per cent. has been calculated; this is, however, a result seldom obtained in practice, where two-thirds, or 66 per cent., is nearer to the general efficiency. Jordan got a maximum efficiency of ·66 from one of the Clausthal engines, making four strokes per minute, and ·71 making three strokes per minute. These results were for the combined engine and pumps, from which it was calculated that the efficiency of the engine alone was, in the first case ·83, and in the second ·85. It would be a great mistake to calculate on such high efficiencies.

Corn mills will grind about a bushel of corn per horse-power per hour, but much depends on the state of the stones and of the grain. The value of the work done in an hour being once known, the value of the standard horse-power can be determined accordingly.

---

* Vol. ii., p. 342.

TABLE I.—Coefficients of Discharge from Square and differently proportioned Rectangular Lateral Orifices in thin Vertical Plates, arranged from Poncelet and Lesbros.

Heads of water measured to the upper sides of the orifices, in English inches.	Ratio of the head to the length of the orifice.	Square orifice 8″ × 8″. Ratio of the sides 1 to 1.		Rectangular orifice 8″ × 4″. Ratio of the sides 2 to 1.		Rectangular orifice 8″ × 2″. Ratio of the sides 4 to 1.	
		Heads taken back from the orifice.	Heads taken at the orifice.	Heads taken back from the orifice.	Heads taken at the orifice.	Heads taken back from the orifice.	Heads taken at the orifice.
0·000			·619		·667		·713
0·197	·025		·597		·630		·668
0·394	·050		·595		·618	·607	·642
0·591	·075		·594	·593	·615	·612	·639
0·787	·100	·572	·594	·596	·614	·615	·638
1·181	·150	·578	·593	·600	·613	·620	·637
1·575	·200	·582	·593*	·603	·612	·623	·636
1·969	·250	·585	·593	·605	·612*	·625	·636
2·362	·300	·587	·594	·607	·613	·627	·635
2·756	·350	·588	·594	·609	·613	·628	·635
3·150	·400	·589	·594	·610	·613	·629	·635
3·545	·450	·591	·595	·610	·614	·629	·634
3·937	·500	·592	·595	·611	·614	·630	·634
4·724	·600	·593	·596	·612	·614	·630	·633
5·512	·700	·595	·597	·613	·614	·630	·632
6·299	·800	·596	·597	·614	·615	·631*	·631
7·087	·900	·597	·598	·615	·615	·630	·631
7·874	1·000	·598	·599	·615	·615	·630	·630
9·843	1·250	·599	·600	·616	·616	·630	·630
11·811	1·500	·600	·601	·616	·616	·629	·629
15·748	2·000	·602	·602	·617	·617	·628	·629
19·685	2·500	·603	·603	·617*	·617*	·628	·628
23·622	3·000	·604	·604	·617	·617	·627	·627
27·560	3·500	·604	·604	·616	·616	·627	·627
31·497	4·000	·605	·605	·616	·616	·627	·627
35·434	4·500	·605*	·605*	·615	·615	·526	·626
39·371	5·000	·605	·605	·615	·615	·626	·626
43·307	5·500	·604	·604	·614	·614	·625	·625
47·245	6·000	·604	·604	·614	·614	·624	·624
51·182	6·500	·603	·603	·613	·613	·622	·622
55·119	7·000	·603	·603	·612	·612	·621	·621
59·056	7·500	·602	·602	·611	·611	·620	·620
62·993	8·000	·602	·602	·611	·611	·618	·618
66·930	8·500	·602	·602	·610	·610	·617	·617
70·867	9·000	·601	·601	·609	·609	·615	·615
74·805	9·500	·601	·601	·608	·608	·614	·614
78·742	10·000	·601	·601	·607	·607	·613	·614
118·112	15·000	·601	·601	·603	·603	·606	·606

* See pages 60, 61, and 62.

TABLE I.—Coefficients of Discharge from Square and differently proportioned Rectangular Lateral Orifices in thin Vertical Plates, arranged from Poncelet and Lesbros.

Rectangular orifice 8″ × 1·18″. Ratio of the sides 7 to 1 nearly.		Rectangular orifice 8″ × 0·8″. Ratio of the sides 10 to 1.		Rectangular orifice 8″ × 0·4″. Ratio of the sides 20 to 1.		Ratio of the head to the length of the orifice.	Heads of water measured to the upper sides of the orifices in English inches.
Heads taken back from the orifice.	Heads taken at the orifice.	Heads taken back from the orifice.	Heads taken at the orifice.	Heads taken back from the orifice.	Heads taken at the orifice.		
	·766		·783		·795		
	·725		·750	·705	·778	·025	0·197
·630	·687	·660	·720	·701	·762	·050	0·394
·632	·674	·660	·707	·697	·745	·075	0·591
·634	·668	·659	·697	·694	·729	·100	0·787
·638	·659	·659	·685	·688	·708	·150	1·181
·640	·654	·658	·678	·683	·695	·200	1·575
·640*	·651	·658	·672	·679	·686	·250	1·969
·640	·647	·657	·668	·676	·681	·300	2·362
·639	·645	·656	·665	·673	·677	·350	2·756
·638	·643	·656	·662	·670	·675	·400	3·150
·637	·641	·655	·659	·668	·672	·450	3·543
·637	·640	·654	·657	·666	·669	·500	3·937
·636	·637	·653	·655	·663	·665	·600	4·724
·635	·636	·651	·653	·660	·661	·700	5·512
·634	·635	·650	·651	·658	·659	·800	6·299
·634	·634	·649	·650	·657	·657	·900	7·087
·633	·633	·648	·649	·655	·656	1·000	7·874
·632	·632	·646	·646	·653	·653	1·250	9·843
·632	·632	·644	·644	·650	·651	1·500	11·811
·631	·631	·642	·642	·647	·647	2·000	15·748
·630	·630	·640	·640	·644	·645	2·500	19·685
·630	·630	·638	·638	·642	·643	3·000	23·622
·629	·629	·637	·637	·640	·640	3·500	27·560
·629	·629	·636	·636	·637	·637	4·000	31·497
·628	·628	·634	·634	·635	·635	4·500	35·434
·628	·628	·633	·633	·632	·632	5·000	39·371
·627	·627	·631	·631	·629	·629	5·500	43·307
·626	·626	·628	·628	·626	·626	6·000	47·245
·624	·624	·625	·625	·622	·622	6·500	51·182
·622	·622	·622	·622	·618	·618	7·000	55·119
·620	·620	·619	·619	·615	·615	7·500	59·056
·618	·618	·617	·617	·613	·613	8·000	62·993
·616	·616	·615	·615	·612	·612	8·500	66·930
·615	·615	·614	·614	·612	·612	9·000	70·867
·613	·613	·612	·612	·611	·611	9·500	74·805
·612	·612	·612	·612	·611	·611	10·000	78·742
·608	·608	·610	·610	·609	·609	15·000	118·112

* See pages 60, 61, and 62.

TABLE II.—For finding the Velocities from the Altitudes, and the Altitudes from the Velocities.—Altitudes 0 feet 0 $\frac{1}{100}$ inch to 0 feet $3\frac{7}{8}$ inches.

Altitudes h in feet and inches.		Coefficients of velocity, and the corresponding velocities of discharge in inches per second.					
		1. Values of $v = 27{\cdot}8\sqrt{h}$, the theoretical velocity in inches.	2. Values of $v = 27{\cdot}077\,h^{\frac{1}{2}}$ Coefficient 974.	3. Values of $v = 26{\cdot}577\,h$ Coefficient ·956.	4. Values of $v = 23{\cdot}908\,h^{\frac{1}{2}}$ Coefficient ·860.	5. Values of $v = 22{\cdot}657\,h^{\frac{1}{2}}$ Coefficient ·815.	6. Values of $v = 22{\cdot}24\,h$ Coefficient ·800.
0′	$0\frac{1}{100}$″	2·78	2·71	2·66	2·39	2·27	2·22
0	$0\frac{1}{64}$	3·48	3·38	3·32	2·99	2·83	2·78
0	$0\frac{1}{16}$	6·95	6·77	6·64	5·98	5·66	5·56
0	$0\frac{1}{8}$	9·829	9·57	9·40	8·45	8·01	7·86
0	$0\frac{3}{16}$	12·038	11·72	11·51	10·35	9·81	9·63
0	$0\frac{1}{4}$	13·900	13·54	13·29	11·95	11·33	11·12
0	$0\frac{5}{16}$	15·541	15·14	14·86	13·36	12·67	12·43
0	$0\frac{3}{8}$	17·024	16·58	16·27	14·64	13·87	13·62
0	$0\frac{7}{16}$	18·388	17·91	17·58	15·81	14·99	14·71
0	$0\frac{1}{2}$	19·658	19·15	18·79	16·91	16·02	15·73
0	$0\frac{9}{16}$	20·850	20·31	19·93	17·93	16·99	16·68
0	$0\frac{5}{8}$	21·978	21·41	21·01	18·90	17·91	17·58
0	$0\frac{11}{16}$	23·051	22·45	22·04	19·82	18·79	18·44
0	$0\frac{3}{4}$	24·076	23·45	23·02	20·70	19·62	19·26
0	$0\frac{13}{16}$	25·059	24·41	24·00	21·55	20·42	20·05
0	$0\frac{7}{8}$	26·005	25·33	24·86	22·36	21·19	20·80
0	$0\frac{15}{16}$	26·917	26·22	25·73	23·15	21·94	21·53
0	1	27·800	27·08	26·58	23·91	22·66	22·24
0	$1\frac{1}{8}$	29·486	28·72	28·19	25·36	24·03	23·59
0	$1\frac{1}{4}$	31·081	30·27	29·71	26·73	25·33	24·87
0	$1\frac{3}{8}$	32·598	31·75	31·16	28·03	26·57	26·08
0	$1\frac{1}{2}$	34·048	33·19	32·58	29·30	27·75	27·26
0	$1\frac{5}{8}$	35·438	34·52	33·88	30·48	28·88	28·35
0	$1\frac{3}{4}$	36·776	35·82	35·16	31·63	29·97	29·42
0	$1\frac{7}{8}$	38·067	37·08	36·39	32·74	31·02	30·45
0	2	39·315	38·29	37·59	33·81	32·04	31·45
0	$2\frac{1}{8}$	40·525	39·47	38·74	34·85	33·03	32·42
0	$2\frac{1}{4}$	41·700	40·62	39·87	35·86	33·99	33·36
0	$2\frac{3}{8}$	42·843	41·73	40·96	36·84	34·92	34·27
0	$2\frac{1}{2}$	43·956	42·81	42·02	37·80	35·82	35·16
0	$2\frac{5}{8}$	45·041	43·87	43·06	38·74	36·71	36·03
0	$2\frac{3}{4}$	46·101	44·90	44·07	39·65	37·57	36·88
0	$2\frac{7}{8}$	47·137	45·90	45·06	40·54	38·42	37·71
0	3	48·151	46·90	46·03	41·41	39·24	38·52
0	$3\frac{1}{8}$	49·144	47·87	46·98	42·26	40·05	39·32
0	$3\frac{1}{4}$	50·117	48·81	47·91	43·10	40·85	40·09
0	$3\frac{3}{8}$	51·072	49·74	48·82	43·92	41·62	40·86
0	$3\frac{1}{2}$	52·009	50·66	49·72	44·73	42·39	41·61
0	$3\frac{5}{8}$	52·930	51·55	50·60	45·52	43·14	42·34
0	$3\frac{3}{4}$	53·834	52·43	51·47	46·30	43·88	43·07
0	$3\frac{7}{8}$	54·725	53·30	52·32	47·06	44·60	43·78

TABLE II.—For finding the Velocities from the Altitudes, and the Altitudes from the Velocities.—Altitudes 0 feet $0\frac{1}{100}$ inch to 0 feet $3\frac{7}{8}$ inches.

Coefficients of velocity, and the corresponding velocities of discharge in inches per second.						Altitudes h in feet and inches.	
7. Values of $v = 19{\cdot}46\, h^{\frac{1}{2}}$ Coefficient ·700.	8. Values of $v = 18{\cdot}515\, h^{\frac{1}{2}}$ Coefficient ·666.	9. Values of $v = 17{\cdot}458\, h^{\frac{1}{2}}$ Coefficient ·628.	10. Values of $v = 17{\cdot}153\, h^{\frac{1}{2}}$ Coefficient ·617.	11. Values of $v = 16{\cdot}847\, h^{\frac{1}{2}}$ Coefficient ·606.	12. Values of $v = 15{\cdot}935\, h^{\frac{1}{2}}$ Coefficient ·584.	′	″
1·95	1·85	1·75	1·72	1·68	1·62	0	$0\frac{1}{100}$
2·43	2·31	2·18	2·15	2·11	2·03	0	$0\frac{1}{64}$
4·87	4·63	4·36	4·29	4·21	4·06	0	$0\frac{1}{16}$
6·88	6·55	6·17	6·06	5·96	5·74	0	$0\frac{1}{8}$
8·43	8·02	7·56	7·43	7·29	7·03	0	$0\frac{3}{16}$
9·73	9·26	8·73	8·58	8·42	8·12	0	$0\frac{1}{4}$
10·88	10·35	9·76	9·59	9·42	9·08	0	$0\frac{5}{16}$
11·92	11·24	10·69	10·50	10·32	9·94	0	$0\frac{3}{8}$
12·87	12·25	11·55	11·35	11·14	10·74	0	$0\frac{7}{16}$
13·76	12·97	12·34	12·13	11·91	11·48	0	$0\frac{1}{2}$
14·60	13·89	13·09	12·86	12·64	12·18	0	$0\frac{9}{16}$
15·38	14·64	13·80	13·56	13·32	12·84	0	$0\frac{5}{8}$
16·14	15·35	14·48	14·22	13·97	13·46	0	$0\frac{11}{16}$
16·85	16·03	15·12	14·85	14·59	14·06	0	$0\frac{3}{4}$
17·54	16·69	15·74	15·46	15·19	14·63	0	$0\frac{13}{16}$
18·20	17·32	16·33	16·04	15·76	15·09	0	$0\frac{7}{8}$
18·84	17·93	16·90	16·61	16·31	15·72	0	$0\frac{15}{16}$
19·46	18·51	17·46	17·15	16·85	16·24	0	1
20·64	19·64	18·52	18·19	17·87	17·22	0	$1\frac{1}{8}$
21·76	20·70	19·52	19·18	18·84	18·15	0	$1\frac{1}{4}$
22·82	21·71	20·47	20·11	19·75	19·04	0	$1\frac{3}{8}$
23·85	22·69	21·38	21·01	20·63	19·88	0	$1\frac{1}{2}$
24·81	23·60	22·26	21·87	21·48	20·70	0	$1\frac{5}{8}$
25·74	24·49	23·10	22·69	22·29	21·48	0	$1\frac{3}{4}$
26·65	25·35	23·91	23·49	23·07	22·23	0	$1\frac{7}{8}$
27·52	26·18	24·69	24·26	23·82	22·96	0	2
28·37	26·99	25·45	25·00	24·50	23·67	0	$2\frac{1}{8}$
29·19	27·77	26·19	25·73	25·27	24·35	0	$2\frac{1}{4}$
29·99	28·53	26·91	26·43	25·96	25·02	0	$2\frac{3}{8}$
30·77	29·27	27·60	27·12	26·64	25·67	0	$2\frac{1}{2}$
31·53	30·00	28·29	27·79	27·29	26·30	0	$2\frac{5}{8}$
32·27	30·70	28·95	28·44	27·94	26·92	0	$2\frac{3}{4}$
33·00	31·39	29·60	29·08	28·57	27·53	0	$2\frac{7}{8}$
33·71	32·07	30·24	29·71	29·18	28·12	0	3
34·40	32·73	30·86	30·32	29·78	28·70	0	$3\frac{1}{8}$
35·08	33·38	31·47	30·92	30·37	29·27	0	$3\frac{1}{4}$
35·75	34·01	32·07	31·51	30·95	29·83	0	$3\frac{3}{8}$
36·41	34·64	32·66	32·09	31·52	30·37	0	$3\frac{1}{2}$
37·05	35·25	33·24	32·66	32·08	30·91	0	$3\frac{5}{8}$
37·68	35·85	33·81	33·22	32·62	31·44	0	$3\frac{3}{4}$
38·31	36·45	34·37	33·77	33·16	31·96	0	$3\frac{7}{8}$

TABLE II.—For finding the Velocities from the Altitudes, and the Altitudes from the Velocities.—Altitudes 0 feet 4 inches to 1 foot.

Altitudes h in feet and inches.		Coefficients of velocity, and the corresponding velocities of discharge in inches per second.					
		1. Values of $v = 27{\cdot}8\sqrt{h}$, the theoretical velocity in inches.	2. Values of $v = 27{\cdot}077\,h^{\frac{1}{2}}$ Coefficient ·974.	3. Values of $v = 26{\cdot}577\,h^{\frac{1}{2}}$ Coefficient ·956.	4. Values of $v = 23{\cdot}908\,h^{\frac{1}{2}}$ Coefficient ·860.	5. Values of $v = 22{\cdot}657\,h^{\frac{1}{2}}$ Coefficient ·815.	6. Values of $v = 22{\cdot}24\,h^{\frac{1}{2}}$ Coefficient ·800.
0′	4″	55·600	54·15	53·15	47·82	45·31	44·48
0	$4\frac{1}{8}$	56·462	54·99	53·98	48·56	46·02	45·17
0	$4\frac{1}{4}$	57·311	55·82	54·79	49·29	46·71	45·85
0	$4\frac{3}{8}$	58·148	56·64	55·59	50·01	47·39	46·52
0	$4\frac{1}{2}$	58·973	57·44	56·38	50·72	48·06	47·18
0	$4\frac{5}{8}$	59·786	58·23	57·16	51·42	48·73	47·83
0	$4\frac{3}{4}$	60·589	59·01	57·92	52·11	49·38	48·47
0	$4\frac{7}{8}$	61·368	59·77	58·67	52·78	50·02	49·09
0	5	62·163	60·55	59·43	53·46	50·66	49·73
0	$5\frac{1}{8}$	62·935	61·30	60·17	54·12	51·29	50·35
0	$5\frac{1}{4}$	63·698	62·04	60·90	54·78	51·91	50·96
0	$5\frac{3}{8}$	64·452	62·78	61·62	55·43	52·53	51·56
0	$5\frac{1}{2}$	65·197	63·50	62·33	56·07	53·14	52·16
0	$5\frac{5}{8}$	65·933	64·22	63·03	56·70	53·74	52·75
0	$5\frac{3}{4}$	66·662	64·93	63·73	57·33	54·33	53·33
0	$5\frac{7}{8}$	67·383	65·63	64·42	57·95	54·92	53·91
0	6	68·096	66·33	65·10	58·56	55·50	54·48
0	$6\frac{1}{4}$	69·500	67·69	66·44	59·77	56·64	55·60
0	$6\frac{1}{2}$	70·876	69·03	67·76	60·95	57·24	56·70
0	$6\frac{3}{4}$	72·227	70·35	69·05	62·11	58·86	57·78
0	7	73·552	71·64	70·32	63·25	59·95	58·84
0	$7\frac{1}{4}$	74·854	72·91	71·56	64·37	61·01	59·88
0	$7\frac{1}{2}$	76·133	74·15	72·78	65·47	62·05	60·91
0	$7\frac{3}{4}$	77·392	75·38	73·99	66·56	63·07	61·91
0	8	78·630	76·59	75·17	67·62	64·08	62·90
0	$8\frac{1}{4}$	79·849	77·77	76·34	68·67	65·08	63·88
0	$8\frac{1}{2}$	81·050	78·94	77·48	69·70	66·06	64·84
0	$8\frac{3}{4}$	82·234	80·10	78·62	70·72	67·02	65·79
0	9	83·40	81·23	79·73	71·72	67·97	66·72
0	$9\frac{1}{4}$	84·550	82·35	80·83	72·71	68·91	67·64
0	$9\frac{1}{2}$	85·685	83·46	81·92	73·69	69·83	68·55
0	$9\frac{3}{4}$	86·805	84·55	82·99	74·65	70·75	69·44
0	10	87·911	85·63	84·04	75·60	71·65	70·33
0	$10\frac{1}{4}$	89·004	86·69	85·09	76·54	72·54	71·20
0	$10\frac{1}{2}$	90·082	87·74	86·12	77·47	73·42	72·07
0	$10\frac{3}{4}$	91·148	88·79	87·14	78·39	74·29	72·92
0	11	92·202	89·80	88·15	79·29	75·14	73·76
0	$11\frac{1}{4}$	93·244	90·82	89·14	80·19	75·99	74·59
0	$11\frac{1}{2}$	94·274	91·82	90·13	81·08	76·83	75·42
0	$11\frac{3}{4}$	95·294	92·82	91·10	81·95	77·66	76·23
1	0	96·302	93·80	92·06	82·82	78·49	77·04

TABLE II.—For finding the Velocities from the Altitudes, and the Altitudes from the Velocities.—Altitudes 0 feet 4 inches to 1 foot.

Coefficients of velocity, and the corresponding velocities of discharge in inches per second.						Altitudes h in feet and inches.	
7. Values of $v = 19{\cdot}46\, h^{\frac{1}{2}}$ Coefficient ·700.	8. Values of $v = 18{\cdot}515\, h^{\frac{1}{2}}$ Coefficient ·666.	9. Values of $v = 17{\cdot}458\, h^{\frac{1}{2}}$ Coefficient ·628.	10. Values of $v = 17{\cdot}153\, h^{\frac{1}{2}}$ Coefficient ·617.	11. Values of $v = 16{\cdot}847\, h^{\frac{1}{2}}$ Coefficient ·606.	12. Values of $v = 15{\cdot}935\, h^{\frac{1}{2}}$ Coefficient ·584.	′	″
38·92	37·03	34·92	34·31	33·69	32·47	0	4
39·52	37·60	35·46	34·84	34·22	32·97	0	4⅛
40·12	38·17	35·99	35·36	34·73	33·47	0	4¼
40·70	38·73	36·52	35·88	35·24	33·96	0	4⅜
41·28	39·28	37·03	36·39	35·74	34·44	0	4½
41·85	39·82	37·55	36·89	36·23	34·92	0	4⅝
42·41	40·35	38·05	37·38	36·72	35·38	0	4¾
42·96	40·87	38·54	37·86	37·19	35·84	0	4⅞
43·51	41·40	39·04	38·35	37·67	36·30	0	5
44·05	41·91	39·52	38·83	38·14	36·75	0	5⅛
44·59	42·42	40·00	39·30	38·60	37·20	0	5¼
45·12	42·92	40·48	39·77	39·06	37·64	0	5⅜
45·64	43·42	40·94	40·23	39·51	38·07	0	5½
46·15	43·91	41·41	40·68	39·96	38·51	0	5⅝
46·66	44·40	41·86	41·13	40·40	38·93	0	5¾
47·17	44·88	42·32	41·58	40·83	39·35	0	5⅞
47·67	45·35	42·76	42·02	41·27	39·77	0	6
48·65	46·29	43·65	42·88	42·12	40·59	0	6¼
49·61	47·20	44·51	43·73	42·95	41·39	0	6½
50·56	48·10	45·36	44·56	43·77	42·18	0	6¾
51·49	48·99	46·19	45·38	44·57	42·95	0	7
52·40	49·85	47·01	46·18	45·36	43·71	0	7¼
53·29	50·70	47·81	46·97	46·14	44·46	0	7½
54·17	51·54	48·60	47·75	46·90	45·20	0	7¾
55·04	52·37	49·38	48·51	47·65	45·92	0	8
55·89	53·18	50·15	49·27	48·39	46·63	0	8¼
56·74	53·98	50·90	50·01	49·12	47·33	0	8½
57·56	54·77	51·64	50·74	49·83	48·02	0	8¾
58·38	55·54	52·38	51·46	50·54	48·71	0	9
59·19	56·31	53·10	52·17	51·24	49·38	0	9¼
59·98	57·07	53·81	52·87	51·93	50·04	0	9½
60·76	57·81	54·51	53·56	52·60	50·69	0	9¾
61·54	58·55	55·22	54·24	53·27	51·34	0	10
62·30	59·28	55·89	54·92	53·94	51·98	0	10¼
63·06	60·00	56·57	55·58	54·59	52·61	0	10½
63·80	60·70	57·24	56·24	55·24	53·23	0	10¾
64·54	61·41	57·90	56·89	55·87	53·85	0	11
65·27	62·10	58·56	57·53	56·51	54·45	0	11¼
65·99	62·79	59·70	58·17	57·13	55·06	0	11½
66·71	63·47	59·84	58·80	57·75	55·65	0	11¾
67·41	64·14	60·48	59·42	58·36	56·24	1	0

TABLE II.—For finding the Velocities from the Altitudes, and the Altitudes from the Velocities.—Altitudes 1 foot 0½ inch to 5 feet 3 inches.

Altitudes h in feet and inches.		Coefficients of velocity, and the corresponding velocities of discharge in inches per second.					
′	″	1. Values of $v = 27{\cdot}8\sqrt{h}$, the theoretical velocity in inches.	2. Values of $v = 27{\cdot}077\, h^{\frac{1}{2}}$ Coefficient ·974.	3. Values of $v = 26{\cdot}577\, h^{\frac{1}{2}}$ Coefficient ·956.	4. Values of $v = 23{\cdot}908\, h^{\frac{1}{2}}$ Coefficient ·860.	5. Values of $v = 22{\cdot}657\, h^{\frac{1}{2}}$ Coefficient ·815.	6. Values of $v = 22{\cdot}24\, h^{\frac{1}{2}}$ Coefficient ·800.
1	0½	98·288	95·73	93·96	84·53	80·10	78·63
1	1	100·234	97·63	95·82	86·20	81·69	80·19
1	1½	102·144	99·49	97·65	87·84	83·25	81·71
1	2	104·018	101·31	99·44	89·46	84·77	83·21
1	2½	105·859	103·11	101·20	91·04	86·28	84·69
1	3	107·669	104·87	102·93	92·60	87·75	86·14
1	3½	109·449	106·60	104·63	94·13	89·20	87·56
1	4	111·200	108·31	106·31	95·63	90·63	88·96
1	4½	112·924	109·99	107·96	97·11	92·03	90·34
1	5	114·622	111·42	109·58	98·58	93·42	91·70
1	5½	116·296	113·27	111·18	100·01	94·78	93·04
1	6	117·945	114·78	112·76	101·43	96·13	94·36
1	7	121·177	118·03	115·85	104·21	98·76	96·94
1	8	124·325	121·09	118·86	106·92	101·33	99·46
1	9	127·396	124·08	121·79	109·56	103·83	101·92
1	10	130·394	127·00	124·66	112·14	106·27	104·31
1	11	133·324	129·86	127·46	114·66	108·66	106·66
2	0	136·192	132·65	130·20	117·12	111·00	108·95
2	1½	140·383	136·73	134·21	120·73	114·41	112·31
2	3	144·453	140·70	138·10	124·23	117·73	115·56
2	4½	148·411	144·55	141·88	127·64	120·96	118·73
2	6	152·267	148·31	145·57	130·95	124·10	121·81
2	7½	156·027	151·97	149·16	134·18	127·16	124·82
2	9	159·699	155·55	152·67	137·34	130·15	127·76
2	10½	163·288	159·04	156·10	140·43	133·80	130·63
3	0	166·800	162·46	159·46	143·45	135·94	133·44
3	1½	170·240	165·81	162·75	146·41	138·75	136·19
3	3	173·611	169·10	165·97	149·31	141·49	138·89
3	4½	176·918	172·32	169·13	152·15	144·19	141·53
3	6	180·165	175·48	172·24	154·94	146·83	144·13
3	7½	183·354	178·59	175·29	157·68	149·43	146·68
3	9	186·488	181·64	178·28	160·38	151·99	149·19
3	10½	189·571	184·64	181·23	163·03	154·50	151·66
4	0	192·604	187·60	184·13	165·64	156·97	154·08
4	2	196·576	191·46	187·93	169·06	160·21	157·26
4	4	200·469	195·26	191·65	172·40	163·38	160·37
4	6	204·287	198·98	195·30	175·69	166·49	163·43
4	8	208·036	202·63	198·88	178·91	169·55	166·43
4	10	211·718	206·21	202·40	182·08	172·55	169·37
5	0	215·338	209·74	205·86	185·19	175·50	172·27
5	3	220·656	214·92	210·95	189·76	179·83	176·52

TABLE II.—For finding the Velocities from the Altitudes, and the Altitudes from the Velocities.—Altitudes 1 foot 0½ inch to 5 feet 3 inches.

Coefficients of velocity, and the corresponding velocities of discharge in inches per second.						Altitudes h in feet and inches.
7. Values of $v = 19{\cdot}46\ h^{\frac{1}{2}}$ Coefficient ·700.	8. Values of $v = 18{\cdot}515\ h^{\frac{1}{2}}$ Coefficient ·666.	9. Values of $v = 17{\cdot}458\ h^{\frac{1}{2}}$ Coefficient ·628.	10. Values of $v = 17{\cdot}153\ h^{\frac{1}{2}}$ Coefficient ·617.	11. Values of $v = 16{\cdot}847\ h^{\frac{1}{2}}$ Coefficient ·606.	12. Values of $v = 15{\cdot}935\ h^{\frac{1}{2}}$ Coefficient ·584.	
68·80	65·46	61·72	60·64	59·56	57·40	1′ 0½″
70·16	66·76	62·95	61·84	60·74	58·54	1 1
71·50	68·03	64·15	63·02	61·90	59·65	1 1½
72·81	69·28	65·32	64·18	63·03	60·75	1 2
74·10	70·50	66·48	65·32	64·15	61·82	1 2½
75·37	71·71	67·62	66·43	65·25	62·88	1 3
76·61	72·89	68·73	67·53	66·33	63·92	1 3½
77·84	74·06	69·83	68·61	67·34	64·94	1 4
79·05	75·21	70·92	69·67	68·43	65·95	1 4½
80·24	76·34	71·98	70·72	69·46	66·94	1 5
81·41	77·45	73·03	71·75	70·48	67·92	1 5½
82·56	78·55	74·07	72·77	71·47	68·88	1 6
84·82	80·70	76·10	74·77	73·43	70·77	1 7
87·03	82·80	78·08	76·71	75·34	72·61	1 8
89·18	84·85	80·00	78·60	77·20	74·40	1 9
91·28	86·84	81·89	80·45	79·02	76·15	1 10
93·33	88·79	83·73	82·26	80·79	77·86	1 11
95·33	90·70	85·53	84·03	82·53	79·54	2 0
98·27	93·50	88·16	86·62	85·07	81·98	2 1½
101·12	96·21	90·72	89·13	87·54	84·36	2 3
103·89	98·84	93·20	91·57	89·94	86·67	2 4½
106·59	101·41	95·62	93·95	92·27	88·92	2 6
109·22	103·91	97·99	96·27	94·55	91·12	2 7½
111·79	106·36	100·29	98·53	96·78	93·26	2 9
114·30	108·75	102·54	100·75	98·95	95·36	2 10½
116·76	111·09	104·75	102·92	101·08	97·41	3 0
119·17	113·38	106·91	105·04	103·17	99·42	3 1½
121·53	115·62	109·03	107·12	105·21	101·39	3 3
123·84	117·83	111·10	109·16	107·21	103·32	3 4½
126·12	119·99	113·14	111·16	109·18	105·22	3 6
128·35	122·11	115·15	113·13	111·11	107·08	3 7½
130·54	124·20	117·11	115·06	113·01	108·91	3 9
132·70	126·25	119·05	116·97	114·88	110·71	3 10½
134·82	128·27	120·96	118·84	116·72	112·48	4 0
137·60	130·92	123·45	121·29	119·12	114·80	4 2
140·33	133·51	125·89	123·69	121·48	117·07	4 4
143·00	136·06	128·29	126·05	123·80	119·30	4 6
145·63	138·55	130·65	128·36	126·07	121·49	4 8
148·20	141·00	132·96	130·63	128·30	123·64	4 10
150·74	143·42	135·23	132·86	130·49	125·76	5 0
154·46	146·96	138·57	136·14	133·72	128·86	5 3

TABLE II.—For finding the Velocities from the Altitudes, and the Altitudes from the Velocities.—Altitudes 5 feet 6 inches to 17 feet.

Altitudes h in feet and inches.		Coefficients of velocity, and the corresponding velocities of discharge in inches per second.					
′	″	1. Values of $v = 27{\cdot}8\sqrt{h}$, the theoretical velocity in inches.	2. Values of $v = 27{\cdot}077\ h^{\frac{1}{2}}$ Coefficient ·974.	3. Values of $v = 26{\cdot}577\ h^{\frac{1}{2}}$ Coefficient ·956.	4. Values of $v = 23{\cdot}908\ h^{\frac{1}{2}}$ Coefficient ·869.	5. Values of $v = 22{\cdot}657\ h^{\frac{1}{2}}$ Coefficient ·815.	6. Values of $v = 22{\cdot}24\ h^{\frac{1}{2}}$ Coefficient ·809.
5	6	225·848	219·98	215·91	194·23	184·07	180·68
5	9	230·924	224·92	220·76	198·59	188·20	184·74
6	0	235·891	229·76	225·51	202·87	192·25	188·71
6	3	240·755	234·50	230·16	207·05	196·22	192·60
6	6	245·524	239·14	234·72	211·15	200·10	196·42
6	9	250·200	243·69	239·19	215·17	203·91	200·16
7	0	254·791	248·17	243·58	219·12	207·65	203·83
7	3	259·301	252·56	247·89	222·99	211·33	207·44
7	6	263·734	256·88	252·13	226·81	214·94	210·99
7	9	268·093	261·12	256·30	230·56	218·50	214·47
8	0	272·383	265·30	260·40	234·25	221·99	217·91
8	3	276·607	269·41	264·44	237·88	225·43	221·29
8	6	280·766	273·47	268·41	241·46	228·82	224·61
8	9	284·865	277·46	272·33	244·98	232·17	227·89
9	0	288·906	281·39	276·19	248·46	235·46	231·12
9	3	292·897	285·28	280·00	251·89	238·71	234·31
9	6	296·823	289·11	283·76	255·27	241·91	237·46
9	9	300·703	292·88	287·47	258·60	245·07	240·56
10	0	304·534	296·62	291·13	261·90	248·19	243·63
10	3	308·317	300·30	294·75	265·15	251·28	245·65
10	6	312·054	303·94	297·32	268·37	254·32	249·64
10	9	315·747	307·54	301·85	271·54	257·33	292·60
11	0	319·398	311·09	305·34	274·68	260·31	255·52
11	3	323·007	314·61	308·79	277·79	262·25	258·41
11	6	326·576	318·09	312·21	280·86	266·16	261·26
11	9	330·107	321·52	315·58	283·89	269·04	264·09
12	0	333·600	324·93	318·92	286·90	271·88	266·88
12	3	337·057	328·29	322·23	289·87	274·70	269·65
12	6	340·479	331·63	325·50	292·81	277·49	272·38
12	9	343·867	334·93	328·74	295·73	280·25	275·09
13	0	347·222	338·19	331·94	298·61	282·99	277·78
13	3	350·545	341·43	335·12	301·47	285·69	280·44
13	6	353·836	344·64	338·27	304·30	288·38	283·07
13	9	357·097	347·81	341·39	307·10	291·03	285·68
14	0	360·329	350·96	344·47	309·88	293·67	288·26
14	6	366·707	357·17	350·57	315·37	298·87	293·37
15	0	372·976	363·28	356·57	320·76	303·98	298·38
15	6	379·141	369·28	362·46	326·06	309·00	303·31
16	0	385·208	375·19	368·26	331·28	313·94	308·17
16	6	391·181	381·01	373·97	336·42	318·81	312·94
17	0	397·063	386·74	379·50	341·47	323·61	317·65

TABLE II.—For finding the Velocities from the Altitudes, and the Altitudes from the Velocities.—Altitudes 5 feet 6 inches to 17 feet.

Coefficients of velocity, and the corresponding velocities of discharge in inches per second.						Altitudes h in feet and inches.	
7. Values of $v = 19{\cdot}46\,h^{\frac{1}{2}}$ Coefficient ·700.	8. Values of $v = 18{\cdot}515\,h^{\frac{1}{2}}$ Coefficient ·666.	9. Values of $v = 17{\cdot}458\,h^{\frac{1}{2}}$ Coefficient ·628.	10. Values of $v = 17{\cdot}153\,h^{\frac{1}{2}}$ Coefficient ·617.	11. Values of $v = 16{\cdot}847\,h^{\frac{1}{2}}$ Coefficient ·606.	12. Values of $v = 15{\cdot}935\,h^{\frac{1}{2}}$ Coefficient ·584.	'	"
158·09	150·41	141·83	139·35	136·86	131·90	5	6
161·65	153·80	145·02	142·48	139·94	134·86	5	9
165·12	157·10	148·14	145·55	142·95	137·76	6	0
168·53	160·34	151·19	148·55	145·90	140·60	6	3
171·87	163·52	154·19	151·49	148·79	143·39	6	6
175·14	166·63	157·13	154·37	151·62	146·12	6	9
178·35	169·69	160·01	157·21	154·40	148·80	7	0
181·51	172·69	162·84	159·99	157·14	151·43	7	3
184·61	175·65	165·62	162·72	159·82	154·02	7	6
187·67	178·55	168·36	165·41	162·46	156·57	7	9
190·67	181·41	171·06	168·06	165·06	159·07	8	0
193·62	184·22	173·71	170·67	167·62	161·54	8	3
196·54	186·99	176·32	173·23	170·14	163·97	8	6
199·41	189·72	178·90	175·76	172·63	166·36	8	9
202·23	192·41	181·43	178·26	175·08	168·72	9	0
205·02	195·07	183·94	180·71	177·49	171·05	9	3
207·78	197·68	186·40	183·14	179·87	173·34	9	6
210·49	200·27	188·84	185·53	182·23	175·61	9	9
213·17	202·82	191·25	187·90	184·55	177·85	10	0
215·82	205·34	193·62	190·23	186·84	180·06	10	3
218·44	207·83	195·97	192·54	189·10	182·24	10	6
221·02	210·29	198·29	194·82	191·34	184·40	10	9
223·58	212·72	200·58	197·07	193·55	186·53	11	0
226·10	215·12	202·85	199·30	195·74	188·64	11	3
228·60	217·50	205·09	201·50	197·91	190·72	11	6
231·07	219·85	207·31	203·68	200·04	192·78	11	9
233·52	222·18	209·50	205·83	202·16	194·82	12	0
235·94	224·48	211·67	207·96	204·26	196·84	12	3
238·34	226·76	213·82	210·08	206·33	198·84	12	6
240·71	229·02	215·95	212·17	208·38	200·82	12	9
243·06	231·25	218·06	214·24	210·42	202·78	13	0
245·38	233·46	220·14	216·29	212·43	204·72	13	3
247·69	235·65	222·21	218·32	214·42	206·64	13	6
249·97	237·83	224·26	220·33	216·40	208·54	13	9
252·23	239·98	226·29	222·32	218·36	210·43	14	0
256·70	244·23	230·29	226·26	222·22	214·16	14	6
261·08	248·40	234·23	230·13	226·02	217·82	15	0
265·40	252·51	238·10	233·93	229·76	221·42	15	6
269·65	256·55	241·91	237·67	233·44	224·96	16	0
273·83	260·53	245·66	241·36	237·06	228·45	16	6
277·94	264·44	249·36	244·99	240·62	231·89	17	0

TABLE II.—For finding the Velocities from the Altitudes, and the Altitudes from the Velocities.—Altitudes 17 feet 6 inches to 40 feet.

Altitudes h in feet and inches.		Coefficients of velocity, and the corresponding velocities of discharge in inches per second.					
		1. Values of $v = 27·8\sqrt{h}$, the theoretical velocity in inches.	2. Values of $v = 27·077\,h^{\frac{1}{2}}$ Coefficients ·974.	3. Values of $v = 26·577\,h^{\frac{1}{2}}$ Coefficient ·956.	4. Values of $v = 23·908\,h^{\frac{1}{2}}$ Coefficient ·860.	5. Values of $v = 22·657\,h^{\frac{1}{2}}$ Coefficient ·815.	6. Values of $v = 22·24\,h^{\frac{1}{2}}$ Coefficient ·800.
′	″						
17	6	402·860	392·39	385·13	346·46	328·33	322·29
18	0	408·575	397·95	390·60	351·37	332·99	326·86
18	6	414·211	403·44	395·99	356·22	337·58	331·37
19	0	419·772	408·86	401·30	361·00	342·11	335·82
19	6	425·258	414·20	406·55	365·72	346·59	340·21
20	0	430·676	419·48	411·73	370·38	351·00	344·54
20	6	436·026	424·69	416·84	374·98	355·36	348·82
21	0	441·311	429·84	421·89	379·53	359·59	353·05
21	6	446·534	434·92	426·89	384·02	363·93	357·23
22	0	451·697	439·95	431·82	388·46	368·13	361·36
22	6	456·801	444·92	436·70	392·85	372·29	365·44
23	0	461·848	449·84	441·53	397·19	376·41	369·48
23	6	466·841	450·70	446·30	401·48	380·48	373·47
24	0	471·782	459·52	451·02	405·73	384·50	377·43
24	6	476·671	464·28	455·70	409·94	388·49	381·34
25	0	481·510	468·99	460·32	414·10	392·43	385·21
25	6	486·301	473·66	464·90	418·22	396·34	389·04
26	0	491·046	478·28	469·44	422·30	400·20	392·84
26	6	495·745	482·86	473·93	426·34	404·03	396·60
27	0	500·400	487·39	478·38	430·34	407·83	400·32
27	6	505·012	491·88	482·79	434·31	411·58	404·01
28	0	509·582	496·33	487·16	438·24	415·31	407·67
28	6	514·112	500·75	491·49	442·14	419·00	411·29
29	0	518·602	505·12	495·78	446·00	422·66	414·88
29	6	523·054	509·45	500·04	449·83	426·29	418·44
30	0	527·468	513·75	504·26	453·62	429·89	421·97
30	6	531·845	518·02	508·44	457·31	433·45	425·48
31	0	536·187	522·25	512·59	461·12	436·99	428·95
31	6	540·494	526·44	516·71	464·82	440·50	432·40
32	0	544·767	530·60	520·80	468·50	443·98	435·81
32	6	549·006	534·73	524·85	472·15	447·44	439·20
33	0	553·213	538·83	528·87	475·76	450·87	442·57
33	6	557·388	542·90	532·86	479·35	454·27	445·91
34	0	561·532	546·93	536·83	482·92	457·65	449·23
34	6	565·646	550·94	540·76	486·46	461·00	452·52
35	0	569·730	554·92	544·66	489·97	464·33	455·78
36	0	577·812	562·79	552·39	496·92	470·92	462·25
37	0	585·782	570·55	560·01	503·77	477·41	468·63
38	0	593·646	578·21	567·53	510·54	483·82	474·92
39	0	601·406	585·77	574·94	517·21	490·15	481·12
40	0	609·067	593·23	582·27	523·80	496·39	487·25

TABLE II.—For finding the Velocities from the Altitudes, and the Altitudes from the Velocities.—Altitudes 17 feet 6 inches to 40 feet.

Coefficients of velocity, and the corresponding velocities of discharge in inches per second.						Altitudes h in feet and inches.	
7. Values of $v = 119{\cdot}46\ h^{\frac{1}{2}}$ Coefficient ·700.	8. Values of $v = 18{\cdot}515\ h^{\frac{1}{2}}$ Coefficient ·666.	9. Values of $v = 17{\cdot}458\ h^{\frac{1}{2}}$ Coefficient ·628.	10. Values of $v = 17{\cdot}153\ h^{\frac{1}{2}}$ Coefficient ·617.	11. Values of $v = 16{\cdot}847\ h^{\frac{1}{2}}$ Coefficient ·606.	12. Values of $v = 15{\cdot}935\ h^{\frac{1}{2}}$ Coefficient ·584.		
282·00	268·30	253·00	248·56	244·34	235·27	17′	6″
286·00	272·11	256·59	252·09	247·60	238·61	18	0
289·95	275·86	260·12	255·57	251·01	241·90	18	6
293·84	279·57	263·32	259·00	254·38	245·14	19	0
297·68	283·22	267·06	262·38	257·71	248·35	19	6
301·47	286·83	270·46	265·73	260·99	251·51	20	0
305·22	290·39	273·82	269·03	264·23	254·64	20	6
308·92	293·91	277·08	272·23	267·37	257·67	21	0
312·57	297·39	280·42	275·51	270·60	260·78	21	6
316·19	300·83	283·67	278·70	273·73	263·79	22	0
319·76	304·23	286·87	281·85	276·82	266·77	22	6
323·29	307·59	290·04	284·96	279·88	269·72	23	0
326·79	310·92	293·18	288·04	282·91	272·64	23	6
330·25	314·21	296·28	291·09	285·90	275·52	24	0
333·67	317·46	299·35	294·11	288·86	278·38	24	6
337·06	320·69	302·39	297·09	291·80	281·20	25	0
340·41	323·88	305·40	300·05	294·70	284·00	25	6
343·73	327·04	308·38	302·98	297·57	286·77	26	0
347·02	330·17	311·33	305·87	300·42	289·52	26	6
350·28	333·13	314·25	308·75	303·24	292·23	27	0
353·51	336·34	317·15	311·59	306·04	294·93	27	6
856·71	339·38	320·02	314·41	308·81	297·60	28	0
359·88	342·40	322·86	317·20	311·55	300·24	28	6
363·02	345·39	325·68	319·98	314·27	302·86	29	0
366·14	348·35	328·48	322·72	316·97	305·46	29	6
369·23	351·29	331·25	325·45	319·65	308·04	30	0
372·29	354·21	334·00	328·15	322·30	310·60	30	6
375·33	357·10	336·73	330·83	324·93	313·13	31	0
378·35	359·97	339·43	333·48	327·54	315·60	31	6
381·34	362·81	342·11	336·12	330·13	318·14	32	0
384·30	365·64	344·78	338·74	332·70	320·62	32	6
387·25	368·44	347·42	341·33	335·25	323·08	33	0
390·17	371·22	350·04	343·01	337·78	325·51	33	6
393·07	373·98	352·64	346·47	340·29	327·93	34	0
395·95	376·72	355·23	349·00	342·78	330·34	34	6
398·81	379·44	357·79	351·52	345·26	332·72	35	0
404·47	384·82	362·87	356·51	350·15	337·44	36	0
410·05	390·13	367·87	361·43	354·98	342·10	37	0
415·55	395·37	372·81	366·28	359·75	346·69	38	0
420·98	400·54	377·68	371·11	364·45	351·22	39	0
426·35	405·64	382·49	375·79	369·09	355·70	40	0

TABLE III.—Square Roots for finding the effects of the Velocity of Approach when the Orifice is small in proportion to the Head. Also for finding the Increase in the Discharge from an Increase of Head. (See pp. 91 to 99).

No.	Square root.	No.	Square root.	No.	Square root.	No.	Square root.
1·000	1·0000	1·115	1·0559	1·475	1·2141	1·975	1·4053
1·001	1·0005	1·120	1·0583	1·49	1·2207	1·99	1·4107
1·002	1·0010	1·125	1·0607	1·5	1·2247	2·00	1·4142
1·004	1·0020	1·13	1·0630	1·51	1·2288	2·01	1·4177
1·005	1·0025	1·135	1·0654	1·525	1·2349	2·025	1·4230
1·006	1·0030	1·14	1·0677	1·54	1·2410	2·04	1·4283
1·008	1·0040	1·145	1·0700	1·55	1·2450	2·05	1·4318
1·009	1·0044	1·15	1·0723	1·56	1·2490	2·06	1·4353
1·010	1·0050	1·155	1·0747	1·575	1·2550	2·075	1·4405
1·011	1·0055	1·16	1·0770	1·58	1·2570	2·09	1·4457
1·012	1·0060	1·165	1·0794	1·59	1·2610	2·10	1·4491
1·014	1·0070	1·17	1·0817	1·6	1·2649	2·11	1·4526
1·015	1·0075	1·175	1·0840	1·61	1·2689	2·125	1·4577
1·016	1·0080	1·18	1·0863	1·625	1·2748	2·14	1·4629
1·018	1·0090	1·185	1·0886	1·64	1·2806	2·15	1·4663
1·019	1·0095	1·19	1·0909	1·65	1·2845	2·16	1·4697
1·020	1·0100	1·195	1·0932	1·66	1·2884	2·175	1·4748
1·0225	1·0112	1·2	1·0954	1·675	1·2942	2·19	1·4799
1·025	1·0124	1·21	1·1000	1·69	1·3000	2·2	1·4832
1·0275	1·0137	1·22	1·1045	1·7	1·3038	2·21	1·4866
1·03	1·0149	1·23	1·1091	1·71	1·3077	2·225	1·4916
1·0325	1·0161	1·24	1·1136	1·725	1·3134	2·24	1·4967
1·035	1·0174	1·25	1·1180	1·74	1·3191	2·25	1·5000
1·0375	1·0186	1·26	1·1225	1·75	1·3229	2·26	1·5033
1·04	1·0198	1·27	1·1269	1·76	1·3267	2·275	1·5083
1·0425	1·0210	1·28	1·1314	1·775	1·3323	2·29	1·5133
1·045	1·0223	1·29	1·1358	1·79	1·3379	2·3	1·5166
1·0475	1·0235	1·30	1·1402	1·80	1·3416	2·31	1·5199
1·05	1·0247	1·31	1·1446	1·81	1·3454	2·325	1·5248
1·055	1·0271	1·325	1·1511	1·825	1·3509	2·34	1·5297
1·06	1·0296	1·34	1·1576	1·84	1·3565	2·35	1·5330
1·065	1·0320	1·35	1·1619	1·85	1.3601	2.36	1·5362
1·07	1·0344	1·36	1·1662	1·86	1·3638	2·375	1·5411
1·075	1·0368	1·375	1·1726	1·875	1·3693	2·39	1·5460
1·08	1·0392	1·39	1·1790	1·89	1·3748	2·4	1·5492
1·085	1·0416	1·40	1·1832	1·9	1·3784	2·41	1·5524
1·09	1·0440	1·41	1·1874	1·91	1·3820	2·425	1·5572
1·095	1·0464	1·425	1·1937	1·925	1·3875	2·44	1·5621
1·1	1·0488	1·44	1·2000	1·94	1·3928	2·45	1·5652
1·105	1·0512	1·45	1·2042	1·95	1·3964	2·46	1·5684
1·110	1·0536	1·46	1·2083	1·96	1·4000	2·475	1·5737

TABLE III.—Square Roots for finding the effects of the Velocity of Approach when the Orifice is small in proportion to the Head. Also for finding the increase in the Discharge from an increase of Head. (See pp. 91 to 99.)

No.	Square root.	No.	Square root.	No.	Square root.	No.	Square root.
2·49	1·5780	3·0000	1·7321	4·5	2·1213	26	5·0990
2·5	1·5811	3·025	1·7393	5·0	2·2361	27	5·1962
2·51	1·5843	3·05	1·7464	5·5	2·3452	28	5·2915
2·525	1·5890	3·075	1·7536	6·0	2·4495	29	5·3852
2·54	1·5937	3·1	1·7607	6·5	2·5495	30	5·4772
2·55	1·5969	3·125	1·7678	7·0	2·6458	31	5·5678
2·56	1·6000	3·15	1·7748	7·5	2·7386	32	5·6569
2·575	1·6047	3·175	1·7819	8·0	2·8284	33	5·7446
2·59	1·6093	3·2	1·7889	8·5	2·9155	34	5·8310
2·6	1·6125	3·225	1·7958	9·0	3·0000	35	5·9161
2·61	1·6155	3·25	1·8028	9·5	3·0822	36	6·0000
2·625	1·6202	3·275	1·8097	10·0	3·1623	37	6·0828
2·64	1·6248	3·3	1·8166	10·5	2·2404	38	6·1644
2·65	1·6279	3·325	1·8235	11·0	3·3166	39	6·2450
2·66	1·6310	3·35	1·8303	11·5	3·3912	40	6·3246
2·675	1·6355	3·375	1·8371	12·0	3·4641	41	6·4031
2·69	1·6401	3·4	1·8439	12·5	3·5355	42	6·4807
2·7	1·6432	3·425	1·8507	13·0	3·6056	43	6·5574
2·71	1·6462	3·45	1·8574	13·5	3·6742	44	6·6332
2·725	1·6508	3·475	1·8641	14·0	3·7417	45	6·7082
2·74	1·6553	3·5	1·8708	14·5	3·8079	46	6·7823
2·75	1·6583	3·525	1·8775	15·0	3·8730	47	6·8557
2·76	1·6613	3·55	1·8841	15·5	3·9370	48	6·9282
2·775	1·6658	3·575	1·8908	16·0	4·0000	49	7·0000
2·79	1·6703	3·6	1·8974	16·5	4·0620	50	7·0711
2·8	1·6733	3·625	1·9039	17·0	4·1231	51	7·1414
2·81	1·6763	3·65	1·9105	17·5	4·1833	52	7·2111
2·825	1·6808	3·675	1·9170	18·0	4·2426	53	7·2810
2·84	1·6852	3·7	1·9235	18·5	4·3012	54	7·3485
2·85	1·6882	3·725	1·9300	19·0	4·3589	55	7·4162
2·86	1·6912	3·75	1·9365	19·5	4·4159	56	7·4833
2·875	1·6956	3·775	1·9429	20·0	4·4721	57	7·5498
2·89	1·7000	3·8	1·9494	20·5	4·5277	58	7·6158
2·9	1·7029	3·825	1·9558	21·0	4·5826	59	7·6811
2·91	1·7059	3·85	1·9621	21·5	4·6368	60	7·7460
2·925	1·7103	3·875	1·9685	22·0	4·6904	61	7·8102
2·94	1·7146	3·9	1·9748	22·5	4·7434	62	7·8740
2·95	1·7176	3·925	1·9812	23·0	4·7958	63	7·9373
2·96	1·7205	3·95	1·9875	23·5	4·8477	64	8·0000
2·975	1·7248	3·975	1·9938	24·0	4·8990	65	8·0623
2·99	1·7292	4·0	2·0000	25·0	5·0000	66	8·1240

TABLE IV.—For finding the Discharge through Rectangular Orifices; in which $n = \frac{h}{d}$. *Also for finding the effects of the Velocity of Approach to Weirs, and the Depression on the Crest.* (See pp. 91 to 99.)

$1+n$.	$n^{\frac{3}{2}}$.	$(1+n)^{\frac{3}{2}}$.	$(1+n)^{\frac{3}{2}}-n^{\frac{3}{2}}$.	$1+n$.	$n^{\frac{3}{2}}$.	$(1+n)^{\frac{3}{2}}$.	$(1+n)^{\frac{3}{2}}-n^{\frac{3}{2}}$.
1·000	·0000	1·0000	1·0000	1·115	·0390	1·1774	1·1384
1·001	·0000	1·0015	1·0015	1·120	·0416	1·1853	1·1437
1·002	·0001	1·0030	1·0029	1·125	·0442	1·1932	1·1491
1·004	·0003	1·0060	1·0058	1·13	·0469	1·2012	1·1543
1·005	·0004	1·0075	1·0072	1·135	·0496	1·2092	1·1596
1·006	·0005	1·0090	1·0086	1·14	·0524	1·2172	1·1648
1·008	·0007	1·0120	1·0113	1·145	·0552	1·2251	1·1700
1·009	·0009	1·0135	1·0127	1·15	·0581	1·2332	1·1751
1·010	·0010	1·0150	1·0140	1·155	·0610	1·2413	1·1803
1·011	·0012	1·0165	1·0154	1·16	·0640	1·2494	1·1854
1·012	·0013	1·0181	1·0167	1·165	·0670	1·2574	1·1904
1·014	·0017	1·0211	1·0194	1·17	·0701	1·2655	1·1955
1·015	·0018	1·0226	1·0207	1·175	·0732	1·2737	1·2005
1·016	·0020	1·0241	1·0221	1·18	·0764	1·2818	1·2054
1·018	·0024	1·0271	1·0247	1·185	·0796	1·2900	1·2104
1·019	·0026	1·0286	1·0260	1·19	·0828	1·2981	1·2153
1·020	·0028	1·0301	1·0273	1·195	·0861	1·3063	1·2202
1·0225	·0034	1·0339	1·0306	1·2	·0894	1·3145	1·2251
1·025	·0040	1·0377	1·0338	1·21	·0962	1·3310	1·2348
1·0275	·0046	1·0415	1·0370	1·22	·1032	1·3475	1·2443
1·03	·0052	1·0453	1·0401	1·23	·1103	1·3641	1·2538
1·0325	·0059	1·0491	1·0433	1·24	·1176	1·3808	1·2632
1·035	·0065	1·0530	1·0464	1·25	·1250	1·3975	1·2725
1·0375	·0073	1·0568	1·0495	1·26	·1326	1·4143	1·2818
1·04	·0080	1·0606	1·0526	1·27	·1403	1·4312	1·2909
1·0425	·0088	1·0644	1·0557	1·28	·1482	1·4482	1·3000
1·045	·0095	1·0683	1·0587	1·29	·1562	1·4652	1·3090
1·0475	·0104	1·0721	1·0617	1·30	·1643	1·4822	1·3179
1·05	·0112	1·0759	1·0648	1·31	·1726	1·4994	1·3268
1·055	·0129	1·0836	1·0707	1·325	·1853	1·5252	1·3399
1·06	·0147	1·0913	1·0766	1·34	·1983	1·5512	1·3529
1·065	·0166	1·0991	1·0825	1·35	·2071	1·5686	1·3615
1·07	·0185	1·1068	1·0883	1·36	·2160	1·5860	1·3700
1·075	·0205	1·1146	1·0940	1·375	·2296	1·6123	1·3827
1·08	·0226	1·1224	1·0997	1·39	·2436	1·6388	1·3952
1·085	·0248	1·1302	1·1054	1·40	·2530	1·6565	1·4035
1·09	·0270	1·1380	1·1110	1·41	·2625	1·6743	1·4118
1·095	·0293	1·1458	1·1166	1·425	·2771	1·7011	1·4240
1·1	·0316	1·1537	1·1221	1·44	·2919	1·7280	1·4361
1·105	·0340	1·1616	1·1275	1·45	·3019	1·7460	1·4442
1·110	·0365	1·1695	1·1330	1·46	·3120	1·7641	1·4521

Values of n from 0 to ·46. [*Continued on next page.*

TABLE IV.—For finding the Discharge through Rectangular Orifices; in which $n = \frac{h}{d}$. *Also for finding the effects of the Velocity of Approach to Weirs, &c.* (See pp. 91 to 99.)

$1 + n$.	$n^{\frac{3}{2}}$.	$(1+n)^{\frac{3}{2}}$.	$(1+n)^{\frac{3}{2}} - n^{\frac{3}{2}}$.	$1 + n$.	$n^{\frac{3}{2}}$.	$(1+n)^{\frac{3}{2}}$.	$(1+n)^{\frac{3}{2}} - n^{\frac{3}{2}}$.
1·475	·3274	1·7914	1·4640	1·975	·9627	2·7756	1·8128
1·49	·3430	1·8188	1·4758	1·99	·9850	2·8072	1·8222
1·5	·3536	1·8371	1·4836	2·	1·0000	2·8284	1·8284
1·51	·3642	1·8555	1·4913	2·01	1·0150	2·8497	1·8346
1·525	·3804	1·8832	1·5028	2·025	1·0377	2·8816	1·8439
1·54	·3968	1·9111	1·5143	2·04	1·0606	2·9137	1·8531
1·55	·4079	1·9297	1·5218	2·05	1·0759	2·9352	1·8592
1·56	·4191	1·9484	1·5294	2·06	1·0913	2·9567	1·8653
1·575	·4360	1·9766	1·5406	2·075	1·1146	2·9890	1·8744
1·58	·4417	1·9860	1·5443	2·09	1·1380	3·0215	1·8835
1·59	·4532	2·0049	1·5517	2·10	1·1537	3·0432	1·8895
1·6	·4648	2·0239	1·5591	2·11	1·1695	3·0650	1·8955
1·61	·4764	2·0429	1·5664	2·125	1·1932	3·0977	1·9045
1·625	·4941	2·0715	1·5774	2·14	1·2172	3·1306	1·9134
1·64	·5120	2·1002	1·5882	2·15	1·2332	3·1525	1·9193
1·65	·5240	2·1195	1·5954	2·16	1·2494	3·1745	1·9252
1·66	·5362	2·1388	1·6026	2·175	1·2737	3·2077	1·9340
1·675	·5546	2·1678	1·6132	2·19	1·2981	3·2409	1·9428
1·69	·5732	2·1970	1·6238	2·2	1·3145	3·2631	1·9486
1·7	·5857	2·2165	1·6309	2·21	1·3310	3·2854	1·9544
1·71	·5983	2·2361	1·6379	2·225	1·3558	3·3189	1·9631
1·725	·6173	2·2656	1·6483	2·24	1·3808	3·3525	1·9717
1·74	·6366	2·2952	1·6586	2·25	1·3975	3·3750	1·9775
1·75	·6495	2·3150	1·6655	2·26	1·4143	3·3975	1·9832
1·76	·6626	2·3349	1·6724	2·275	1·4397	3·4314	1·9917
1·775	·6823	2·3648	1·6826	2·29	1·4652	3·4654	2·0002
1·79	·7022	2·3949	1·6927	2·3	1·4822	3·4881	2·0059
1·80	·7155	2·4150	1·6994	2·31	1·4994	3·5109	2·0115
1·81	·7290	2·4351	1·7061	2·325	1·5252	3·5451	2·0200
1·825	·7493	2·4654	1·7161	2·34	1·5512	3·5795	2·0284
1·84	·7699	2·4959	1·7260	2·35	1·5686	3·6025	2·0339
1·85	·7837	2·5163	1·7326	2·36	1·5860	3·6255	2·0395
1·86	·7975	2·5367	1·7392	2·375	1·6123	3·6601	2·0478
1·875	·8185	2·5674	1·7490	2·39	1·6388	3·6948	2·0561
1·89	·8396	2·5983	1·7587	2·4	1·6565	3·7181	2·0616
1·9	·8538	2·6190	1·7652	2·41	1·6743	3·7413	2·0670
1·91	·8681	2·6397	1·7716	2·425	1·7011	3·7763	2·0752
1·925	·8896	2·6709	1·7813	2·44	1·7280	3·8114	2·0834
1·94	·9114	2·7021	1·7907	2·45	1·7460	3·8349	2·0888
1·95	·9259	2·7230	1·7971	2·46	1·7641	3·8584	2·0942
1·96	·9406	2·7440	1·8034	2·475	1·7914	3·8937	2·1023

Values of n from ·475 to 1·475

[*Continued on next page.*

TABLE IV.—For finding the Discharge through Rectangular Orifices; in which $n = \frac{h}{d}$. *Also for finding the effects of the Velocity of Approach to Weirs, &c.* (See pp. 91 to 99.)

$1 + n$.	$n^{\frac{3}{2}}$.	$(1+n)^{\frac{3}{2}}$.	$(1+n)^{\frac{3}{2}} - n^{\frac{3}{2}}$.	$1 + n$.	$n^{\frac{3}{2}}$.	$(1+n)^{\frac{3}{2}}$.	$(1+n)^{\frac{3}{2}} - n^{\frac{3}{2}}$.
2·49	1·3188	3·9292	2·1104	3·	2·8284	5·1962	2·3677
2·5	1·8371	3·9528	2·1157	3·025	2·8816	5·2612	2·3796
2·51	1·8555	3·9766	2·1211	3·05	2·9352	5·3626	2·3914
2·525	1·8832	4·0123	2·1291	3·075	2·9890	5·3922	2·4032
2·54	1·9111	4·0481	2·1370	3·1	3·0432	5·4581	2·4149
2·55	1·9297	4·0720	2·1423	3·125	3·0977	5·5243	2·4266
2·56	1·9484	4·0960	2·1476	3·15	3·1525	5·5907	2·4382
2·575	1·9766	4·1321	2·1554	3·175	3·2077	5·6574	2·4497
2·59	2·0049	4·1682	2·1633	3·2	3·2631	5·7243	2·4612
2·6	2·0239	4·1924	2·1685	3·225	3·3189	5·7915	2·4726
2·61	2·0429	4·2166	2·1737	3·25	3·3750	5·8590	2·4840
2·625	2·0715	4·2530	2·1815	3·275	3·4314	5·9268	2·4953
2·64	2·1002	4·2895	2·1893	3·3	3·4881	5·9947	2·5066
2·65	2·1195	4·3139	2·1944	3·325	3·5451	6·0630	2·5179
2·66	2·1388	4·3383	2·1996	3·35	3·6025	6·1315	2·5290
2·675	2·1678	4·3751	2·2073	3·375	3·6601	6·2003	2·5401
2·69	2·1970	4·4119	2·2149	3·4	3·7181	6·2693	2·5512
2·7	2·2165	4·4366	2·2200	3·425	3·7763	6·3386	2·5623
2·71	2·2361	4·4612	2·2251	3·45	3·8349	6·4081	2·5732
2·725	2·2656	4·4983	2·2327	3·475	3·8937	6·4779	2·5842
2·74	2·2952	4·5355	2·2403	3·5	3·9528	6·5479	2·5951
2·75	2·3150	4·5604	2·2453	3·525	4·0123	6·6182	2·6059
2·76	2·3349	4·5853	2·2504	3·55	4·0720	6·6887	2·6167
2·775	2·3648	4·6227	2·2579	3·575	4·1321	6·7595	2·6274
2·79	2·3949	4·6602	2·2654	3·6	4·1924	6·8305	2·6381
2·8	2·4150	4·6853	2·2703	3·625	4·2530	6·9018	2·6488
2·81	2·4351	4·7104	2·2753	3·65	4·3139	6·9733	2·6594
2·825	2·4654	4·7482	2·2827	3·675	4·3751	7·0451	2·6700
2·84	2·4959	4·7861	2·2902	3·7	4·4366	7·1171	2·6805
2·85	2·5163	4·8114	2·2951	3·725	4·4983	7·1893	2·6910
2·86	2·5367	4·8367	2·3000	3·75	4·5604	7·2618	2·7015
2·875	2·5674	4·8748	2·3074	3·775	4·6227	7·3346	2·7119
2·89	2·5983	4·9130	2·3147	3·8	4·6853	7·4076	2·7223
2·9	2·6190	4·9385	2·3196	3·825	4·7482	7·4808	2·7326
2·91	2·6397	4·9641	2·3244	3·85	4·8114	7·5542	2·7429
2·925	2·6708	5·0025	2·3317	3·875	4·8748	7·6279	2·7531
2·94	2·7021	5·0411	2·3389	3·9	4·9385	7·7019	2·7634
2·95	2·7230	5·0668	2·3438	3·925	5·0025	7·7761	2·7735
2·96	2·7440	5·0926	2·3486	3·95	5·0668	7·8505	2·7837
2·975	2·7756	5·1313	2·3558	3·975	5·1313	7·9251	2·7838
2·99	2·8072	5·1702	2·3630	4·	5·1962	8·	2·8038

Values of n from 1·49 to 3.

[*Continued on next page.*

TABLE IV.—*For finding the Discharge through Rectangular Orifices; in which* $n = \frac{h}{d}$. *Also for finding the effects of the Velocity of Approach to Weirs, &c.* (See pp. 91 to 99.)

$1+n$.	$n^{\frac{3}{2}}$.	$(1+n)^{\frac{3}{2}}$	$(1+n)^{\frac{3}{2}}-n^{\frac{3}{2}}$.	$1+n$.	$n^{\frac{3}{2}}$.	$(1+n)^{\frac{3}{2}}$.	$(1+n)^{\frac{3}{2}}-n^{\frac{3}{2}}$.
4·5	6·5479	9·5459	2·9980	26·	125·0000	132·5745	7·5745
5·0	8·0000	11·1803	3·1803	27·	132·5745	140·2961	7·7216
5·5	9·5459	12·8986	3·3527	28·	140·2961	148·1621	7·8660
6·0	11·1803	14·6969	3·5166	29·	148·1621	156·1698	8·0077
6·5	12·8986	16·5718	3·6732	30·	156·1698	164·3168	8·1470
7·0	14·6969	18·5203	3·8234	31·	164·3168	172·6007	8·2839
7·5	16·5718	20·5396	3·9678	32·	172·6007	181·0193	8·4186
8·0	18·5203	22·6274	4·1071	33·	181·0193	189·5706	8·5513
8·5	20·5396	24·7815	4·2419	34·	189·5706	198·2524	8·6818
9·0	22·6274	27·0000	4·3726	35·	198·2524	207·0628	8·8104
9·5	24·7815	29·2810	4·4995	36·	207·0628	216·0000	8·9372
10·0	27·0000	31·6228	4·6228	37·	216·0000	225·0622	9·0622
10·5	29·2810	34·0239	4·7429	38·	225·0622	234·2477	9·1855
11·0	31·6228	36·4829	4·8601	39·	234·2477	243·5549	9·3072
11·5	34·0239	38·9984	4·9745	40·	243·5549	252·9822	9·4273
12·0	36·4829	41·5692	5·0863	41·	252·9822	262·5281	9·5459
12·5	38·9984	44·1942	5·1958	42·	262·5281	272·1911	9·6630
13·0	41·5692	46·8722	5·3030	43·	272·1911	281·9699	9·7788
13·5	44·1942	49·6022	5·4080	44·	281·9699	291·8630	9·8931
14·0	46·8722	52·3832	5·5110	45·	291·8630	301·8692	10·0062
14·5	49·6022	55·2144	5·6122	46·	301·8692	311·9872	10·1180
15·0	52·3832	58·0947	5·7115	47·	311·9872	322·2158	10·2286
15·5	55·2144	61·0236	5·8092	48·	322·2158	332·5538	10·3380
16·0	58·0947	64·	5·9053	49·	332·5538	343·0000	10·4462
16·5	61·0236	67·0247	6·0011	50·	343·0000	353·5534	10·5534
17·0	64·	70·0928	6·0928	51·	353·5534	364·2128	10·6594
17·5	67·0247	73·2078	6·1831	52·	364·2128	374·9773	10·7645
18·0	70·0928	76·3675	6·2747	53·	374·9773	385·8458	10·8685
18·5	73·2078	79·5715	6·3637	54·	385·8458	396·8173	10·9715
19·0	76·3675	82·8191	6·4516	55·	396·8173	407·8909	11·0736
19·5	79·5715	86·1097	6·5382	56·	407·8909	419·0656	11·1747
20·0	82·8191	89·4427	6·6236	57·	419·0656	430·3406	11·2750
20·5	86·1097	92·8177	6·7080	58·	430·3406	441·7148	11·3742
21·0	89·4427	96·2341	6·7914	59·	441·7148	453·1876	11·4728
21·5	92·8177	99·6914	6·8737	60·	453·1876	464·7580	11·5704
22·0	96·2341	103·1892	6·9551	61·	464·7580	476·4252	11·6672
22·5	99·6914	106·7269	7·0355	62·	476·4252	488·1885	11·7633
23·	103·1892	110·3041	7·1149	63·	488·1885	500·0470	11·8585
23·5	106·7269	113·9205	7·1936	64·	500·0470	512·0000	11·9530
24·	110·3041	117·5755	7·2714	65·	512·0000	524·0468	12·0468
25·	117·5755	125·	7·4245	66·	524·0468	536·1865	12·1397

Values of n from 3·5 to 65.

TABLE V.—Coefficients of Discharge for Different Ratios of the Channel to the Orifice.—Coefficients for Heads in still water ·550 and ·573. See equations (44) and (44*a*) and the observations thereon at p. 99.

Ratio of the channel to the orifice.	Coefficient ·550 for heads in still water.			Coefficient ·573 for heads in still water.		
	Ratio of the height due to the velocity of approach to the head.	Coefficients for orifices: the heads measured to the centres.	Coefficients for weirs: the heads measured the full depth.	Ratio of the height due to the velocity of approach to the head.	Coefficients for orifices: the heads measured to the centres.	Coefficients for weirs: the heads measured the full depth.
30·	·000	·550	·550	·000	·573	·573
20·	·001	·550	·551	·001	·573	·574
15·	·001	·550	·551	·001	·573	·574
10·	·003	·551	·552	·003	·574	·576
9·	·004	·551	·553	·004	·574	·576
8·	·005	·551	·554	·005	·574	·577
7·	·006	·552	·555	·007	·575	·578
6·	·008	·552	·557	·009	·576	·580
5·5	·010	·553	·558	·011	·576	·582
5·0	·012	·553	·559	·013	·577	·584
4·5	·015	·554	·562	·016	·578	·586
4·0	·019	·555	·565	·021	·579	·589
3·75	·022	·556	·566	·024	·580	·592
3·50	·025	·557	·569	·028	·581	·594
3·25	·029	·558	·572	·032	·582	·598
3·0	·035	·559	·575	·038	·584	·602
2·75	·042	·561	·580	·045	·586	·607
2·50	·051	·564	·586	·055	·589	·614
2·25	·064	·567	·594	·069	·593	·623
2·00	·082	·572	·606	·089	·598	·636
1·95	·086	·573	·609	·094	·599	·639
1·90	·091	·575	·612	·100	·601	·643
1·85	·097	·576	·615	·106	·603	·647
1·80	·103	·578	·619	·113	·604	·651
1·75	·110	·579	·623	·120	·606	·655
1·70	·117	·581	·627	·128	·609	·660
1·65	·125	·583	·632	·137	·611	·666
1·60	·134	·586	·637	·147	·614	·671
1·55	·144	·588	·643	·158	·617	·678
1·50	·155	·591	·649	·171	·620	·685
1·45	·168	·594	·656	·185	·624	·694
1·40	·183	·598	·664	·201	·628	·703
1·35	·199	·602	·673	·220	·633	·713
1·30	·218	·607	·683	·241	·638	·724
1·25	·240	·612	·695	·266	·645	·737
1·20	·265	·619	·707	·295	·652	·753
1·15	·297	·626	·723	·330	·661	·770
1·10	·333	·635	·741	·372	·671	·791
1·05	·378	·646	·762	·424	·684	·816
1·00	·434	·659	·787	·489	·699	·845

See the auxiliary tables, pp. 104, 108, and 111.

TABLE V.—Coefficients of Discharge for different Ratios of the Channel to the Orifice.—Coefficients for heads in still water ·584 and ·595. See equations (44) and (44*a*) and the observations thereon at p. 99.

Ratio of the channel to the orifice.	Coefficient ·584 for heads in still water.			Coefficient ·595 for heads in still water.		
	Ratio of the height due to the velocity of approach to the head.	Coefficients for orifices: the heads measured to the centres.	Coefficients for weirs: the heads measured the full depth.	Ratio of the height due to the velocity of approach to the head.	Coefficients for orifices: the heads measured to the centres.	Coefficients for weirs: the heads measured the full depth.
30·	·000	·584	·584	·000	·595	·595
20·	·001	·584	·585	·001	·595	·596
15·	·002	·584	·585	·002	·595	·596
10·	·003	·585	·587	·004	·596	·598
9·0	·004	·585	·588	·004	·596	·599
1·0	·005	·586	·588	·006	·597	·600
7·0	·007	·586	·590	·007	·597	·601
6·0	·010	·587	·592	·010	·598	·603
5·5	·011	·587	·593	·012	·599	·605
5·0	·014	·588	·595	·014	·599	·607
4·5	·017	·589	·598	·018	·600	·610
4·0	·022	·590	·601	·023	·602	·613
3·75	·025	·591	·604	·026	·603	·616
3·50	·029	·592	·606	·030	·604	·619
3·25	·033	·594	·610	·035	·605	·622
3·0	·039	·595	·614	·041	·607	·627
2·75	·047	·598	·620	·049	·609	·633
2·50	·058	·601	·627	·060	·613	·641
2·25	·072	·605	·637	·075	·617	·651
2·0	·093	·611	·651	·097	·623	·666
1·95	·099	·612	·654	·103	·625	·669
1·90	·104	·614	·660	·109	·227	·673
1·85	·111	·615	·662	·115	·628	·678
1·80	·118	·617	·666	·123	·630	·682
1·75	·125	·620	·671	·131	·633	·687
1·70	·134	·622	·676	·140	·635	·693
1·65	·143	·624	·682	·149	·638	·699
1·60	·154	·627	·689	·160	·641	·706
1·55	·166	·631	·696	·173	·644	·713
1·50	·179	·634	·703	·187	·648	·721
1·45	·194	·638	·712	·202	·652	·730
1·40	·211	·643	·722	·220	·657	·741
1·35	·230	·648	·732	·241	·663	·752
1·30	·253	·654	·745	·265	·669	·765
1·25	·279	·661	·759	·293	·677	·780
1·20	·310	·669	·775	·325	·685	·797
1·15	·348	·678	·794	·366	·695	·818
1·10	·393	·689	·816	·414	·707	·842
1·05	·448	·703	·842	·473	·722	·870
1·00	·518	·719	·874	·548	·740	·905

See the auxiliary tables, pp. 104, 108, and 111.

TABLE V.—Coefficients of Discharge for different Ratios of the Channel to the Orifice.—Coefficients for heads in still water ·606 and ·617. See equations (44) and (44*a*) and the observations thereon at p. 99.

Ratio of the channel to the orifice.	Coefficient ·606 for heads in still water.			Coefficient ·617 for heads in still water.		
	Ratio of the height due to the velocity of approach to the head.	Coefficients for orifices: the heads measured to the centres.	Coefficients for weirs: the heads measured the full depth.	Ratio of the height due to the velocity of approach to the head.	Coefficients for orifices: the heads measured to the centres.	Coefficients for weirs: the heads measured the full depth.
30·	·000	·606	·606	·000	·617	·617
20·	·001	·606	·607	·001	·617	·618
15·	·002	·607	·607	·002	·618	·619
10·	·004	·607	·609	·004	·618	·620
9·0	·005	·607	·610	·005	·618	·621
8·0	·006	·608	·611	·006	·619	·622
7·0	·008	·608	·612	·008	·619	·624
6·0	·010	·609	·615	·011	·620	·626
5·5	·012	·610	·616	·013	·621	·628
5·0	·015	·611	·619	·015	·622	·630
4·5	·018	·612	·621	·019	·623	·633
4·0	·023	·613	·625	·024	·624	·637
3·75	·027	·614	·628	·028	·626	·640
3·50	·031	·615	·631	·032	·627	·643
3·25	·036	·617	·635	·037	·628	·647
3·00	·043	·619	·640	·044	·630	·653
2·75	·051	·621	·646	·053	·633	·660
2·50	·062	·625	·654	·065	·637	·668
2·25	·078	·629	·665	·081	·642	·679
2·00	·101	·636	·681	·105	·649	·696
1·95	·107	·638	·685	·111	·650	·700
1·90	·113	·639	·689	·118	·652	·704
1·85	·119	·641	·693	·125	·654	·709
1·80	·128	·644	·698	·133	·657	·714
1·75	·136	·646	·703	·142	·659	·720
1·70	·146	·649	·709	·552	·662	·726
1·65	·156	·652	·716	·163	·665	·733
1·60	·167	·655	·723	·175	·669	·741
1·55	·180	·658	·731	·188	·673	·749
1·50	·195	·662	·739	·204	·677	·759
1·45	·212	·667	·749	·221	·681	·768
1·40	·231	·672	·760	·241	·687	·780
1·35	·252	·678	·772	·264	·694	·793
1·30	·278	·685	·786	·291	·701	·808
1·25	·307	·693	·803	·322	·709	·825
1·20	·342	·702	·821	·359	·719	·845
1·15	·384	·713	·843	·404	·731	·868
1·10	·436	·726	·868	·459	·745	·895
1·05	·499	·742	·898	·527	·763	·928
1·00	·580	·762	·936	·615	·784	·969

See the auxiliary tables, pp. 104, 108, and 111.

TABLE V.—Coefficients of Discharge for different Ratios of the Channel to the Orifice.—Mean Coefficient ·628. Coefficients for heads in still water ·628 and ·639. See equations (44) and (44*a*) and the observations thereon at p. 99.

Ratio of the channel to the orifice.	Coefficient ·628 for heads in still water.			Coefficient ·639 for heads in still water.		
	Ratio of the height due to the velocity of approach to the head.	Coefficients for orifices: the heads measured to the centres.	Coefficients for weirs: the heads measured the full depth.	Ratio of the height due to the velocity of approach to the head.	Coefficients for orifices: the heads measured to the centres.	Coefficients for weirs: the heads measured the full depth.
30·	·000	·628	·628	·000	·639	·639
20·	·001	·628	·629	·001	·639	·640
15·	·002	·629	·630	·002	·640	·641
10·	·004	·629	·632	·004	·640	·643
9·0	·005	·630	·632	·005	·641	·644
8·0	·006	·630	·634	·006	·641	·645
7·0	·008	·631	·635	·008	·642	·647
6·0	·011	·631	·638	·011	·643	·649
5·5	·013	·632	·640	·014	·643	·651
5·0	·016	·633	·642	·017	·644	·654
4·5	·020	·634	·645	·021	·646	·657
4·0	·025	·636	·649	·026	·647	·662
3·75	·029	·637	·652	·030	·648	·665
3·50	·033	·638	·656	·034	·650	·668
3·25	·039	·639	·659	·040	·652	·673
3·0	·046	·642	·666	·048	·654	·678
2·75	·055	·645	·672	·057	·657	·686
2·50	·067	·649	·682	·070	·661	·695
2·25	·084	·654	·694	·088	·666	·708
2·0	·109	·661	·711	·114	·674	·727
1·95	·116	·663	·715	·120	·676	·731
1·90	·123	·665	·720	·128	·679	·736
1·85	·130	·668	·725	·135	·681	·741
1·80	·139	·670	·731	·144	·684	·747
1·75	·148	·673	·737	·154	·686	·753
1·70	·158	·676	·743	·165	·690	·760
1·65	·169	·679	·750	·176	·693	·768
1·60	·182	·683	·758	·190	·697	·776
1·55	·196	·687	·767	·205	·701	·786
1·50	·213	·692	·777	·222	·706	·796
1·45	·231	·697	·788	·241	·712	·808
1·40	·252	·703	·800	·262	·718	·820
1·35	·276	·709	·814	·289	·725	·836
1·30	·304	·717	·830	·319	·734	·853
1·25	·338	·726	·846	·354	·743	·872
1·20	·377	·734	·866	·396	·755	·895
1·15	·425	·750	·894	·447	·769	·921
1·10	·484	·765	·924	·509	·785	·953
1·05	·557	·784	·959	·588	·805	·991
1·00	·651	·807	1·002	·690	·831	1·038

See the auxiliary tables, pp. 104, 108, and 111.

TABLE V.—Coefficients of Discharge for different Ratios of the Channel to the Orifice.—Coefficients for heads in still water ·650 and ·667. See equations (44) and (44*a*) and the observations thereon at p. 99.

Ratio of the channel to the orifice.	Coefficient ·650 for heads in still water.			Coefficient ·667 for heads in still water.		
	Ratio of the height due to the velocity of approach to the head.	Coefficients for orifices: the heads measured to the centres.	Coefficients for weirs: the heads measured the full depth.	Ratio of the height due to the velocity of approach to the head.	Coefficients for orifices: the heads measured to the centres.	Coefficients for weirs: the heads measured the full depth.
30·	·000	·650	·650	·000	·667	·667
20·	·001	·650	·651	·001	·667	·668
15·	·002	·651	·652	·002	·667	·669
10·	·004	·651	·654	·004	·668	·671
9·	·005	·652	·655	·006	·669	·672
8·	·007	·652	·656	·007	·669	·673
7·0	·009	·653	·658	·009	·670	·675
6·0	·012	·654	·661	·012	·671	·678
5·5	·014	·655	·663	·015	·672	·680
5·0	·017	·656	·665	·018	·673	·682
4·5	·021	·657	·669	·022	·674	·687
4·0	·027	·659	·674	·029	·676	·692
3·75	·031	·660	·677	·033	·678	·696
3·50	·036	·662	·681	·038	·679	·700
3·25	·042	·663	·686	·044	·681	·705
3·0	·049	·666	·692	·052	·684	·711
2·75	·059	·669	·699	·062	·687	·720
2·50	·073	·673	·709	·077	·692	·731
2·25	·091	·679	·723	·096	·698	·745
2·0	·118	·687	·742	·125	·707	·766
1·95	·125	·689	·747	·132	·709	·771
1·90	·133	·692	·752	·140	·712	·777
1·85	·141	·694	·758	·149	·715	·783
1·80	·150	·697	·764	·159	·718	·790
1·75	·160	·700	·771	·170	·721	·797
1·70	·172	·704	·779	·182	·725	·805
1·65	·184	·707	·786	·195	·729	·814
1·60	·198	·711	·795	·210	·733	·823
1·55	·213	·716	·805	·227	·738	·833
1·50	·231	·721	·816	·246	·744	·846
1·45	·251	·727	·828	·268	·751	·859
1·40	·275	·734	·842	·293	·758	·874
1·35	·302	·742	·858	·322	·764	·888
1·30	·333	·751	·876	·356	·776	·911
1·25	·371	·761	·896	·398	·788	·934
1·20	·415	·773	·920	·446	·802	·961
1·15	·469	·788	·949	·506	·818	·992
1·10	·537	·806	·983	·580	·838	1·030
1·05	·621	·828	1·024	·675	·863	1·076
1·00	·732	·855	1·074	·800	·894	1·133

See the auxiliary tables, pp. 104, 108, and 111.

TABLE V.—Coefficients of Discharge for different Ratios of the Channel to the Orifice.—Coefficients for heads in still water $\sqrt{\cdot 5} = \cdot 7071$ and 1. See equations (44) and (44*a*) and the observations thereon at pp. 98 and 99.

Ratio of the channel to the orifice.	Coefficient ·707 for heads in still water.			Coefficient 1·000 for heads in still water, and multipliers of c_d in equations (45*a*) and (46*a*), which see.		
	Ratio of the height due to the velocity of approach to the head.	Coefficients for orifices: the heads measured to the centres.	Coefficients for weirs: the heads measured the full depth.	Ratio of the height due to the velocity of approach to the head.	Coefficients for orifices: the heads measured to the centres.	Coefficients for weirs: the heads measured the full depth.
30·	·001	·707	·708	·001	1·001	1·002
20·	·001	·708	·708	·003	1·001	1·004
15·	·001	·708	·709	·005	1·002	1·006
10·	·005	·709	·712	·010	1·005	1·014
9·	·006	·709	·713	·013	1·006	1·017
8·	·008	·710	·714	·016	1·008	1·021
7·	·010	·711	·717	·021	1·010	1·028
6·	·014	·712	·721	·029	1·014	1·038
5·5	·017	·713	·723	·034	1·017	1·045
5·0	·020	·714	·727	·041	1·021	1·055
4·5	·025	·716	·731	·052	1·026	1·067
4·0	·032	·718	·737	·067	1·033	1·084
3·75	·037	·720	·742	·077	1·038	1·096
3·50	·043	·722	·747	·089	1·044	1·110
3·25	·050	·724	·753	·105	1·051	1·127
3·00	·059	·728	·760	·125	1·061	1·149
2·75	·071	·732	·770	·152	1·073	1·178
2·50	·087	·737	·783	·190	1·091	1·216
2·25	·110	·745	·801	·246	1·116	1·269
2·00	·143	·756	·826	·333	1·155	1·347
1·95	·151	·759	·832	·356	1·165	1·367
1·90	·161	·762	·839	·383	1·176	1·389
1·85	·171	·765	·846	·412	1·188	1·413
1·80	·182	·769	·854	·446	1·203	1·441
1·75	·195	·773	·863	·484	1·218	1·471
1·70	·209	·778	·873	·529	1·237	1·505
1·65	·225	·783	·883	·579	1·257	1·543
1·60	·243	·788	·895	·641	1·281	1·589
1·55	·263	·795	·908	·711	1·308	1·638
1·50	·286	·802	·923	·800	1·342	1·699
1·45	·312	·810	·939	·903	1·379	1·767
1·40	·342	·819	·958	1·042	1·429	1·854
1·35	·378	·830	·980	1·216	1·489	1·958
1·30	·421	·842	1·003	1·449	1·565	2·088
1·25	·471	·857	1·033	1·778	1·667	2·259
1·20	·532	·875	1·066	2·273	1·810	2·499
1·15	·608	·897	1·107	3·100	2·025	2·844
1·10	·704	·923	1·155	4·762	2·400	3·440
1·05	·830	·957	1·216	9·756	3·280	4·803
1·00	1·000	1·000	1·293	infinite.	infinite.	infinite.

See the auxiliary table, p. 111, also pp. 112, 113.

TABLE VI.—The Discharge over Weirs or Notches of one foot in length, in Cubic feet per minute.—Depths ¼ inch to 10 inches. GREATER COEFFICIENTS ·667 to ·617.—*The Formulæ at the heads of the Columns give the Value of the Discharge, D, in Cubic feet per minute, when l, the length of the Weir, is taken in feet, and the head, h, in inches. For* $l \sqrt{h^3}$ *we may substitute* $l\, h \sqrt{h}$, *retaining the same standards.*

Heads in inches.	Theoretical discharge, $D = 7{\cdot}72\, l \sqrt{h^3}$.	Coefficient ·667. $D = 5{\cdot}15\, l \sqrt{h^3}$.	Coefficient ·650. $D = 5{\cdot}02\, l \sqrt{h^3}$.	Coefficient ·639. $D = 4{\cdot}93\, l \sqrt{h^3}$.	Coefficient ·628. $D = 4{\cdot}85\, l \sqrt{h^3}$.	Coefficient ·617. $D = 4{\cdot}76\, l \sqrt{h^3}$.
·25	·965	·644	·627	·617	·606	·596
·5	2·730	1·821	1·775	1·744	1·714	1·684
·75	5·016	3·345	3·260	3·205	3·150	3·095
1·	7·722	5·151	5·019	4·934	4·849	4·764
1·25	10·792	7·198	7·015	6·896	6·777	6·659
1·5	14·186	9·462	9·221	9·065	8·909	8·753
1·75	17·877	11·924	11·620	11·423	11·227	11·030
2·	21·842	14·569	14·197	13·957	13·717	13·477
2·25	26·062	17·383	16·940	16·654	16·367	16·080
2·5	30·524	20·360	19·841	19·505	19·169	18·833
2·75	35·215	23·489	22·890	22·503	22·115	21·728
3·	40·125	26·763	26·081	25·640	25·199	24·757
3·25	45·244	30·178	29·408	28·911	28·413	27·915
3·5	50·563	33·726	32·866	32·310	31·754	31·197
3·75	56·077	37·403	36·450	35·833	35·216	34·599
4·	61·777	41·205	40·155	39·476	38·796	38·116
4·25	67·658	45·128	43·978	43·233	42·489	41·745
4·5	73·714	49·167	47·914	47·103	46·292	45·482
4·75	79·942	53·321	51·962	51·083	50·203	49·324
5·	86·335	57·585	56·118	55·168	54·218	53·269
5·25	92·891	61·958	60·379	59·357	58·335	57·314
5·5	99·604	66·436	64·743	63·647	62·551	61·456
5·75	106·472	71·017	69·207	68·036	66·864	65·693
6·	113·491	75·698	73·769	72·521	71·272	70·024
6·25	120·657	80·478	78·427	77·100	75·772	74·445
6·5	127·969	85·355	83·180	81·772	80·365	78·957
6·75	135·422	90·326	88·024	86·535	85·045	83·555
7·	143·015	95·391	92·960	91·387	89·813	88·240
7·25	150·744	100·546	97·983	96·325	94·667	93·009
7·5	158·608	105·792	103·095	101·350	99·606	97·861
7·75	166·604	111·125	108·292	106·460	104·627	102·795
8·	174·731	116·546	113·575	111·653	109·731	107·809
8·25	182·984	122·051	118·940	116·927	114·914	112·901
8·5	191·365	127·640	124·387	122·282	120·177	118·072
8·75	199·869	133·313	129·915	127·716	125·518	123·319
9·	208·496	139·067	135·522	133·229	130·935	128·642
9·25	217·243	144·901	141·207	138·818	136·428	134·039
9·5	226·111	150·816	146·972	144·485	141·997	139·510
9·75	235·093	156·807	152·810	150·225	147·639	145·053
10·	244·193	162·877	158·725	156·039	153·353	150·666

See pp. 114 to 133.

TABLE VI.—The Discharge over Weirs or Notches of one foot in length, in Cubic feet per minute.—Depths 10·25 inches to 32 inches. GREATER COEFFICIENTS ·667 to ·617.—*The Formulæ at the heads of the Columns give the Value of the Discharge, D, in Cubic feet per minute, when l, the length of the Weir, is taken in feet, and the head, h, in inches. For* $l\sqrt{h^3}$ *we may substitute* $l\,h\sqrt{h}$, *retaining the same standards.*

Heads in inches.	Theoretical discharge, D = $7{\cdot}72\,l\sqrt{h^3}$.	Coefficient ·667. D = $5{\cdot}15\,l\sqrt{h^3}$.	Coefficient ·650. D = $5{\cdot}02\,l\sqrt{h^3}$.	Coefficient ·639. D = $4{\cdot}93\,l\sqrt{h^3}$.	Coefficient ·628. D = $4{\cdot}85\,l\sqrt{h^3}$.	Coefficient ·617. D = $4{\cdot}76\,l\sqrt{h^3}$.
10·25	253·407	169·023	164·715	161·927	159·140	156·352
10·5	262·734	175·244	170·777	167·887	164·997	162·107
10·75	272·173	181·450	176·913	173·919	170·925	167·931
11·	281·723	187·909	183·120	180·021	176·922	173·823
11·25	291·382	194·352	189·398	186·193	182·988	179·782
11·5	301·148	200·866	195·746	192·434	189·121	185·808
11·75	311·024	207·451	202·164	198·743	195·321	191·900
12·	321·	214·107	208·650	205·119	201·588	198·057
12·5	341·275	227·628	221·826	218·072	214·318	210·564
13·	361·950	241·421	235·268	231·286	227·305	223·323
13·5	383·031	255·482	248·970	244·757	240·543	236·330
14·	404·507	269·806	262·930	258·480	254·030	249·581
14·5	426·368	284·387	277·139	272·449	267·759	263·069
15·	448·611	299·223	291·597	286·662	281·728	276·793
15·5	471·228	314·309	306·298	301·115	295·931	290·748
16·	494·212	329·639	321·238	315·801	310·365	304·929
16·5	517·558	345·211	336·413	330·720	325·026	319·333
17·	541·261	361·021	351·820	345·866	339·912	333·958
17·5	565·315	377·065	367·455	361·236	355·018	348·799
18·	589·715	393·340	383·315	376·828	370·341	363·854
18·5	614·443	409·833	399·388	392·629	385·870	379·111
19·	639·533	426·569	415·696	408·662	401·627	394·592
19·5	664·944	443·518	432·214	424·899	417·585	410·270
20·	690·682	460·685	448·943	441·346	433·748	426·151
20·5	716·737	478·064	465·879	457·995	450·111	442·227
21·	743·125	495·664	483·031	474·857	466·683	458·508
21·5	769·823	513·472	500·385	491·917	483·449	474·981
22·	796·832	531·487	517·941	509·176	500·410	491·645
22·5	824·151	549·709	535·698	526·632	517·567	508·501
23·	851·775	568·134	553·654	544·284	534·915	525·545
23·5	879·700	586·760	571·805	562·128	552·452	542·775
24·	907·925	605·586	590·151	580·164	570·177	560·190
25·	965·253	643·824	627·414	616·797	606·179	595·561
26·	1023·748	682·840	665·436	654·175	642·914	631·653
27·	1083·375	722·611	704·194	692·277	680·360	668·442
28·	1144·116	763·125	743·675	731·090	718·505	705·920
29·	1205·950	804·369	783·868	770·602	757·337	744·071
30·	1268·864	846·332	824·762	810·804	796·847	782·889
31·	1332·833	889·000	866·341	851·680	837·019	822·358
32·	1397·842	932·361	908·597	893·221	877·845	862·469

See pp. 114 to 133.

TABLE VI.—The Discharge over Weirs or Notches of one foot in length, in Cubic feet per minute.—Depths 33 inches to 72 inches. **Greater coefficients** ·667 to ·617.—*The Formulæ at the heads of the Columns give the Value of the Discharge, D, in Cubic feet per minute, when l, the length of the Weir, is taken in feet, and the head, h, in inches. For* $l\sqrt{h^3}$ *we may substitute* $l\,h\sqrt{h}$, *retaining the same standards.*

Heads in inches.	Theoretical discharge, D = $7{\cdot}72\,l\sqrt{h^3}$.	Coefficient ·667. D = $5{\cdot}15\,l\sqrt{h^3}$.	Coefficient ·650. D = $5{\cdot}02\,l\sqrt{h^3}$	Coefficient ·639. D = $4{\cdot}93\,l\sqrt{h^3}$.	Coefficient ·628. D = $4{\cdot}85\,l\sqrt{h^3}$.	Coefficient ·617. D = $4{\cdot}76\,l\sqrt{h^3}$.
33·	1463·875	976·405	951·519	935·416	919·314	903·211
34·	1530·917	1021·122	995·096	978·256	961·416	944·576
35·	1598·951	1066·500	1039·318	1021·730	1004·141	986·553
36·	1667·964	1112·532	1084·177	1065·829	1047·481	1029·134
37·	1737·943	1159·208	1129·663	1110·546	1091·428	1072·311
38·	1808·875	1206·520	1175·769	1155·871	1135·974	1116·076
39·	1880·746	1254·458	1222·485	1201·797	1181·108	1160·420
40·	1953·544	1303·014	1269·804	1248·315	1226·826	1205·337
41·	2027·258	1352·181	1317·718	1295·418	1273·118	1250·818
42·	2101·876	1401·951	1366·219	1343·099	1319·978	1296·857
43·	2177·387	1452·317	1415·302	1391·350	1367·399	1343·448
44·	2253·783	1503·273	1464·959	1440·167	1415·376	1390·584
45·	2331·052	1554·812	1515·184	1489·542	1463·901	1438·259
46·	2409·183	1606·925	1565·969	1539·468	1512·967	1486·466
47·	2488·170	1659·609	1617·311	1589·941	1562·571	1535·201
48·	2568·	1712·856	1669·200	1640·952	1612·704	1584·456
49·	2648·666	1766·660	1721·633	1692·498	1663·362	1634·227
50·	2730·160	1821·021	1774·604	1744·572	1714·540	1684·509
51·	2812·474	1875·920	1828·108	1797·171	1766·234	1735·296
52·	2895·597	1931·363	1882·138	1850·286	1818·435	1786·583
53·	2979·525	1987·343	1936·691	1903·916	1871·142	1838·367
54·	3064·253	2043·857	1991·764	1958·058	1924·351	1890·644
55·	3149·755	2100·887	2047·341	2012·693	1978·046	1943·399
56·	3236·050	2158·445	2103·433	2067·836	2032·239	1996·643
57·	3323·117	2216·519	2160·026	2123·472	2086·917	2050·363
58·	3410·946	2275·101	2217·115	2179·594	2142·074	2104·554
59·	3499·542	2334·195	2274·702	2236·207	2197·712	2159·217
60·	3588·889	2393·789	2332·778	2293·300	2253·822	2214·344
61·	3678·984	2453·882	2391·340	2350·871	2310·402	2269·933
62·	3769·825	2514·473	2450·386	2408·918	2367·450	2325·982
63·	3861·393	2575·549	2509·905	2467·430	2424·955	2382·479
64·	3953·694	2637·114	2569·901	2526·410	2482·920	2439·429
65·	4046·720	2699·162	2630·368	2585·854	2541·340	2496·826
66·	4140·465	2761·690	2691·302	2645·757	2600·212	2554·667
67·	4234·922	2824·693	2752·699	2706·115	2659·531	2612·947
68·	4330·086	2888·167	2814·556	2766·925	2719·294	2671·663
69·	4425·954	2952·111	2876·870	2828·185	2779·499	2730·814
70·	4522·516	3016·518	2939·635	2889·888	2840·140	2790·392
71·	4619·774	3081·389	3002·853	2952·036	2901·218	2850·401
72·	4717·718	3146·718	3066·518	3014·622	2962·727	2910·832

See pp. 114 to 133.

TABLE VI.—The Discharge over Weirs or Notches of one foot in length, in Cubic feet per minute.—Depths ¼ inch to 10 inches. LESSER COEFFICIENTS ·606 to ·518.—*The Formulæ at the heads of the Columns give the Value of the Discharge, D, in Cubic feet per minute, when l, the length of the Weir, is taken in feet, and the head, h, in inches. For $l\sqrt{h^3}$ we may substitute $l\,h\sqrt{h}$, retaining the same standards.*

Heads in inches.	Coefficient ·606. D = $4{\cdot}68\,l\sqrt{h^3}$.	Coefficient ·595. D = $4{\cdot}59\,l\sqrt{h^3}$.	Coefficient ·584. D = $4{\cdot}51\,l\sqrt{h^3}$.	Coefficient ·562. D = $4{\cdot}34\,l\sqrt{h^3}$.	Coefficient ·540. D = $4{\cdot}17\,l\sqrt{h^3}$.	Coefficient ·518. D = $4\,l\sqrt{h^3}$.
·25	·585	·574	·564	·542	·521	·500
·5	1·654	1·624	1·504	1·534	1·474	1·414
·75	3·039	2·985	2·929	2·819	2·708	2·598
1·	4·680	4·595	4·510	4·340	4·170	4·000
1·25	6·540	6·421	6·303	6·065	5·828	5·590
1·5	8·597	8·441	8·284	7·973	7·660	7·348
1·75	10·833	10·637	10·440	10·047	9·653	9·260
2·	13·236	12·996	12·756	12·275	11·795	11·314
2·25	15·794	15·507	15·220	14·647	14·073	13·500
2·5	18·498	18·162	17·826	17·155	16·483	15·811
2·75	21·340	20·953	20·556	19·791	19·016	18·241
3·	24·316	23·874	23·433	22·550	21·668	20·785
3·25	27·418	26·920	26·422	25·427	24·432	23·436
3·5	30·641	30·085	29·529	28·416	27·304	26·192
3·75	33·982	33·366	32·749	31·515	30·281	29·048
4·	37·437	36·757	36·078	34·719	33·360	32·000
4·25	41·001	40·256	39·512	38·024	36·535	35·047
4·5	44·671	43·860	43·049	41·427	39·806	38·184
4·75	48·445	47·565	46·686	44·927	43·169	41·410
5·	52·319	51·369	50·420	48·520	46·621	44·722
5·25	56·292	55·270	54·248	52·205	50·161	48·117
5·5	60·360	59·264	58·169	55·977	53·786	51·595
5·75	64·522	63·351	62·180	59·837	57·495	55·153
6·	68·776	67·527	66·279	63·782	61·285	58·788
6·25	73·118	71·791	70·464	67·809	65·155	62·500
6·5	77·549	76·142	74·734	71·919	69·103	66·288
6·75	82·066	80·576	79·086	76·107	73·128	70·149
7·	86·667	85·094	83·521	80·374	77·228	74·082
7·25	91·351	89·693	88·034	84·718	81·402	78·085
7·5	96·116	94·372	92·627	89·138	85·648	82·159
7·75	100·962	99·129	97·297	93·631	89·966	86·301
8·	105·887	103·965	102·043	98·199	94·355	90·511
8·25	110·889	108·876	106·863	102·837	98·812	94·786
8·5	115·967	113·862	111·757	107·547	103·337	99·127
8·75	121·121	118·922	116·723	112·326	107·929	103·532
9·	126·349	124·055	121·762	117·175	112·588	108·001
9·25	131·649	129·259	126·870	122·090	117·311	112·532
9·5	137·023	134·535	132·048	127·074	122·100	117·125
9·75	142·467	139·881	137·294	132·122	126·950	121·778
10·	147·991	145·295	142·609	137·237	131·864	126·492

See pp. 114 to 133.

TABLE VI.—The Discharge over Weirs or Notches of one foot in length in Cubic feet per minute.—Depths 10·25 inches to 32 inches. LESSER COEFFICIENTS ·606 to ·518.—*The Formulæ at the heads of the Columns give the Value of the Discharge, D, in Cubic feet per minute, when l, the length of the Weir, is taken in feet, and the head, h, in inches. For* $l\sqrt{h^3}$ *we may substitute* $l\,h\sqrt{h}$, *retaining the same standards.*

Heads in inches.	Coefficient ·606. D = 4·68 $l\sqrt{h^3}$.	Coefficient ·595. D = 4·59 $l\sqrt{h^3}$.	Coefficient ·584. D = 4·51 $l\sqrt{h^3}$.	Coefficient ·562. D = 4·34 $l\sqrt{h^3}$.	Coefficient ·540. D = 4·17 $l\sqrt{h^3}$.	Coefficient ·518. D = 4 $l\sqrt{h^3}$.
10·25	153·565	150·777	147·990	142·415	136·840	131·265
10·5	159·217	156·327	153·437	147·657	141·876	136·096
10·75	164·937	161·943	158·949	152·961	146·974	140·986
11·	170·724	167·625	164·526	158·328	152·130	145·933
11·25	176·577	173·372	170·167	163·756	157·346	150·936
11·5	182·496	179·183	175·870	169·245	162·620	155·995
11·75	188·479	185·059	181·636	174·794	167·952	161·109
12·	194·526	190·995	187·464	180·402	173·340	166·278
12·5	206·810	203·056	199·302	191·794	184·286	176·778
13·	219·342	215·360	211·379	203·415	195·453	187·490
13·5	232·117	227·903	223·690	215·263	206·837	198·410
14·	245·131	240·682	236·232	227·333	218·434	209·535
14·5	258·379	253·689	248·999	239·619	230·239	220·859
15·	271·858	266·924	261·989	252·119	242·250	232·380
15·5	285·564	280·381	275·197	264·830	254·463	244·096
16·	299·492	294·056	288·620	277·747	266·875	256·001
16·5	313·640	307·947	302·253	290·868	279·481	268·095
17·	328·004	322·050	316·096	304·189	292·281	280·373
17·5	342·581	336·362	330·144	317·707	305·270	292·833
18·	357·367	350·880	344·394	331·420	318·446	305·472
18·5	372·352	365·594	358·835	345·317	331·799	318·241
19·	387·557	380·522	373·487	359·418	345·348	331·278
19·5	402·956	395·642	388·327	373·699	359·070	344·441
20·	418·553	410·959	403·358	388·163	372·968	357·773
20·5	434·343	426·458	418·574	402·806	387·038	371·270
21·	450·334	442·159	433·985	417·636	401·288	384·939
21·5	466·513	458·045	449·577	432·641	415·704	398·768
22·	482·880	474·115	465·350	447·819	430·289	412·759
22·5	499·436	490·370	481·304	463·173	445·042	426·910
23·	517·176	506·806	497·437	478·698	459·959	441·219
23·5	533·098	523·421	513·745	494·391	475·038	455·685
24·	550·203	540·215	530·228	510·254	490·280	470·305
25·	584·943	574·326	563·708	542·472	521·237	500·001
26·	620·391	609·130	597·869	575·346	552·824	530·301
27·	656·525	644·608	632·691	608·857	585·023	561·188
28·	693·334	680·749	668·164	642·993	617·823	592·652
29·	730·806	717·540	704·275	677·744	651·213	624·682
30·	768·932	754·974	741·017	713·102	685·187	657·272
31·	807·697	793·036	778·374	749·052	719·730	690·407
32·	847·092	831·716	816·340	785·587	754·835	724·082

See pp. 114 to 133.

TABLE VI.—The Discharge over Weirs or Notches of one foot in length, in Cubic feet per minute.—Depths 33 inches to 72 inches. LESSER COEFFICIENTS ·606 to ·518.—*The Formulæ at the heads of the Columns give the Value of the Discharge, D, in Cubic feet per minute, when l, the length of the Weir, is taken in feet, and the head, h, in inches. For* $l\sqrt{h^3}$ *we may substitute* $l\,h\sqrt{h}$, *retaining the same standards.*

Heads in inches.	Coefficient ·606. D = $4{\cdot}68\,l\sqrt{h^3}$.	Coefficient ·595. D = $4{\cdot}59\,l\sqrt{h^3}$.	Coefficient ·584. D = $4{\cdot}51\,l\sqrt{h^3}$.	Coefficient ·562. D = $4{\cdot}34\,l\sqrt{h^3}$.	Coefficient ·540. D = $4{\cdot}17\,l\sqrt{h^3}$.	Coefficient ·518 D = $4\,l\sqrt{h^3}$.
33·	887·108	871·006	854·903	822·698	790·493	758·287
34·	927·736	910·896	894·056	860·375	826·695	793·015
35·	968·964	951·376	933·787	898·610	863·434	828·257
36·	1010·786	992·439	974·091	937·396	900·701	864·005
37·	1053·193	1034·076	1014·959	976·724	938·489	900·254
38·	1096·178	1076·281	1056·383	1016·588	976·793	936·997
39·	1139·732	1119·044	1098·356	1056·979	1015·603	974·226
40·	1183·848	1162·359	1140·870	1097·892	1054·914	1011·936
41·	1228·518	1206·219	1183·919	1139·319	1094·719	1050·120
42·	1273·737	1250·616	1227·496	1181·254	1135·013	1088·772
43·	1319·497	1295·545	1271·594	1223·691	1175·789	1127·886
44·	1365·792	1341·001	1316·209	1266·626	1217·043	1167·460
45·	1412·618	1386·976	1361·334	1310·051	1258·768	1207·485
46·	1459·965	1433·464	1406·963	1353·961	1300·959	1247·957
47·	1507·831	1480·461	1453·091	1398·352	1343·612	1288·872
48·	1556·208	1527·960	1499·712	1443·216	1386·720	1330·224
49·	1605·092	1575·956	1546·821	1488·550	1430·280	1372·009
50·	1654·477	1624·445	1594·413	1534·350	1474·286	1414·223
51·	1704·359	1673·422	1642·485	1580·610	1518·736	1456·862
52·	1754·732	1722·880	1691·029	1627·326	1563·622	1499·919
53·	1805·592	1772·817	1740·043	1674·493	1608·944	1543·394
54·	1856·937	1823·231	1789·524	1722·110	1654·697	1587·283
55·	1908·751	1874·104	1839·457	1770·162	1700·868	1631·573
56·	1961·046	1925·450	1889·853	1818·660	1747·467	1676·274
57·	2013·809	1977·255	1940·700	1867·592	1794·483	1721·375
58·	2067·033	2029·513	1991·992	1916·952	1841·911	1766·870
59·	2120·722	2082·227	2043·733	1966·743	1889·753	1812·763
60·	2174·867	2135·389	2095·911	2016·956	1938·000	1859·045
61·	2229·464	2188·995	2148·527	2067·589	1986·651	1905·714
62·	2284·514	2243·046	2201·578	2118·642	2035·706	1952·769
63·	2340·004	2297·529	2255·054	2170·103	2085·152	2000·202
64·	2395·939	2352·448	2308·957	2221·976	2134·995	2048·013
65·	2452·312	2407·798	2363·284	2274·257	2185·229	2096·201
66·	2509·122	2463·577	2418·032	2326·941	2235·851	2144·761
67·	2566·363	2519·779	2473·194	2380·026	2286·858	2193·690
68·	2624·032	2576·401	2528·770	2433·508	2338·246	2242·985
69·	2682·128	2633·443	2584·757	2487·386	2390·015	2292·644
70·	2740·645	2690·897	2641·149	2541·654	2442·159	2342·663
71·	2799·583	2748·766	2697·948	2596·313	2494·678	2393·043
72·	2858·937	2807·042	2755·147	2651·358	2547·568	2443·778

See pp. 114 to 133.

TABLE VII.—For finding the Mean Velocity from the Maximum Velocity at the Surface, in Mill Races, Streams, and Rivers with uniform Channels; and the Maximum Velocity from the Mean Velocity.

For the Velocity in feet per minute, multiply by 5.

Maximum velocity at the surface in inches per second.	Mean velocity in large channels in inches per second.	Mean velocity in smaller channels in inches per second.	Maximum velocity at the surface in inches per second.	Mean velocity in large channels in inches per second.	Mean velocity in smaller channels in inches per second.	Maximum velocity at the surface in inches per second.	Mean velocity in large channels in inches per second.	Mean velocity in smaller channels in inches per second.
1	·84	·75	41	34·24	33·37	81	67·64	68·86
2	1·67	1·51	42	35·07	34·23	82	68·47	69·77
3	2·51	2·27	43	35·91	35·09	83	69·31	70·68
4	3·34	3·04	44	36·74	35·95	84	70·14	71·59
5	4·18	3·81	45	37·58	36·82	85	70·98	72·50
6	5·01	4·58	46	38·41	37·69	86	71·81	73·42
7	5·85	5·36	47	39·25	38·56	87	72·65	74·33
8	6·68	6·14	48	40·08	39·43	88	73·48	75·24
9	7·52	6·92	49	40·92	40·30	89	74·32	76·16
10	8·35	7·71	50	41·75	41·17	90	75·15	77·08
11	9·19	8·50	51	42·59	42·05	91	75·99	77·99
12	10·02	9·29	52	43·42	42·92	92	76·82	78·91
13	10·86	10·09	53	44·26	43·80	93	77·66	79·83
14	11·69	10·88	54	45·09	44·68	94	78·49	80·75
15	12·53	11·69	55	45·93	45·56	95	79·33	81·67
16	13·36	12·49	56	46·76	46·44	96	80·16	82·59
17	14·20	13·30	57	47·60	47·32	97	81·00	83·51
18	15·03	14·11	58	48·43	48·21	98	81·83	84·43
19	15·87	14·92	59	49·27	49·09	99	82·67	85·36
20	16·70	15·73	60	50·10	49·98	100	83·50	86·28
21	17·54	16·55	61	50·94	50·87	101	84·34	87·20
22	18·37	17·37	62	51·77	51·76	102	85·17	88·13
23	19·21	18·19	63	52·61	52·65	103	86·01	89·06
24	20·04	19·02	64	53·44	53·54	104	86·84	89·98
25	20·88	19·85	65	54·28	54·43	105	87·68	90·91
26	21·71	20·68	66	55·11	55·33	106	88·51	91·84
27	22·55	21·51	67	55·95	56·22	107	89·35	92·77
28	23·38	22·34	68	56·78	57·12	108	90·18	93·69
29	24·22	23·18	69	57·62	58·02	109	91·02	94·62
30	25·05	24·02	70	58·45	58·91	110	91·85	95·55
31	25·89	24·86	71	59·29	59·81	111	92·69	96·49
32	26·72	25·70	72	60·12	60·71	112	93·52	97·42
33	27·56	26·54	73	60·96	61·61	113	94·36	98·35
34	28·39	27·39	74	61·79	62·52	114	95·19	99·28
35	29·23	28·24	75	62·63	63·42	115	96·03	100·21
36	30·06	29·09	76	63·46	64·32	116	96·86	101·15
37	30·90	29·94	77	64·30	65·23	117	97·70	102·08
38	31·73	30·79	78	65·13	66·13	118	98·53	103·02
39	32·57	31·65	79	65·97	67·04	119	99·37	103·95
40	33·40	32·51	80	66·80	67·95	120	100·20	104·89

See pp. 188 to 191.

TABLE VIII.—For finding the Mean Velocities of Water flowing in Pipes, Drains, Streams, and Rivers.—For a full cylindrical pipe, divide the diameter by 4 to find the hydraulic mean depth.

Diameters of pipes $\frac{1}{2}$ *inch to* 2 *inches. Falls per mile* 1 *inch to* 12 *feet.*

Falls per mile in feet and inches, and the hydraulic inclinations.			"Hydraulic mean depths," or "mean radii," and velocities in inches per second.				
Falls.		Inclinations one in	$\frac{1}{16}$ inch.	$\frac{1}{8}$ inch.	$\frac{1}{4}$ inch.	$\frac{3}{8}$ inch.	$\frac{1}{2}$ inch.
F.	I.						
0	1	63360	·14	·24	·38	·49	·57
0	2	31680	·22	·37	·59	·76	·90
0	3	21120	·28	·48	·75	·97	1·15
0	4	15840	·34	·57	·89	1·15	1·36
0	5	12672	·38	·65	1·02	1·30	1·55
0	6	10560	·42	·72	1·13	1·45	1·72
0	7	9051	·46	·78	1·24	1·58	1·88
0	8	7920	·50	·85	1·33	1·71	2·02
0	9	7040	·53	·90	1·43	1·83	2·16
0	10	6336	·57	·96	1·51	1·94	2·30
0	11	5760	·60	1·01	1·60	1·96	2·42
1	0	5280	·63	1·06	1·68	2·15	2·54
1	3	4224	·71	1·20	1·90	2·43	2·88
1	6	3520	·79	1·33	2·10	2·69	3·19
1	9	3017	·87	1·45	2·29	2·94	3·48
2	0	2640	·93	1·56	2·47	3·16	3·75
2	3	Interpolated.	·99	1·67	2·63	3·37	3·99
2	6	2112	1·05	1·77	2·79	3·58	4·24
2	9	Interpolated.	1·11	1·87	2·94	3·77	4·47
3	0	1760	1·16	1·96	3·09	3·96	4·69
3	3	Interpolated.	1·21	2·05	3·23	4·14	4·91
3	6	1508	1·26	2·14	3·37	4·32	5·12
3	9	Interpolated.	1·31	2·22	3·50	4·48	5·31
4	0	1320	1·36	2·30	3·63	4·65	5·51
4	6	Interpolated.	1·45	2·45	3·87	4·96	5·88
5	0	1056	1·54	2·61	4·11	5·27	6·24
5	6	Interpolated.	1·62	2·75	4·33	5·55	6·58
6	0	880	1·71	2·89	4·55	5·83	6·91
6	6	Interpolated.	1·78	3·02	4·76	6·10	7·22
7	0	754	1·86	3·15	4·97	6·36	7·54
7	6	Interpolated.	1·93	3·27	5·16	6·61	7·83
8	0	660	2·01	3·39	5·35	6·86	8·12
8	6	Interpolated.	2·07	3·51	5·53	7·09	8·40
9	0	587	2·14	3·62	5·72	7·32	8·68
9	6	Interpolated.	2·20	3·74	5·89	7·55	8·94
10	0	528	2·28	3·85	6·07	7·77	9·21
10	6	Interpolated.	2·33	3·95	6·24	7·99	9·47
11	0	480	2·40	4·06	6·40	8·20	9·72
11	6	Interpolated.	2·46	4·16	6·57	8·41	9·97
12	0	440	2·52	4·27	6·73	8·62	10·21

See p. 208.

TABLE VIII.—For finding the Mean Velocities of Water flowing in Pipes, Drains, Streams, and Rivers.—For a full cylindrical pipe, divide the diameter by 4 to find the hydraulic mean depth.

Diameters of pipes ½ inch to 2 inches. Falls per mile 13 *feet to* 5280 *feet.*

Falls per mile in feet, and the hydraulic inclination.		"Hydraulic mean depths," or "mean radii," and velocities in inches per second.				
Falls.	Inclinations one in	$\frac{1}{16}$ inch.	$\frac{1}{8}$ inch.	$\frac{1}{4}$ inch.	$\frac{3}{8}$ inch.	$\frac{1}{2}$ inch.
F.						
13·2	400	2·66	4·50	7·10	9·10	10·78
13·6	Interpolated.	2·71	4·59	7·24	9·27	10·98
14·1	375	2·76	4·67	7·37	9·44	11·18
14·6	Interpolated.	2·82	4·76	7·52	9·63	11·41
15·1	350	2·87	4·85	7·66	9·82	11·63
15·6	Interpolated.	2·94	4·96	7·83	10·03	11·88
16·2	325	3·00	5·07	7·99	10·24	12·13
17·6	300	3·14	5·30	8·37	10·72	12·70
19·2	275	3·30	5·58	8·80	11·27	13·35
21·1	250	3·48	5·89	9·39	11·90	14·10
23·5	225	3·70	6·26	9·87	12·65	14·99
26·4	200	3·96	6·70	10·57	13·54	16·04
30·2	175	4·28	7·24	11·42	14·63	17·33
35·2	150	4·68	7·92	12·49	16·00	18·96
37·7	140	4·88	8·24	13·00	16·66	19·74
42·2	125	5·21	8·81	13·90	17·80	21·09
48·	110	5·62	9·50	14·98	19·19	22·74
52·8	100	5·94	10·05	15·85	20·30	24·06
58·7	90	6·33	10·69	16·87	21·61	25·60
66·	80	6·78	11·47	18·10	23·17	27·46
75·4	70	7·35	12·42	19·59	25·09	29·73
88·	60	8·05	13·61	21·48	27·51	32·60
105·6	50	8·99	15·19	23·96	30·69	36·37
117·3	45	9·57	16·18	25·53	32·70	38·75
132·0	40	10·28	17·37	27·41	35·11	41·60
150·8	35	11·14	18·84	29·71	38·06	45·10
176·	30	12·23	20·68	32·62	41·78	49·51
212·2	25	13·66	23·09	36·43	46·67	55·30
264.	20	15·64	26·44	41·71	53·43	63·30
352·	15	18·61	31·46	49·63	63·57	75·33
528·	10	23·73	40·11	63·28	81·06	96·05
586·7	9	25·26	42·70	67·37	86·29	102·25
660·	8	27·08	45·78	72·22	92·51	109·61
754·3	7	29·29	49·51	78·10	100·04	118·54
880·0	6	32·05	54·15	85·43	109·43	129·66
1056·	5	35·08	60·15	94·89	121·54	144·02
1320·	4	40·40	68·29	107·73	137·99	163·51
1760·	3	47·48	80·25	126·61	162·17	192·16
2640·	2	59·47	100·53	158·59	203·14	240·70
5280·	1	88·13	148·97	235·02	301·04	356·70

See p. 208.

TABLE VIII.—For finding the Mean Velocities of Water flowing in Pipes, Drains, Streams, and Rivers.—For a full cylindrical pipe, divide the diameter by 4 to find the hydraulic mean depth.

Diameters of pipes 2½ inches to 5 inches. Falls per mile 1 *inch to* 12 *feet.*

Falls per mile in feet and inches, and the hydraulic inclinations.		"Hydraulic mean depths." or "mean radii," and velocities in inches per second.				
Falls.	Inclinations one in	⅝ inch.	¾ inch.	⅞ inch.	1 inch.	1¼ in. interpolated.
F. I.						
0 1	63360	·65	·73	·79	·85	·96
0 2	31680	1·02	1·13	1·23	1·33	1·49
0 3	21120	1·30	1·45	1·58	1·70	1·91
0 4	15840	1·54	1·71	1·87	2·01	2·26
0 5	12672	1·76	1·95	2·13	2·29	2·58
0 6	10560	1·95	2·17	2·36	2·55	2·86
0 7	9051	2·13	2·37	2·58	2·78	3·13
0 8	7920	2·30	2·55	2·78	3·00	3·37
0 9	7040	2·46	2.73	2·98	3·21	3·61
0 10	6336	2·61	2·90	3·16	3·40	3·83
0 11	5760	2·76	3·06	3·33	3·59	4·04
1 0	5280	2·89	3·21	3·50	3·77	4·24
1 3	4224	3·28	3·64	3·97	4·27	4·81
1 6	3520	3·63	4·03	4·39	4·73	5·32
1 9	3017	3·96	4.39	4·79	5·16	5·80
2 0	2640	4·26	4·73	5·16	5·55	6·25
2 3	Interpolated.	4·55	5·04	5·50	5·92	6·66
2 6	2112	4·83	5·35	5·84	6·29	7·07
2 9	Interpolated.	5·09	5·64	6·15	6·12	7·46
3 0	1760	5·34	5·92	6·46	6·96	7·83
3 3	Interpolated.	5·58	6·19	6·75	7·27	8·18
3 6	1508	5·82	6·46	7·04	7·59	8·53
3 9	Interpolated.	6·05	6·71	7·31	7·88	8·86
4 0	1320	6·27	6·95	7·58	8·17	9·19
4 6	Interpolated.	6·69	7·42	8·09	8·71	9·80
5 0	1056	7·10	7·88	8·59	9·25	10·41
5 6	Interpolated.	7·48	8·30	9·05	9·76	10·97
6 0	880	7·86	8·72	9·51	10·25	11·53
6 6	Interpolated.	8·22	9·12	9.94	10·71	12·05
7 0	754	8·57	9·51	10·37	11·17	12·57
7 6	Interpolated.	8·92	9·89	10·78	11·62	13·06
8 0	660	9·24	10·25	11·18	12·04	13·54
8 6	Interpolated.	9·55	10·60	11·56	12·45	14·01
9 0	587	9·87	10·95	11·94	12·86	14·47
9 6	Interpolated.	10·18	11·28	12·31	13·26	14.91
10 0	528	10·48	11·62	12·67	13·65	15·36
10 6	Interpolated.	10·77	11·95	13·03	14·03	15·78
11 0	480	11·06	12·27	13·38	14·41	16·21
11 6	Interpolated.	11·34	12·58	13·72	14·82	16·64
12 0	440	11·62	12·89	14·05	15·22	17·07

TABLE VIII.—For finding the Mean Velocity of Water flowing in Pipes, Drains, Streams, and Rivers.—For a full cylindrical pipe, divide the diameter by 4 to find the hydraulic mean depth.

Diameters of pipes 2½ *inches to* 5 *inches. Falls per mile* 13 *feet to* 5280 *feet.*

Falls per mile in feet and inches, and the hydraulic inclinations.		“Hydraulic mean depths,” or “mean radii,” and velocities in inches per second.				
Falls.	Inclinations one in	⅝ inch.	¾ inch.	⅞ inch.	1 inch.	1¼ in. interpolated.
F.						
13·2	400	12·26	13·60	14·83	15·98	17·98
13·6	Interpolated.	12·49	13·86	15·11	16·28	18·31
14·1	375	12·72	14·11	15·39	16·58	18·65
14·6	Interpolated.	12·98	14·39	15·70	16·91	19·02
15·1	350	13·23	14·68	16·00	17·24	19·40
15·6	Interpolated.	13·52	14·99	16·35	17·62	19·81
16·2	325	13·80	15·31	16·79	17·99	20·23
17·6	300	14·45	16·02	17·48	18·83	21·18
19·2	275	15·19	16·85	18·37	19·79	22·26
21·1	250	16·04	17·80	19·40	20·91	23·52
23·5	225	17·05	18·91	20·62	22·21	24·99
26·4	200	18·25	20·24	22·07	23·78	26·75
30·2	175	19·71	21·87	23·85	25·69	28·90
35·2	150	21·57	23·92	26·09	28·11	31·62
37·7	140	22·45	24·91	27·16	29·26	32·92
42·2	125	23·99	26·62	29·03	31·27	35·18
48·	110	25·87	28·69	31·29	33·71	37·92
52·8	100	27·36	30·35	33·10	35·66	40·11
58·7	90	29·12	32·31	35·23	37·96	42·69
66·	80	31·23	34·64	37·78	40·70	45·79
75·4	70	33·82	37·51	40·91	44·07	49·58
88·0	60	37·08	41·13	44·86	48·33	54·36
105·6	50	41·37	45·78	50·04	53·91	60·65
117·3	45	44·08	48·89	53·32	57·44	64·62
132·	40	47·32	52·49	57·25	61·67	69·37
150·8	35	51·30	56·90	62·06	66·86	75·20
176·	30	56·32	62·47	68·13	73·40	82·56
211·2	25	62·90	69·77	76·09	81·97	92·21
264·	20	72·01	79·87	87·11	93·84	105·56
352·	15	85·68	95·05	103·66	111·67	125·61
528·	10	109·26	121·19	132·17	142·39	160·17
586·7	9	116·31	129·01	140·70	151·58	170·50
660·	8	124·68	138·30	150·83	162·49	182·78
754·3	7	134·84	149·57	163·12	175·73	197·67
880·	6	147·69	163·60	178·42	192·22	216·22
1056·	5	163·82	181·71	198·17	213·50	240·15
1320·	4	185·99	206·31	225·00	242·39	272·66
1760·	3	218·58	242·46	264·42	284·86	320·43
2640·	2	273·79	303·70	331·22	356·82	401·37
5280·	1	405·74	450·07	490·84	528·76	594·82

TABLE VIII.—For finding the Mean Velocities of Water flowing in Pipes, Drains, Streams, and Rivers.—For a full cylindrical pipe, divide the diameter by 4 to find the hydraulic mean depth. OPEN DRAINS AND PIPES.—*Diameters of pipes* 6 *inches to* 12 *inches. Falls per mile* 1 *inch to* 12 *feet.*

Falls per mile in feet and inches, and the hydraulic inclinations.			"Hydraulic mean depths," or "mean radii," and velocities in inches per second.				
Falls.		Inclinations one in	1½ inch.	1¾ in. interpolated.	2 inches.	2½ inches	3 inches.
F.	I.						
0	1	63360	1·07	1·15	1·24	1·40	1·55
0	2	31680	1·66	1·80	1·94	2·19	2·41
0	3	21120	2·12	2·30	2·48	2·80	3·08
0	4	15840	2·52	2·73	2·94	3·34	3·65
0	5	12672	2·86	3·11	3·35	3·77	4·16
0	6	10560	3·18	3·45	3·72	4·19	4·62
0	7	9051	3·47	3·77	4·06	4·58	5·04
0	8	7920	3·75	4·06	4·38	4·94	5·44
0	9	7040	4·01	4·34	4·68	5·28	5·81
0	10	6336	4·25	4·61	4·97	5·60	6·17
0	11	5760	4·49	4·86	5·24	5·91	6·51
1	0	5280	4·71	5·11	5·51	6·21	6·84
1	3	4224	5·34	5·79	6·24	7·03	7·75
1	6	3520	5·91	6·41	6·91	7·79	8·58
1	9	3017	6·44	6·99	7·53	8·49	9·35
2	0	2640	6·94	7·53	8·11	9·14	10·07
2	3	Interpolated.	7·40	8·03	8·65	9·74	10·74
2	6	2112	7·86	8·52	9·18	10·35	11·40
2	9	Interpolated.	8·28	8·98	9·67	10·90	12·01
3	0	1760	8·70	9·43	10·16	11·45	12·62
3	3	Interpolated.	9·09	9·85	10·62	11·97	13·19
3	6	1508	9·48	10·28	11·08	12·48	13·76
3	9	Interpolated.	9·84	10·67	11·50	12·96	14·29
4	0	1320	10·21	11·07	11·93	13·44	14·81
4	6	Interpolated.	10·89	11·80	12·72	14·34	15·80
5	0	1056	11·56	12·54	13·51	15·23	16·78
5	6	Interpolated.	12·18	13·21	14·24	16·04	17·68
6	0	880	12·80	13·88	14·96	16·86	18·58
6	6	Interpolated.	13·38	14·51	15·64	17·62	19·42
7	0	754	13·96	15·14	16·32	18·39	20·26
7	6	Interpolated.	14·51	15·73	16·95	19·10	21·05
8	0	660	15·05	16·32	17·58	19·82	21·84
8	6	Interpolated.	15·56	16·87	18·18	20·49	22·58
9	0	587	16·07	17·43	18·78	21·17	23·32
9	6	Interpolated.	16.57	17·97	19·36	21·82	24·04
10	0	528	17·06	18·50	19·94	22·47	24·76
10	6	Interpolated.	17·54	19·01	20·49	23·09	25·45
11	0	480	18·01	19·53	21·04	23·72	26·13
11	6	Interpolated.	18·47	20·02	21·57	24·32	26·79
12	0	440	18·92	20·51	22·11	24·91	27·45

TABLE VIII.—For finding the Mean Velocities of Water flowing in Pipes, Drains, Streams, and Rivers.—For a full cylindrical pipe, divide the diameter by 4 to find the hydraulic mean depth.

Diameters of pipes 6 *inches to* 14 *inches.* *Falls per mile* 13 *feet to* 5280 *feet.*

Falls per mile in feet, and the hydraulic inclinations.		"Hydraulic mean depths," or "mean radii," and velocities in inches per second.				
Falls.	Inclinations, one in	1½ inches	2 inches.	2½ inches	3 inches.	3½ inches
F.						
13·2	400	19·97	23·34	26·30	28·98	31·44
13·6		20·34	23·77	26·79	29·52	32·03
14·1	375	20·72	24·21	27·28	30·06	32·62
14·6		21·13	24·69	27·83	30·67	33·27
15·1	350	21·55	25·18	28·38	31·27	33·93
15·6		22·01	25·72	28·99	31·94	34·66
16·2	325	22·48	26·27	29·60	32·62	35·39
17·6	300	23·53	27·50	30·99	34·15	37·05
19·2	275	24·74	28·90	32·57	35·89	38·94
21·1	250	26·13	30·53	34·41	37·91	41·14
23·5	225	27·76	32·44	36·56	40·28	43·71
26·4	200	29·72	34·72	39·13	43·12	46·79
30·2	175	32·11	37·52	42·28	46·59	50·55
35·2	150	35·13	41·04	46·26	50·97	55·30
37·7	140	36·57	42·73	48·16	53·07	57·58
42·2	125	39·08	45·66	51·46	56·71	61·53
48·	110	42·13	49·23	55·48	61·13	66·33
52·8	100	44·57	52·07	58·69	64·67	70·17
58·7	90	47·43	55·42	62·46	68·83	74·68
66·	80	50·87	59·44	66·99	73·81	80·09
75·4	70	55·08	64·36	72·50	79·92	86·72
88·	60	60·39	70·57	79·53	87·63	95·09
105·6	50	67·38	78·73	88·73	97·77	106·08
117·3	45	71·79	83·88	94·54	104·17	113·03
132·	40	77·07	90·06	101·50	118·84	121·35
150·8	35	83·55	97·63	110·03	121·24	131·55
176·	30	91·72	107·18	120·79	133·10	144·41
211·2	25	102·44	119·70	134·90	148·65	161·29
264·	20	117·28	137·03	154·44	170·18	184·65
352·	15	139·56	163·06	183·78	202·50	219·72
528·	10	177·95	207·92	234·33	258·21	280·16
586·7	9	189·43	221·34	249·45	274·87	298·24
660·	8	203·07	237·28	267·42	294·67	319·72
754·3	7	219·61	256·61	289·20	318·67	345·77
880·	6	240·22	281·36	316·33	348·57	378·20
1056·	5	266·81	311·75	351·35	387·15	420·07
1320·	4	302·92	353·95	398·91	439·55	476·93
1760·	3	356·00	415·96	468·80	516·57	560·49
2640·	2	445·93	521·04	587·22	647·06	702·08
5280·	1	660·84	772·16	870·23	958·91	1040·44

TABLE VIII.—For finding the Mean Velocities of Water flowing in Pipes, Drains, Streams, and Rivers.—For a full cylindrical pipe, divide the diameter by 4 to find the hydraulic mean depth.

Diameters of pipes 14 *inches to* 22 *inches.* *Falls per mile* 1 *inch to* 12 *feet.*

Falls per mile in feet and inches, and the hydraulic inclinations.			" Hydraulic mean depths," or " mean radii," and velocities in inches per second.				
Falls.		Inclinations, one in	3½ inches	4 inches.	4½ inches	5 inches.	5½ inches
F.	I.						
0	1	63360	1·68	1·80	1·91	2·02	2·13
0	2	31680	2·61	2·81	2·98	3·15	3·32
0	3	21120	3·34	3·59	3·82	4·03	4·24
0	4	15840	3·96	4·25	4·52	4·78	5·02
0	5	12672	4·51	4·84	5·15	5·44	5·72
0	6	10560	5·01	5·37	5·72	6·04	6·35
0	7	9051	5·47	5·87	6·24	6·60	6·94
0	8	7920	5·90	6·33	6·74	7·12	7·48
0	9	7040	6·31	6·77	7·20	7·61	8·00
0	10	6336	6·70	7·18	7·64	8·08	8·49
0	11	5760	7·06	7·58	8·06	8·52	8·96
1	0	5280	7·42	7·96	8·47	8·95	9·41
1	3	4224	8·41	9·02	9·60	10·14	10·66
1	6	3520	9·31	9·99	10·63	11·23	11·80
1	9	3017	10·15	10·89	11·58	12·24	12·86
2	0	2640	10·93	11·73	12·47	13·18	13·86
2	3	Interpolated.	11·65	12.50	13·30	14·05	14·77
2	6	2112	12·37	13·28	14·12	14·93	15·69
2	9	Interpolated.	13·03	13·68	14·88	15·72	16·53
3	0	1760	13·69	14·69	15·63	16·52	17·36
3	3	Interpolated.	14·31	15·35	16·33	17·26	18·14
3	6	1508	14·92	16·01	17·03	18·00	18·92
3	9	Interpolated.	15·50	16·63	17·69	18·70	19·65
4	0	1320	16·07	17·25	18·35	19·39	20·38
4	6	Interpolated.	17·14	18·39	19·56	20·68	21·73
5	0	1056	18·21	19·53	20·78	21·96	23·08
5	6	Interpolated.	19·18	20·58	21·90	23·14	24·32
6	0	880	20·16	21·63	23·01	24·32	25·56
6	6	Interpolated.	21·07	22·61	24·05	25·42	26·72
7	0	754	21·98	23·59	25·09	26·52	27·87
7	6	Interpolated.	22·84	24·50	26·07	27·55	28·96
8	0	660	23·69	25·42	27·04	28·58	30·04
8	6	Interpolated.	24·50	26·29	27·97	29·55	31·06
9	0	587	25·31	27·54	28·89	30·53	32·09
9	6	Interpolated.	26·09	27·99	29·78	31·47	33·08
10	0	528	26·87	28·83	30·67	32·41	34·06
10	6	Interpolated.	27·61	29·62	31·52	33·31	35·01
11	0	480	28·35	30·42	32·37	34·20	35·95
11	6	Interpolated.	29·07	31·19	33·18	35·07	36·86
12	0	440	29·79	31·96	34·00	35·93	37·77

TABLE VIII.—For finding the Mean Velocities of Water flowing in Pipes, Drains, Streams, and Rivers.—For a full cylindrical pipe, divide the diameter by 4 to find the hydraulic mean depth.
Diameters of pipes 16 *inches to* 2 *feet. Falls per mile* 13 *feet to* 5280 *feet.*

Falls per mile in feet and the hydraulic inclinations.		"Hydraulic mean depths," or "mean radii," and velocities in inches per second.				
Falls.	Inclination, one in	4 inches.	4½ inches	5 inches.	5½ inches	6 inches.
F.						
13·2	400	33·74	35·89	37·93	39·87	41·72
13·6	Interpolated.	34·37	36·56	38·64	40·61	42·50
14·1	375	35·00	37·23	39·35	41·36	43·28
14·6	Interpolated.	35·70	37·98	40·14	42·19	44·15
15·1	350	36·40	38·73	40·92	43·02	45·01
15·6	Interpolated.	37·19	39·56	41·81	43·94	45·99
16·2	325	37·97	40·40	42·69	44·87	46·96
17·6	300	39·75	42·29	44·69	46·97	49·16
19·2	275	41·78	44·45	46·97	49·38	51·67
21·1	250	44·14	46·95	49·62	52·16	54·58
23·5	225	46·90	49·90	52·72	55·42	58·00
26·4	200	50·20	53·41	56·44	59·32	62·08
30·2	175	54·24	57·71	60·98	64·10	67·07
35·2	150	59·34	63·13	66·71	70·12	73·37
37·7	140	61·78	65·72	69·45	73·00	76·39
42·2	125	66·02	70·23	74·22	78·01	81·64
48·	110	71·17	75·72	80·01	84·10	88·00
52·8	100	75·29	80·09	84·64	88·97	93·10
58·7	90	80·13	85·25	90·08	94·69	99·09
66·	80	85·93	91·42	96·61	101·54	106·26
75·4	70	93·04	98·98	104·60	109·95	115·05
88·	60	102·02	108·54	114·70	120·56	126·16
105·6	50	113·82	121·09	127·96	134·50	140·74
117·3	45	121·27	129·01	136·34	143·30	149·96
132·	40	130·20	138·51	146·38	153·86	161·00
150·8	35	141·14	150·16	158·68	166·79	174·53
176·	30	154·95	164·84	174·20	183·10	191·61
211·2	25	173·05	184·10	194·56	204·50	214·00
264·	20	198·12	210·77	222·73	234·11	244·98
352·	15	235·75	250·80	265·04	278·58	291·52
528·	10	300·60	319·80	337·95	355·22	371·71
586·7	9	320·00	340·43	359·76	378·14	395·70
660·	8	343·04	359·65	385·67	405·37	424·20
754·3	7	370·99	394·68	417·08	438·39	458·76
880·	6	405·79	431·70	456·21	479·52	501·79
1056·	5	450·71	479·49	506·71	532·60	557·34
1320·	4	511·72	544·39	575·30	604·69	632·78
1760·	3	601·38	639·78	676·10	710·64	743·65
2640·	2	753·29	801·39	846·89	890·16	931·50
5280·	1	1116·35	1187·62	1255·04	1319·17	1380·44

TABLE VIII.—For finding the Mean Velocities of Water flowing in Pipes, Drains, Streams, and Rivers.—The hydraulic mean depth is found for all channels, by dividing the wetted perimeter into the area.

Hydraulic mean depths 6 *inches to* 10 *inches. Falls per mile* 1 *inch to* 12 *feet.*

Falls per mile in feet and inches, and the hydraulic inclinations.			"Hydraulic mean depths," or "mean radii," and velocities in inches per second.				
Falls.		Inclinations, one in	6 inches.	7 inches.	8 inches.	9 inches.	10 inches.
F.	I.						
0	1	63360	2·23	2·41	2·58	2·75	2·90
0	2	31680	3·47	3·76	4·03	4·28	4·52
0	3	21120	4·43	4·80	5·15	5·47	5·78
0	4	15840	5·26	5·69	6·10	6·49	6·85
0	5	12672	5·98	6·48	6·95	7·39	7·80
0	6	10560	6·65	7·20	7·72	8·20	8·66
0	7	9051	7·26	7·86	8·43	8·96	9·46
0	8	7920	7·83	8·48	9·09	9·67	10·21
0	9	7040	8·37	9·07	9·72	10·33	10·91
0	10	6336	8·88	9·63	10·32	10·97	11·58
0	11	5760	9·37	10·16	10·89	11·57	12·22
1	0	5280	9·84	10·67	11·43	12·15	12·83
1	3	4224	11·16	12·09	12·95	13·77	14·54
1	6	3520	12·35	13·38	14·34	15·25	16·10
1	9	3017	13·46	14·58	15·63	16·61	17·54
2	0	2640	14·50	15·71	16·84	17·90	18·90
2	3	Interpolated.	15·45	16·75	18·24	19·08	20·15
2	6	2112	16·42	17·79	19·64	20·26	21·40
2	9	Interpolated.	17·29	18·74	20·37	21·34	22·54
3	0	1760	18·17	19·69	21·10	22·42	23·68
3	3	Interpolated.	18·99	20·57	22·05	23·43	24·75
3	6	1508	19·80	21·46	23·00	24·44	25·81
3	9	Interpolated.	20·56	22·28	23·88	25·38	26·80
4	0	1320	21·33	23·11	24·77	26·32	27·80
4	6	Interpolated	22·74	24·64	26·41	28·07	29·64
5	0	1056	24·16	26·17	28·05	29·81	31·48
5	6	Interpolated.	25·45	27·58	29·56	31·42	33·17
6	0	880	26·75	28·98	31·06	33·02	34·86
6	6	Interpolated.	27·96	30·29	32·47	34·51	36·44
7	0	754	29·17	31·60	33·87	36·00	38·02
7	6	Interpolated.	30·30	32·83	35·19	37·40	39·50
8	0	660	31·43	34·06	36·50	38·80	40·97
8	6	Interpolated.	32·51	35·22	37·75	40·12	42·37
9	0	587	33·58	36·39	38·99	41·45	43·77
9	6	Interpolated.	34·61	37·50	40·20	42·72	45·11
10	0	528	35·65	38·63	41·40	44·00	46·46
10	6	Interpolated.	36·63	39·69	42·54	45·22	47·75
11	0	480	37·62	40·76	43·69	46·44	49·03
11	6	Interpolated.	38·57	41·79	44·79	47·61	50·27
12	0	440	39·52	42·82	45·90	48·78	51·51

TABLE VIII.—For finding the Mean Velocities of Water flowing in Pipes, Drains, Streams, and Rivers.—The hydraulic mean depth is found for all channels by dividing the wetted perimeter into the area.

Hydraulic mean depths 11 *inches to* 21 *inches. Falls per mile* 1 *inch to* 12 *feet.*

Falls per mile in feet and inches, and the hydraulic inclinations.			"Hydraulic mean depths," or "mean radii," and velocities in inches per second.				
Falls.		Inclinations, one in	11 inches	12 inches.	15 inches.	18 inches.	21 inches.
F.	I.						
0	1	63360	3·05	3·19	3·57	3·92	4·25
0	2	31680	4·75	4·97	5·57	6·12	6·62
0	3	21120	6·07	6·35	7·12	7·82	8·46
0	4	15840	7·19	7·53	8·44	9·27	10·03
0	5	12672	8·19	8·57	9·61	10·55	11·42
0	6	10560	9·10	9·52	10·67	11·72	12·68
0	7	9051	9·94	10·39	11·66	12·80	13·85
0	8	7920	10·72	11·21	12·57	13·81	14·94
0	9	7041	11·46	11·99	13·44	14·76	15·97
0	10	6336	12·16	12·72	14·27	15·66	16·95
0	11	5760	12·83	13·42	15·05	16·53	17·88
1	0	5280	13·48	14·09	15·81	17·36	18·78
1	3	4224	15·27	15·97	17·91	19·67	21·28
1	6	3520	16·91	17·68	19·83	21·78	23·56
1	9	3017	18·23	19·27	21·62	23·73	25·68
2	0	2640	19·85	20·76	23·28	25·63	27·66
2	3	Interpolated.	21·16	22·13	24·82	27·29	29·49
2	6	2112	22·48	23·51	26·36	28·95	31·32
2	9	Interpolated.	23·68	24·76	27·77	30·49	32·99
3	0	1760	24·88	26·02	29·18	32·04	34·67
3	3	Interpolated.	25·99	27·18	30·47	33·48	36·22
3	6	1508	27·11	28·35	31·77	34·92	37·78
3	9	Interpolated.	28·15	29·45	33·01	36·26	39·23
4	0	1320	29·20	30·54	34·25	37·60	40·69
4	6	Interpolated.	31·13	32·56	36·52	40·10	43·39
5	0	1056	33·07	34·59	38·79	42·59	46·09
5	6	Interpolated.	34·85	36·44	40·87	44·88	48·56
6	0	880	36·62	38·30	42·95	47·16	51·03
6	6	Interpolated.	38·28	40·03	44·90	49·30	53·34
7	0	754	39·93	41·76	46·84	51·43	55·65
7	6	Interpolated.	41·48	43·39	48·66	53·43	57·81
8	0	660	43·04	45·01	50·48	55·42	59·97
8	6	Interpolated.	44·50	46·54	52·20	57·32	62·02
9	0	587	45·97	48·08	53·92	59·21	64·06
9	6	Interpolated.	47·39	49·56	55·58	61·03	66·04
10	0	528	48·80	51·04	57·24	62·85	68·01
10	6	Interpolated.	50·15	52·45	58·83	64·59	69·89
11	0	480	51·51	53·87	60·41	66·33	71·78
11	6	Interpolated.	52·81	55·23	61·94	68·01	73·59
12	0	440	54·11	56·59	63·47	69·68	75·40

TABLE VIII.—For finding the Mean Velocities of Water flowing in Pipes, Drains, Streams, and Rivers.—The hydraulic mean depth is found for all channels by dividing the wetted perimeter into the area.

Hydraulic mean depths 24 *inches to* 4 *feet. Falls per mile* 1 *inch to* 12 *feet.*

Falls per mile in feet and inches, and the hydraulic inclinations.			"Hydraulic mean depths," or "mean radii," and velocities in inches per second.				
Falls.		Inclinations, one in	24 inch.	30 inches.	36 inches.	42 inches.	48 inches.
F.	I.						
0	1	63360	4·54	5·09	5·59	6·04	6·47
0	2	31680	7·09	7·94	8·71	9·42	10·08
0	3	21120	9·06	10·15	11·14	12·04	12·89
0	4	15840	10·73	12·03	13·20	14·27	15·27
0	5	12672	12·22	13·69	15·03	16·25	17·39
0	6	10560	13·57	15·21	16·69	18·05	19·31
0	7	9051	14·83	16·61	18·23	19·71	21·09
0	8	7920	15·99	17·92	19·66	21·27	22·76
0	9	7041	17·10	19·16	21·02	22·73	24·33
0	10	6336	18·15	20·33	22·31	24·13	25·82
0	11	5760	19·15	21·45	23·54	25·46	27·24
1	0	5280	20·11	22·53	24·72	26·73	28·61
1	3	4224	22·78	25·53	28·01	30·29	32·42
1	6	3520	25·23	28·27	31·02	33·54	35·90
1	9	3017	27·49	30·81	33·80	36·55	39·12
2	0	2640	29·62	33·18	36·41	39·38	42·14
2	3	Interpolated.	31·57	35·38	38·82	41·98	44·92
2	6	2112	33·53	37·57	41·22	44·58	47·71
2	9	Interpolated.	35·32	39·58	43·43	46·96	50·26
3	0	1760	37·11	41·58	45·63	49·34	52·81
3	3	Interpolated.	38·78	43·45	47·68	51·56	55·18
3	6	1508	40·45	45·32	49·73	53·78	57·55
3	9	Interpolated.	42·00	47·07	51·64	55·85	59·77
4	0	1320	43·56	48·81	53·56	57·92	61·98
4	6	Interpolated.	46·45	52·05	57·11	61·76	66·09
5	0	1056	49·34	55·28	60·66	65·60	70·20
5	6	Interpolated.	51·99	58·25	63·91	69·12	73·97
6	0	880	54·63	61·22	67·17	72·64	77·74*
6	6	Interpolated.	57·11	63·99	70·21	75·93*	81·25
7	0	754	59·58	66·76	73·25	79·21	84·77
7	6	Interpolated.	61·89	69·35	76·09*	87·29	88·06
8	0	660	64·21	71·94	78·94	85·37	91·35
8	6	Interpolated.	66·40	74·40	81·63	88·26	94·47
9	0	587	68·59	76·85*	84·32	91·19	97·59
9	6	Interpolated.	70·60	79·22	86·92	94·00	100·59
10	0	528	72·81	81·58	89·52	96·81	103·60
10	6	Interpolated.	74·83	83·84	91·99	99·49	106·47
11	0	480	76·84*	86·10	94·47	102·17	109·33
11	6	Interpolated.	78·78	88·28	96·86	104·75	112·10
12	0	440	80·72	90·45	99·25	107·33	114·86

TABLE VIII.—For finding the Mean Velocities of Water flowing in Pipes, Drains, Streams, and Rivers.—The hydraulic mean depth is found for all channels by dividing the wetted perimeter into the area.

Hydraulic mean depths 4 *feet* 6 *inches to* 7 *feet. Falls per mile* 1 *inch to* 12 *feet.*

Falls per mile in feet and inches, and the hydraulic inclinations.			"Hydraulic mean depths," or "mean radii," and velocity in inches per second.				
Falls.		Inclinations, one in	54 inch.	60 inches.	66 inches.	72 inches.	84 inches.
F.	I.						
0	1	63360	6·86	7·24	7·60	7·94	8·58
0	2	31680	10·70	11·29	11·85	12·38	13·39
0	3	21120	13·68	14·62	15·14	15·83	17·11
0	4	15840	16·21	17·10	17·95	18·76	20·28
0	5	12672	18·46	19·47	20·43	21·35	23·13
0	6	10560	20·50	21·63	22·70	23·72	25·64
0	7	9051	22·39	23·62	24·79	25·90	28·00
0	8	7920	24·16	25·48	26·74	27·95	30·21
0	9	7041	25·83	27·24	28·59	29·88	32·30
0	10	6336	27·41	28·91	30·34	31·71	34·28
0	11	5760	28·92	30·51	32·01	33.46	36·17
1	0	5280	30·37	32·03	33·62	35·13	37·98
1	3	4224	34·41	36·30	38·10	39·81	43·04
1	6	3520	38·10	40·19	42·18	44·08	47·65
1	9	3017	41·52	43·80	45·97	48·04	51·93
2	0	2640	44·73	47·18	49·52	51·75	55·94
2	3	Interpolated.	47·69	50·30	52·79	55·17	59·64
2	6	2112	50·65	53·42	56·07	58·59	63·34
2	9	Interpolated.	53·35	56·28	59·06	61·72	66·72
3	0	1760	56·06	59·13	62·05	64·85	70·10
3	3	Interpolated.	58·57	61·79	64·84	67·76	73·25
3	6	1508	61·09	64·44	67·63	70·67	76·40*
3	9	Interpolated.	63·44	66·92	70·23	73·39	79·35
4	0	1320	65·80	69·41	72·84	76·11*	82·29
4	6	Interpolated.	70·16	74·01	77·67*	81·16	87·74
5	0	1056	74·52	78·61*	82·50	86·21	93·20
5	6	Interpolated.	78·52*	82·83	86·92	90·84	98·20
6	0	880	82·52	87·05	91·35	95·46	103·20
6	6	Interpolated.	86·25	90·98	95·58	99·78	107·87
7	0	754	89·99	94·92	99·62	104·10	112·54
7	6	Interpolated.	93·48	98·61	103·48	108·14	116·91
8	0	660	96·98	102·30	107·35	112·19	121·28
8	6	Interpolated.	100·29	105·79	111·02	116·01	125·42
9	0	587	103-59	109·27	114·68	119·84	129·56
9	6	Interpolated.	106·78	112·64	118·21	123·53	133·55
10	0	528	109·97	116·01	121·74	127·22	137·54
10	6	Interpolated.	113·02	119·22	125·11	130·74	141·34
11	0	480	116·06	122·43	128·48	134·27	145·15
11	6	Interpolated.	119·00	125·52	131·73	137·66	148·82
12	0	440	121·93	128·61	134·97	141·05	152·49

TABLE VIII.—For finding the Mean Velocities of Water flowing in Pipes, Drains, Streams, and Rivers.—The hydraulic mean depth is found for all channels by dividing the wetted perimeter into the area.

Hydraulic mean depths 8 *feet to* 12 *feet. Falls per mile* 1 *inch to* 12 *feet.*

Falls per mile in feet and inches, and the hydraulic inclinations.			"Hydraulic mean depths," or "mean radii," and velocities in inches per second.				
Falls.		Inclinations, one in	96 inch.	108 inches.	120 inches.	132 inches.	144 inches.
F.	I.						
0	1	63360	9·18	9·75	10·28	10·79	11·27
0	2	31680	14·32	15·20	16·03	16·82	17·57
0	3	21120	18·30	19·43	20·49	21·50	22·46
0	4	15840	21·69	23·02	24·28	25·47	26·62
0	5	12672	24·70	26·21	27·64	29·00	30·31
0	6	10560	27·43	29·11	30·70	32·21	33·66
0	7	9051	29·96	31·80	33·53	35·18	36·76
0	8	7920	32·32	34·30	36·18	37·96	39·66
0	9	7041	34·55	36·67	38·67	40·58	42·40
0	10	6336	36·67	38·92	41·04	43·07	45·00
0	11	5760	38·69	41·06	43·31	45·44	47·48
1	0	5280	40·63	43·12	45·48	47·72	49·86
1	3	4224	46·04	48·87	51·54	54·07	56·50
1	6	3520	50·98	54·11	57·06	59·87	62·56
1	9	3017	55·60	58·96	62·18	65·25	68·17
2	0	2640	59·85	63·52	66·98	70·28	73·44*
2	3	Interpolated.	63·80	67·72	71·41	74·93*	78·29
2	6	2112	67·76	71·91	75·84*	79·58	83·15
2	9	Interpolated.	71·38	75·75*	79·89	83·83	87·59
3	0	1760	75·00*	79·59	83·94	88·08	92·03
3	3	Interpolated.	78·37	83·17	87·71	92·03	96·16
3	6	1508	81·74	86·75	91·48	95·99	100·30
3	9	Interpolated.	84·88	90·09	95·01	99·69	104·16
4	0	1320	88·03	93·43	98·53	103·38	108·02
4	6	Interpolated.	93·87	99·62	105·06	110·24	115·18
5	0	1056	99·70	105·82	111·59	117·09	122·34
5	6	Interpolated.	105·06	111·49	117·58	123·38	128·91
6	0	880	110·41	117·17	123·57	129·66	135·48
6	6	Interpolated.	115·40	122·47	129·16	135·53	141·61
7	0	754	120·40	127·76	134·75	141·39	147·73
7	6	Interpolated.	125·07	132·74	139·99	146·88	153·47
8	0	660	129·75	137·70	145·22	152·38	159·21
8	6	Interpolated.	134·18	142·40	150·18	157·57	164·64
9	0	587	138·60	147·10	155·13	162·77	170·07
9	6	Interpolated.	142·87	151·63	159·91	167·78	175·31
10	0	528	147·14	156·16	164·68	172·80	180·55
10	6	Interpolated.	151·21	160·48	169·24	177·58	185·55
11	0	480	155·29	164·80	173·80	182·36	190·54
11	6	Interpolated.	159·21	168·97	178·19	186·97	195·36
12	0	440	163·13	173·13	182·59	191·58	200·17

TABLE IX.—For finding the Discharge in Cubic Feet per Minute, when the Diameter of a Pipe, or Orifice, and the Velocity of Discharge are known; and vice versâ.

Diameters of pipes in inches.	Discharge in cubic feet per minute, for different velocities.				
	Velocity of 100 inches per second.	Velocity of 200 inches per second.	Velocity of 300 inches per second.	Velocity of 400 inches per second.	Velocity of 500 inches per second.
¼	·170442	·3409	.5113	·6818	·8522
½	·68177	1·3635	2·0453	2·7271	3·4089
¾	1·53398	3·0679	4·6019	6·1359	7·6699
1	2·727077	5·4541	8·1812	10·9083	13·6354
1¼	4·26106	8·5221	12·7832	17·0442	21·3053
1½	6·13593	12·2718	18·4080	24·5437	30·6797
1¾	8·35167	16·7033	25·0550	33·4067	41·7584
2	10·90831	21·1817	32·7249	43·6332	54·5415
2¼	13·80583	27·6117	41·4175	55·2233	69·0291
2½	17·04423	34·0885	51·1327	68·1769	85·2212
2¾	20·62352	41·2470	61·8706	82·4941	103·1176
3	24·54369	49·0874	73·6311	98·1748	122·7185
3¼	28·80475	57·6095	86·4143	115·2190	144·0238
3½	33·40669	66·8134	100·2201	133·6268	167·0335
3¾	38·34952	76·6990	115·0486	153·3981	191·7476
4	43·63323	87·2665	130·8997	174·5329	218·1662
4¼	49·25783	98·5157	147·7735	197·0313	246·2892
4½	55·22331	110·4466	165·6699	220·8932	276·1166
4¾	61·52968	123·0594	184·5890	246·1187	307·6484
5	68·17692	136·3539	204·5308	272·7077	340·8846
5¼	75·16506	150·3301	225·4952	300·6603	375·8253
5½	82·49408	164·9882	247·4822	329·9763	412·4704
5¾	90·16399	180·3280	270·4920	360·6560	450·8200
6	98·17478	196·3495	294·5243	392·6991	490·8739
6¼	106·52645	213·0529	319·5794	426·1058	532·6323
6½	115·2190	230·4380	345·6570	460·8760	576·0950
6¾	124·25245	248·5049	372·7574	497·0098	621·2623
7	133·6268	267·2536	400·8804	534·5072	668·1340
7¼	143·34199	286·6840	430·0260	573.3680	716·7100
7½	153·39809	306·7962	460·1943	613·5924	766·9905
7¾	163·79507	327·5901	491·3852	655·1803	818·9753
8	174·53293	349·0659	523·5988	698·1317	872·6647
8½	197·03132	394·0626	591·0940	788·1253	985·1566
9	220·89325	441·7865	662·6798	883·5730	1104·4663
9½	246·11871	492·2374	738·3561	984·4784	1230·5936
10	272·70771	545·4154	818·1231	1090·8308	1363·5386
10½	300·66025	601·3205	901·9808	1202·6410	1503·3013
11	329·97633	659·9527	989·9290	1319·9053	1649·8817
11½	360·65595	721·3119	1081·9679	1442·6238	1803·2798
12	392·69910	785·3982	1178·0973	1570·7964	1963·4955

TABLE IX.—For finding the Discharge in Cubic Feet per Minute, when the Diameter of a Pipe, or Orifice, and the Velocity of Discharge are known; and vice versâ.

Discharge in cubic feet per minute, for different velocities.					Diameters of pipes in inches.
Velocity of 600 inches per second.	Velocity of 700 inches per second.	Velocity of 800 inches per second.	Velocity of 900 inches per second.	Velocity in 1000 inches per second.	
1·0227	1·1931	1·3635	1·5340	1·7044	¼
4·0906	4·7724	5·4542	6.1359	6·8177	½
9·2039	10·7379	12·2718	13·8058	15·3398	¾
16·3625	19·0895	21·8166	24·5437	27·2708	1
25·5664	29·8274	34·0885	38·3495	42·6106	1¼
36·8155	42·9515	49·0874	55·2234	61·3593	1½
50·1100	58·4617	66·8134	75·1650	83·5167	1¾
65·4499	76·3582	87·2665	98·1748	109·0831	2
82·8350	96·6408	110·4466	124·2525	138·0583	2¼
102·2654	119·3096	136·3538	153·3981	170·4423	2½
123·7411	144·3646	164·9882	185·6117	206·2352	2¾
147·2621	171·8059	196·3496	220·8933	245·4369	3
172·8285	201·6333	230·4380	259·2428	288·0475	3¼
200·4401	233·8468	267·2535	300·6602	334·0669	3½
230·0971	268·4467	306·7962	345·1457	383·4952	3¾
261·7994	305·4326	349·0659	392·6991	436·3323	4
295·5470	344·8048	394·0626	443·3205	492·5783	4¼
331·3399	386·5632	441·7865	497·0098	552·2331	4½
369·1781	430·7077	492·2374	553·7671	615·2968	4¾
409·0615	477·2384	545·4154	613·5923	681·7692	5
450·9904	526·1554	601·3205	676·4855	751·6506	5¼
494·9645	577·4586	659·9526	742·4467	824 9408	5½
540·9839	631·1479	721·3119	811·4759	901·6399	5¾
589·0486	687·2235	785·3982	883·5730	981·7478	6
639·1587	745·6852	852·2116	958·7381	1065·2645	6¼
691·3141	806·5330	921·7520	1036·9710	1152·1900	6½
745·5147	869·7672	994·0196	1118·2721	1242·5245	6¾
801·7608	935·3876	1069·0144	1202·6412	1336·2680	7
860·0519	1003·3939	1146·7359	1290·0779	1433·4199	7¼
920·3885	1073·7866	1227·1847	1380·5828	1533·9809	7½
982·7704	1146·5655	1310·3605	1474·1556	1637·9507	7¾
1047·1976	1221·7305	1396·2634	1570·7964	1745·3293	8
1182·1879	1379·2192	1576·2506	1773·2819	1970·3132	8½
1325·3595	1546·2528	1767·1460	1988·0393	2208·9325	9
1476·7123	1722·8310	1968·9497	2215·0684	2461·1871	9½
1636·2463	1908·9540	2181·6617	2454·3694	2727·0771	10
1803·9615	2104·6218	2405·2820	2705·9423	3006·6025	10½
1979·8580	2309·8343	2639·8106	2969·7870	3299·7633	11
2163·9357	2524·5917	2885·2476	3245·9936	3606·5595	11½
2356·1946	2748·8937	3141·5928	3534·2919	3926·9910	12

TABLE X.—For finding the depths of Weirs of different lengths, the quantity discharged over each being supposed constant. See pages 289 and 290.

Ratios of lengths.	Coefficients.	Ratios of lengths.	Coefficients.	Ratios of lengths.	Coefficients.	Ratios of lengths.	Coefficients.
·01	·0464	·405	·5474	·605	·7153	·805	·8654
·02	·0737	·410	·5519	·610	·7193	·810	·8689
·03	·0965	·415	·5564	·615	·7232	·815	·8725
·04	·1170	·420	·5608	·620	·7271	·820	·8761
·05	·1357	·425	·5653	·625	·7310	·825	·8796
·06	·1533	·430	·5697	·630	·7349	·830	·8832
·07	·1699	·435	·5741	·635	·7388	·835	·8867
·08	·1857	·440	·5785	·640	·7427	·840	·8903
·09	·2008	·445	·5829	·645	·7465	·845	·8938
·10	·2154	·450	·5872	·650	·7504	·850	·8973
·11	·2296	·455	·5916	·655	·7542	·855	·9008
·12	·2433	·460	·5959	·660	·7580	·860	·9043
·13	·2566	·465	·6002	·665	·7619	·865	·9078
·14	·2696	·470	·6045	·670	·7657	·870	·9113
·15	·2823	·475	·6088	·675	·7695	·875	·9148
·16	·2947	·480	·6130	·680	·7733	·880	·9183
·17	·3069	·485	·6173	·685	·7771	·885	·9218
·18	·3188	·490	·6215	·690	·7808	·890	·9253
·19	·3305	·495	·6258	·695	·7846	·895	·9287
·20	·3420	·500	·6300	·700	·7884	·900	·9322
·21	·3533	·505	·6342	·705	·7921	·905	·9356
·22	·3644	·510	·6383	·710	·7959	·910	·9391
·23	·3754	·515	·6425	·715	·7996	·915	·9425
·24	·3862	·520	·6466	·720	·8033	·920	·9459
·25	·3969	·525	·6508	·725	·8070	·925	·9494
·26	·4074	·530	·6549	·730	·8107	·930	·9528
·27	·4177	·535	·6590	·735	·8144	·935	·9562
·28	·4280	·540	·6631	·740	·8181	·940	·9596
·29	·4381	·545	·6672	·745	·8218	·945	·9630
·30	·4481	·550	·6713	·750	·8255	·950	·9664
·31	·4580	·555	·6754	·755	·8291	·955	·9698
·32	·4678	·560	·6794	·760	·8328	·960	·9732
·33	·4775	·565	·6834	·765	·8365	·965	·9762
·34	·4871	·570	·6875	·770	·8401	·970	·9799
·35	·4966	·575	·6915	·775	·8437	·975	·9833
·36	·5061	·580	·6955	·780	·8474	·980	·9866
·37	·5154	·585	·6995	·785	·8510	·985	·9900
·38	·5246	·590	·7035	·790	·9546	·990	·9933
·39	·5338	·595	·7074	·795	·8582	·995	·9967
·40	·5429	·600	·7114	·800	·8618	1·000	1·0000

TABLE XI.—Mean relative Dimensions of equally Discharging Trapezoidal Channels, with Side Slopes varying from 0 *to* 1, *up to* 2 *to* 1.—Half sum of the top and bottom is the mean width, The ratio of the slope, multiplied by the depth, subtracted from the mean width, will give the bottom; and if added, will give the top. TABLE XII. gives the discharge in cubic feet per minute from the primary channel, 70 wide, and the corresponding depths taken in feet. For lesser or greater channels and discharges, see Rules, pp. 243 to 251, and 267 to 271.

The mean widths are given in the top horizontal line, and the corresponding depths in the other horizontal lines. They may be taken in inches, feet, yards, fathoms, or any other measures, whatever.

70	60	50	40	35	30	25	20	15	10
·125	·13	·15	·17	·20	·23	·26	·29	·35	·48
·25	·27	·30	·35	·40	·45	·52	·58	·71	·98
·375	·41	·46	·54	·60	·67	·76	·88	1·09	1·51
·5	·55	·62	·73	·80	·89	1·02	1·19	1·48	2·04*
·625	·68	·78	·91	1·00	1·12	1·29	1·50	1·88	2·62
·75	·82	·94	1·10	1·20	1·35	1·56	1·82	2·28	3·22*
·875	·96	1·10	1·29	1·41	1·58	1·83	2·14	2·69	3·86
1·	1·10	1·26	1·48	1·62	1·81	2·10	2·46	3·11	4·50
1·125	1·24	1·42	1·67	1·83	2·04	2.37	2·79	3·54	5·19*
1·25	1·39	1·58	1·86	2·04	2·28	2·65	3·12	3·98	5·89
1·375	1·53	1·74	2·05	2·25	2·51	2·92	3·46	4·43	6·60
1·5	1·67	1·90	2·24	2·46	2·75	3·20	3·80	4·88	7·31
1·625	1·81	2·06	2·43	2·67	2·99	3·47	4·15	5·34	8·08
1·75	1·95	2·22	2·62	2·88	3·23	3·75	4·50	5·80	8·86
1·875	2·09	2·38	2·81	3·09	3·47	4·03	4·86	6·29	9·68
2·	2·23	2·54	3·00	3·31	3·72	4·32	5·22	6·78	10·50
2·125	2·37	2·70	3·19	3·52	3·96	4·61	5·58	7·29	11·37
2·25	2·51	2·86	3·38	3·73	4·21	4·91	5·95	7·81*	12·25
2·375	2·65	3·02	3·57	3·94	4·45	5·20	6·31	8·32	13·12
2·5	2·79	3·18	3·76	4·16	4·70	5·50	6·68	8·84	14·00
2·625	2·93	3·34	3·95	4·38	4·95	5·79	7·06	9·38	14·92
2·75	3·07	3.51	4·15	4·60	5·21	6·09	7·45	9·93	15·84
2·875	3·21	3·67	4·34	4·82	5·46	6·39	7·83	10·48	16·76
3·	3·35	3·84	4·54	5·04	5·72	6·69	8·22	11·03	17·68
3·125	3·49	4·00	4·73	5·26	5·97	7·00	8·62	11·60	18·68
3·25	3·63	4·17	4·93	5·49	6·23	7·31	9·02	12·17	19·68
3·375	3·77	4·33	5·13	5·72	6·49	7·62	9·42	12·74	20·68
3·5	3·91	4·50	5·33	5·95	6·75	7·93	9·82	13·32	21·68
3·625	4·05	4·66	5·53	6·17	7·01	8·25	10·23*	13·92	22·76
3·75	4·19	4·82	5·73	6·40	7·28	8·57	10·65	14·53	23·84
3·875	4·33	4·98	5·93	6·62	7·54	8·89	11·06	15·14	24·92
4·	4·48	5·14	6·13	6·85	7·81	9·21	11·48	15·75	26·00
4·25	4·76	5·46	6·54	7·30	8·35	9·85	12·33	16·98	28·18
4·5	5·05	5·79	6·95	7·75	8·90	10·50	13·19	18·22	30·36
4·75	5·33	6·12	7·35	8·20	9·45	11·14	14·07	19·50	32·68
5·	5·62	6·45	7·75	8·66	10·00	11·79	14·96	20·80	35·00
5·25	5·90	6·78	8·16	9·14	10·55	12·51*	15·86	22·13	37·40
5·5	6·18	7·12	8·57	9·62	11·10	13·24	16·77	23·47	39·81
5·75	6·46	7·46	8·98	10·11	11·66	13·94	17·71	24·86	42·33
6·	6·75	7·80	9·40	10·60	12·22	14·65	18·65	26·25	44·86

For a similar Table, see p. 270.

TABLE XII.—Discharges from the Primary Channel in the first column of Table XI. If the dimensions of the primary channel be in inches, divide the discharges in this table by 500; if in yards, multiply by 15·6; if in quarters, multiply by 32; and if in fathoms, by 88·2, &c.: see pp. 243 to 251. The final figures in the discharges may be rejected when they do not exceed one-half per cent., or 0·5 in 100. See pages 267 to 271.

Depths of a channel whose mean width is 70:—in feet.	Falls, inclinations, and discharges in cubic feet per minute. Interpolate for intermediate falls; divide greater falls by 4, and double the corresponding discharges.						
	1 inch per mile, 1 in 63360.	2 inches per mile, 1 in 31680.	3 inches per mile, 1 in 21120.	6 inches per mile, 1 in 10560.	9 inches per mile, 1 in 7040.	12 inches per mile, 1 in 5280.	15 inches per mile, 1 in 4224.
·125	47	72	93	139	175	205	233
·25	136	210	268	403	506	596	675
·375	249	389	498	746	940	1105	1252
·50	387	603	770	1155	1454	1709	1935
·625	541	849	1078	1617	2036	2395	2714
·75	714	1112	1420	2128	2681	3153	3573
·875	900	1401	1791	2685	3382	3978	4507
1·	1100	1714	2190	3283	4134	4862	5507
1·125	1310	2042	2614	3909	4927	5792	6577
1·25	1534	2384	3058	4581	5766	6780	7690
1·375	1767	2757	3521	5279	6661	7823	8863
1·50	2013	3142	4006	6016	7588	8915	10099
1·625	2268	3540	4525	6781	8541	10044	11381
1·75	2534	3950	5053	7570	9537	11210	12703
1·875	2812	4384	5599	8386	10570	12429	14083
2·	3090	4821	6161	9230	11628	13675	15513
2·125	3377	5273	6738	10092	12718	14956	16943
2·25	3674	5736	7331	10981	13833	16281	18435
2·375	3977	6210	7937	11889	14981	17645	19960
2·50	4293	6699	8563	12829	16161	19045	21534
2·625	4616	7203	9204	13800	17380	20434	23135
2·75	4947	7716	9865	14782	18624	21886	24800
2·875	5280	8233	10525	15773	19887	23360	26473
3·	5621	8762	11204	16788	21165	24833	28176
3·125	5972	9310	11900	17830	22454	26410	29925
3·25	6329	9862	12614	18897	23780	27994	31714
3·375	6689	10420	13320	19963	25145	29570	33507
3·50	7049	10995	14048	21052	26509	31262	35329
3·625	7418	11574	14785	22153	27906	32860	37186
3·75	7794	12163	15526	23284	29321	34479	39080
3·875	8178	12753	16283	24416	30756	36170	41013
4·	8566	13354	17070	25592	32225	37898	42954
4·25	9355	14582	18643	27936	35191	41368	46916
4·50	10173	15849	20267	30366	38254	44982	50973
4·75	11001	17140	21908	32818	41356	48630	55102
5·	11833	18454	23595	35355	44546	52378	59346
5·25	12696	19802	25362	37939	47795	56209	63688
5·50	13576	21172	27248	40564	51097	60079	68097
5·75	14478	22580	29160	43253	54478	64058	72591
6·	15393	23995	31122	45969	57897	68082	77154

For a similar Table, see p. 271.

TABLE XII.—Discharges from the Primary Channel in the first column of Table XI. If the dimensions of the primary channel be in inches, divide the discharges in this table by 500; if in yards, multiply by 15·6, if in quarters, multiply by 32, and if in fathoms, by 88·2, etc.: see pp. 243 to 251. The final figures in the discharges may be rejected when they do not exceed one-half per cent., or 0·5 in 100. See pages 267 to 271.

Falls, inclinations, and discharges in cubic feet per minute. Interpolate for intermediate falls; divide greater falls by 4, and double the corresponding discharges.							Depths of a channel whose mean width is 70:—in feet.
18 inches per mile, 1 in 3520.	21 inches per mile, 1 in 3017.	24 inches per mile, 1 in 2640.	27 inches per mile, 1 in 2347.	30 inches per mile, 1 in 2112.	33 inches per mile, 1 in 1920.	36 inches per mile, 1 in 1760.	
258	281	303	323	343	362	380	·125
748	815	877	936	993	1049	1100	·25
1387	1511	1627	1736	1843	1952	2037	·375
2145	2336	2515	2684	2852	3023	3155	.50
3004	3274	3527	3753	4021	4207	4414	·625
3957	4311	4645	4966	5287	5553	5817	·75
4991	5422	5859	6274	6650	6992	7342	·875
6097	6622	7159	7631	8107	8540	8974	1·
7266	7920	8531	9124	9660	10200	10693	1·125
8514	9284	9995	10658	11318	11923	12520	1·25
9816	10697	11539	12307	13045	13741	14479	1·375
11182	12185	13152	14007	14862	15656	16448	1·50
12601	13730	14821	15786	16750	17657	18552	1·625
14069	15331	16525	17616	18700	19698	20696	1·75
15593	16997	18306	19517	20728	21840	22944	1·875
17157	18697	20141	21469	22803	24017	25242	2·
18766	20446	22030	23480	24938	26269	27601	2·125
20410	22247	23965	25547	27129	28578	30027	2·25
22104	24087	25947	27662	29395	30934	32512	2·375
23848	25988	27992	29841	31701	33381	35096	2·50
25669	27953	30100	32069	34086	35910	37725	2·625
27479	29933	32247	34384	36512	38471	40415	2·75
29318	31947	34408	36697	38958	41055	43135	2·875
31206	34002	36624	39050	41464	43680	45896	3·
33141	36112	38897	41482	44048	46398	48747	3·125
35126	38266	41223	43954	46672	49174	51664	3·25
37109	40438	43556	46438	49330	51951	54586	3·375
39140	42631	45925	48963	51993	54775	57550	3·50
41184	44872	48343	51537	54728	57659	60580	3·625
43273	47158	50807	54162	57514	60585	63656	3·75
45407	49468	53300	56840	60341	63560	66784	3·875
47551	51818	55832	59414	63200	66576	69951	4·
51911	56586	60973	64974	69013	72694	76383	4·25
56448	61508	66176	70623	75017	79017	82994	4·50
61014	66500	71625	76408	81097	85426	89767	4·75
65713	71628	77140	82250	87351	92015	96653	5·
70509	76863	82779	88200	93731	98729	103745	5·25
75383	82159	88434	94344	100200	105550	110905	5·50
80379	87590	94348	100616	106823	112540	118254	5·75
85407	93093	100275	106911	113505	119616	125664	6·

For a similar Table, see p. 271.

TABLE XIII.—The Square Roots of the fifth powers of numbers for finding the Diameter of a Pipe, or dimensions of a Channel from the Discharge, or the Reverse; showing the relative Discharging Powers of pipes of different Diameters, and of any similar Channels whatever, closed or open. See pages 17, 18, 245, etc.—If d be the diameter of a pipe, in feet, and D the discharge in cubic feet per minute, then for long straight pipes we shall have for velocities of nearly 3 feet per second, $D = 2400\,(d^5 s)^{\frac{1}{2}}$, and $d = \cdot 044 \left(\frac{D^2}{s}\right)^{\frac{1}{5}}$; or if D be the discharge per second, $D = 40\,(d^5 s)^{\frac{1}{2}}$, and $d = \cdot 228 \left(\frac{D^2}{s}\right)^{\frac{1}{5}}$.

Relative dimensions or diameters of pipes.	Relative discharging powers.	Relative dimensions or diameters of pipes.	Relative discharging powers.	Relative dimensions or diameters of pipes.	Relative discharging powers.	Relative dimensions or diameters of pipes.	Relative discharging powers.
·25	·031	10·5	357·2	30·5	5138·	61·	29062·
·5	·177	11·	401·3	31·	5351·	62·	30268·
·75	·485	11·5	448·5	31·5	5569·	63·	31503·
1·	1·	12·	498·8	32·	5793·	64·	32768·
1·25	1·747	12·5	552·4	32·5	6022·	65·	34063·
1·5	2·756	13·	609·3	33·	6256·	66·	35388·
1·75	4·051	13·5	669·6	33·5	6496·	67·	36744·
2·	5·657	14·	733·4	34·	6741·	68·	38131·
2·25	7·594	14·5	800·6	34·5	6991·	69·	39548·
2·5	9·882	15·	871·4	35·	7247·	70·	40996·
2·75	12·541	15·5	945·9	35·5	7509·	71·	42476·
3·	15·588	16·	1024·	36·	7776·	72·	43988·
3·25	19·042	16·5	1105·9	36·5	8049·	73·	45531·
3·5	22·918	17·	1191·6	37·	8327·	74·	47106·
3·75	27·232	17·5	1281·1	37·5	8611·	75·	48714·
4·	32·	18·	1374·6	38·	8901·	76·	50354·
4·25	37·24	18·5	1472·1	38·5	9197·	77·	52027·
4·5	42·96	19·	1573·6	39·	9498·	78·	53732·
4.75	49·17	19·5	1679·1	39·5	9806·	79·	55471·
5·	55·90	20·	1788·9	40·	10119·	80·	57243·
5·25	63·15	20·5	1902·8	41·	10764·	81·	59049·
5·5	70·94	21·	2020·9	42·	11432·	82·	60888·
5·75	79·28	21·5	2143·4	43·	12125·	83·	62762·
6·	88·18	22·	2270·2	44·	12842·	84·	64669·
6·25	97·66	22·5	2401·4	45·	13584·	85·	66611·
6·5	107·72	23·	2537·	46·	14351·	86·	68588·
6·75	118·38	23·5	2677·1	47·	15144·	87·	70599·
7·	129·64	24·	2821·8	48·	15963·	88·	72645·
7·25	141·53	24·5	2971·1	49·	16807·	89·	74727·
7·5	154·05	25·	3125·	50·	17678·	90·	76843·
7·75	167·21	25·5	3283·6	51·	18575·	91·	78996·
8·	181·02	26·	3446·9	52·	19499·	92·	81184·
8·25	195·50	26·5	3615·1	53·	20450·	93·	83408·
8·5	210·64	27·	3788·	54·	21428·	94·	85668·
8·75	226·48	27·5	3965·8	55·	22434·	95·	87965·
9·	243·	28·	4148·5	56·	23468·	96·	90298·
9·25	260·23	28·5	4336·2	57·	24529·	97·	92668·
9·5	278·17	29·	4528·9	58·	25620·	98·	95075·
9·75	296·83	29·5	4726·7	59·	26738·	99·	97519·
10·	316·23	30·	4929·5	60·	27886·	100·	100000·

See Tables at pp. 28 and 29.

TABLE XIV.—Weights and Measures, English and French, with their relative values.

MEASURES OF LENGTH.

12 inches		1 foot.
7·92 inches		1 link.
3 feet		1 yard.
5½ yards	= 16½ feet	1 pole or perch.
100 links	= 22 yards.	1 chain = 4 perches.
40 perches	= 220 yards	1 furlong.
8 furlongs	= 1760 yards	1 mile.
6 feet		1 fathom.
120 fathoms		1 cable's length.
1 Nautical mile		6082·7 feet.
69·12 miles		1 Geographical deg.
3 miles		1 league.

The Irish perch is 21 feet, or seven yards. Three inches make a palm; 4 inches a hand; 5 feet a pace. In cloth measure 2¼ inches =1 nail; 4 nails=1 quarter; 4 quarters=1 yard. 11 Irish miles = 14 English.

MEASURES OF SURFACE.

144 square inches		1 square foot.
62·7264 ,,		1 square link.
9 square feet		1 square yard.
30¼ square yards	= 272¼ square feet	1 square perch.
10,000 square links	= 4,356 ,,	1 square chain.
10 square chains	= 160 square perches	1 acre.
1 rood	= 210 square yards	40 perches.
4 roods	= 4,840 ,,	1 acre.
640 acres	= 3,097,600 ,,	1 square mile.

The Irish perch is 49 square yards, or 441 square feet; 1 Irish acre = 1a. 2r. 19·17p. statute; and 1 statute acre = 0a. 2r. 18·77p. Irish. The Irish acre is to the English acre as 196 is to 121. 100 square feet is a square of roofing, slating, or flooring. The Cunningham acre is = 1a. 1r. 6·61p. English; and 1 English acre is = 0a. 3r. 3·904p. Cunningham measure.

TABLE XIV.—*Continued.*

CUBIC MEASURES, AND MEASURES OF CAPACITY AND WEIGHT.

1728 cubic inches	1 cubic foot.
27 cubic feet	1 cubic yard.
$16\frac{1}{2} \times 1\frac{1}{2} \times 1 = 24·75$ cubic feet	1 perch of masonry.
$16\frac{1}{2} \times 16\frac{1}{2} \times 1\frac{1}{8} = 306$ cubic feet	1 rod of brickwork.
$21 \times 1\frac{1}{2} \times 1 = 30\frac{1}{2}$ cubic feet	1 Irish perch of masonry.

The standard gallon, imperial measure, contains 10 lbs. avoirdupois, of distilled water at 62° Fahrenheit, the barometer standing at 30 inches.

6·232 gallons	1 cubic foot.
8·665 cubic inches	1 gill.
4 gills 34·659 cubic inches	1 pint.
2 pints 69·318 cubic inches	1 quart.
2 quarts 138·637 cubic inches	1 pottle.
2 pottles 277·274 cubic inches	1 gallon.
2 gallons 554·548 cubic inches	1 peck.
4 pecks 2218·191 cubic inches	1 bushel.

The old Irish gallon contained 217·6 cubic inches, nearly, and 1 Irish gallon is therefore = ·7850 imperial gallon. The Irish barrel of lime still measures 40 Irish gallons, or 31·321 imperial gallons, or 4 bushels, very nearly. It is measured by a cylindrical measure 12 inches high, and about 21½ inches in diameter, containing half the Irish barrel. In the old English liquid measures for ale and beer, 36 gallons = 1 barrel = 36 gallons, 3½ quarts, imperial measure, nearly.

For old dry measures, 32 bushels = 1 chaldron = 31 bushels, 1 pint, imperial measure, nearly.

And 36 bushels of coal = 1 chaldron of coal = 34 bushels, 3 pecks, and 1 gallon, imperial measure. The Irish barrel of wheat is 20 stone; barley, 16 stone; oats, 14 stone.

TROY WEIGHT.

24 grains	1 pennyweight.
20 pennyweights	1 ounce.
12 ounces	1 pound.

One pound Troy = 22·816 cubic inches of distilled water, barometer 0 inches; thermometer 62°.

TABLE XIV.—*Continued.*

APOTHECARY'S WEIGHT.

20 Troy grains	1 scruple.
3 scruples	1 drachm.
8 drachms	1 ounce.
12 ounces	1 pound.

The ounce weighs 480 grains, and the pound 5760 grains, both in Troy and Apothecary's weight.

AVOIRDUPOIS OR COMMERCIAL WEIGHT.

One pound Avoirdupois = 27·7274 cubic inches, when the barometer stands at 30 inches, and Fahrenheit's thermometer at 62°.

16 drachms	= 437·5 Troy grains	1 ounce.
16 ounces	= 7,000 Troy grains	1 pound.
14 pounds	= 98,000 Troy grains	1 stone.
8 stone	= 112 pounds	1 cwt.
20 cwt.	= 2,240 pounds	1 ton.

One pound Troy = ·82286 pounds Avoirdupois, and one pound Avoirdupois is equal to 1·2153 pounds Troy. One ton of water contains about 36 cubic feet, equal to 224 imperial gallons, nearly. Ten pounds of distilled water is equal to one gallon, the barometer and thermometer being as above stated.

FRENCH MEASURES AND WEIGHTS COMPARED WITH ENGLISH.

MEASURES OF LENGTH.

1 mètre	3·2808992 feet	1 foot English	·3047945 mètre
1 decimètre	·3280899 ,,	1 inch	·0253995 ,,
1 centimètre	·0328090 ,,	1 yard	·9143835 ,,
1 millimètre	·0032809 ,,	1 perch $5\frac{1}{2}$ yds.	5·0291092 ,,
1 kilomètre (or 1000 mètres)	·621383 mile	1 mile	1·60932 kilomètre

1000 mètres = 100 decimètres = 10 hectomètres = 1 kilomètre = 3280·849 feet. The mètre is the 10,000,000th part of a quadrental arc of the meridian or 39·3708 inches English.

TABLE XIV.—*Continued.*

MEASURES OF SURFACE.

1 centiare (one square mètre)	10·7643 sq. ft.	119·6033 sq. yds	1 are
1 deciare	107·6430 ,,	11·9603 ,,	1 deciare
1 are	1076·430 ,,	1·1960 ,,	1 centiare or sq. mètre.

100 ares = 10 deciares = 1 hectare = 2·471143 English acres, and 17 hectares are nearly equal to 42 English acres.

The old Paris foot is equal 1·06578 English feet; the French inch = 1·06578 English inches; the French line ·08882 of an English inch; the toise is equal to 6 French feet = 76·736 English inches = 6·39468 feet. The perch is 18 French feet; and the perch royal 22 French feet. The French square foot or inch = 1·13581 English square feet or inches, and the cubic foot or inch = 1·21061 English.

MEASURES OF SOLIDITY AND CAPACITY.

	Cubic feet.		Eng. cub. in.
1 millistere	·0353166	1 millilitre	·0610279
1 centistere	·353166	1 centilitre	·610279
1 decistere	3·53166	1 decilitre	6·10279
1 stere (one cubic mètre)	35·3166	1 litre	61·0279
1 decastere	353·166	1 decalitre	610·279
1 hectostere	3531·66	1 hectolitre	6102·79
1 kilostere	35316·6	1 kilolitre	61027·9

The stere and kilolitre are each a cubic mètre, and the litre is a cubic decimètre; 50 litres are nearly 11′ English gallons, and 1 hectolitre 2·751207 English bushels.

MEASURES OF WEIGHT.

·0648 gramme = 1 grain, and 7000 grains = 1 lb. Avoirdupois.

1 milligramme	·015432 grains	1 gramme	15·432 grains
1 centigramme	·15432 ,,	1 decagramme	·02205 lb. avoir.
1 decigramme	1·5432 ,,	1 hectogramme	·2205 ,,
1 gramme	15·432 ,,	1 killogramme	2·2046 ,,

1·01605 tonnes = 1 ton: and 1 tonne = ·984206 ton.

A gramme is the weight of a cubic centimètre of water and its maximum density in vacuo: 1 kilogramme = 2·6795 lbs. Troy = 2·2046 lbs. Avoirdupois. 1 metrical quintal 220·46 lbs. Avoirdupois, and 10 quintals is equal to the weight of a cubic mètre of water.

TABLE XIV.—*Continued.*

CIRCULAR FRENCH AND BRITISH DIVISIONS.

French to British.

$$\left\{\begin{array}{lll} 1^\circ \text{ circular measure French} & & = \cdot 9 \text{ degree British.} \\ 1 \text{ minute} & = \dfrac{\cdot 9 \times 60}{100} & = \cdot 54 \text{ British minutes.} \\ 1 \text{ second} & = \dfrac{\cdot 9 \times 60 \times 60}{100 \times 100} & = \cdot 324 \text{ British seconds.} \end{array}\right.$$

British to French.

$$1^\circ \text{ circular} = \frac{100}{90} = 1\cdot\dot{1} = 1\tfrac{1}{9} \text{ French degrees.}$$

$$1 \text{ minute} = \frac{100}{\cdot 9 \times 60} = 1\cdot\dot{8}5\dot{1} = 1\tfrac{23}{27} \text{ French minutes.}$$

$$1 \text{ second} = \frac{100}{\cdot 9 \times 60} \times \frac{100}{60} = \frac{100 \times 100}{\cdot 9 \times 60 \times 60} = 3\cdot\dot{0}864\dot{2} = 3\tfrac{14}{162} = 3\tfrac{7}{81} \text{ seconds.}$$

TO REDUCE FRENCH TO BRITISH CIRCULAR MEASURES AND THE REVERSE.

Put n = number of British degrees, &c. and n_f = French, &c.

Then $n_e = n_f - \dfrac{n_f}{10}$ and $n_f = n_e + \dfrac{n_e}{9}$.

EXAMPLE 1. Given 309° 57′ 90″ French. Reduce to British

309·57·90
deduct one-tenth 30·95·79

n_e = 278°·62·11 in decimal measures English
60

37·2660 English minutes and decimals
60

15·9600 English seconds and decimals.

[*Ans.* 278°·37′·15″·96 according to the sexagesimal division.

EXAMPLE 2. Reduce 278° : 37′ : 15″·96 British to French.

TABLE XIV.—*Continued.*

Make the seconds decimal parts of the minutes by dividing them by 6 and multiplying by 10. This then becomes 278°·37′·266. In degrees, minutes, and decimals of a minute. Make now the minutes into degrees and decimals of a degree by dividing *them* by 6 and multiplying by 10. The last figures then become

278°·6211 = English degrees and decimals
Add one-ninth 30 ·9579

309 ·5790.

DECIMAL 309° 57′ 90″ French measure.

And so on for others.

FRENCH MEASURES COMPARED WITH ENGLISH.

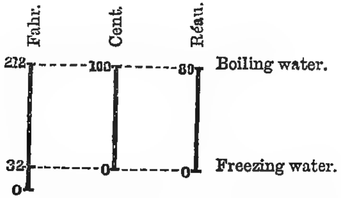
1 MÈTRE = 3·281 feet = 39·371 inches = 1·0936 yds. = ·1988 perch.
1 SQUARE MÈTRE = 10·764 sq. feet = 1·196 yds. = ·03954 sq. perch.
1 FOOT = ·3048 mètre and 1 inch = ·0254 mètre.
1 SQUARE FOOT = ·0929 sq. mètre.
1 PERCH 16½ feet = 5·029 mètres.
1 SQ. PERCH 272¼ ft. = 25·292 sq. mètres = 30¼ sq. yds.
ARE = 100 sq. mètres = 3·954 sq. perches.
A. R. P.
HECTARE—100 *Ares* = 395·4 perches = 2 1 35·4.
1 lb. avoirdupois = ·4536 kilogramme.
LITRE = cubic decimètre = 1·76 pint = ·22 gallon.
DECALITRE = 2·2 gallons = 1·1 peck
HECTOLITRE = 22 gallons = 11 pecks = 2·75 bushels.
KILOGRAMME = weight of one cubic decimètre of distilled water = 2·2046 lbs. avoirdupois = 2·679 troy pounds = 15432·35 grains.
1000 KILOGRAMMES = 2204·6 lbs. = nearly 1 *ton British*, is called a MILLIER or TONNE = 19·68 cwt.
KILOGRAMME = 1000 grammes = 10,000 decigrammes.
7000 grains = 1 lb. avoir. 437·5 grs. = 1 oz. avoir.
5760 grains = 1 lb. troy. 480 grs. = 1 oz. troy.
FORCE OF GRAVITY g = 9·809 mètres = 32·18 feet.

TABLE XIV.—*Continued.*

Length of a pendulum
Vibrating seconds
Latitude of Paris
} = 39·13 inches = ·994 mètre.

Mean barometer at 30 inches = 76·2 centimetres.
1 cubic foot of air weighs 536 grains or 1·23 ozs. avoirdupois = 1·1 oz. troy.
Average height of an atmosphere of equal density about 5 miles.

Thermometers 9° Fahrenheit = 5° centigrade = 4° Réaumur.
Freezing point 32° Fahrenheit = 0° centigrade = 0° Réaumur.
Boiling point 212° ,, = 100° ,, = 80° ,,



$\frac{1}{237}$ increase or decrease in volume of *a gas* for each degree centigrade.
$\frac{1}{480}$ increase or decrease in volume of *a gas* for each degree Fahrenheit.

10 lbs. of distilled water = 1 gallon.
62·32 lbs. of distilled water = 1 cube foot.
1 cubic foot = 6·232 gallons.

CIRCULAR.
100° French = 90° British. French division 1° = 100 minutes, 1 minute = 100 seconds.
100 minutes French = 54 minutes British.
100 seconds French = 32·4 seconds British. Or
1° French = ·9° British.
1′ French = ·54 British.
1″ French = ·324 British.

TABLE XV. Showing the Weight, Specific Gravity, strength, and elasticity of various materials employed by the Physicist and Engineer. When used by the Engineer only about one-fourth of the ultimate strengths here given should be calculated from.

MATERIALS.	Moduli of Rupture.	Moduli of Elasticity.	Crushing forces per square inch, in lbs.	Tenacities per sq. in. in lbs.	Weights of a cubic foot in lbs.	Specific gravities.
Acacia, English Growth	11,200	1,150,000	..	16,000	44·3	·71
Ash	12,000	1,600,000	9,000	17,000	48·0	·77
Brass, Cast	..	8,900,000	10,300	18,000	525·0	8·40
Beech	9,300	1,350,000	8,500	16,000	48·0	·77
Brick, Red	..	..	800	280	135·5	2·20
Brickwork	..	..	..	..	112·5	1·80
Do. Pale Red	..	..	550	300	130·3	2·08
Cedar, American, Fresh	..	490,000	5,600	11,400	56·8	0·91
Do. do. Seasoned	..	..	4,900	..	47·0	0·75
Copper, Cast	..	..	..	19,000	538·0	8·61
Do. Sheet	..	..	..	30,000	549·0	8·80
Do. Wire-drawn	..	..	..	60,000	560·0	8·88
Deal, Christiana	9,900	1,670,000	..	12,400	43·6	0·70
Do. Memel	10,400	1,530,000	..	..	37·0	0·60
Do. Norway Spruce	..	..	..	17,600	21·2	0·34
Elm, Seasoned	6,100	700,000	10,300	13,500	36·8	0·59
Fir, New England	6,600	2,190,000	..	10,000	34·5	0·55
Do. Riga	7,600	1,100,000	6,100	12,000	47·0	0·75
Glass	..	8,000,000	33,000	2,400	153·3	2·45
Iron, Wrought, English	..	..	..	57,000	481·2	7·70
Do. in Bars	..	..	..	57,000	487·0	7·80
Do. rolled in Sheets and Rivetted	..	..	..	31,000	487	7·8
Cast Iron Carron, cold blast	38,500	17,270,000	106,000	16,700	441	7·07
Do. Hot Blast	37,500	16,080,000	108,000	13,500	440·0	7·04
Do. Buffery	37,500	14,000,000	90,000	17,500	441·0	7·06
Larch, green	5,000	900,000	3,200	10,200	36·6	0·52
Do. dry	6,900	1,050,000	5,500	8,900	35·0	0·56
Lead, cast English	..	720,000	..	1,800	717·4	11·44
Do. milled sheet	..	..	..	3,300	712·9	11·40
Marble, white Italian	1,100	2,520,000	..	..	165·0	2·64
Do. black Galway	2,700	..	..	..	168·4	2·70
Mortar, old, good	..	..	250	80	107·1	1·75
Oak, English	10,000	1,450,000	6,600	17,300	58·3	0·93
Do. Canadian	10,500	2,150,000	6,500	10,200	54·5	0·87
Do. Dantzic	8,700	1,190,000	..	12,700	47·4	0·76
Do. African	13,600	2,280,000	..	..	60·7	0·97
Do. Adriatic	8,300	970,000	..	..	62·0	0·99
Pine, pitch	9,800	1,230,000	..	7,800	41·2	0·66
Do. red	8,900	1,840,000	5,300	..	41·2	0·66
Silver, Standard	..	..	..	40,900	644·5	10·31
Slate, Welsh	11,800	15,800,000	..	12,800	180·5	2·89
Do. Westmoreland	..	1,290,000	..	..	174·4	2·70
Do. Valentia	5,200	..	..	..	180·0	2·88
Steel, soft	..	..	..	120,000	486·2	7·80
Do., razor tempered	..	29,000,000	..	150,000	490·0	7·84
Stone, granite average	5,500	..	8,000	..	168·0	2·70
Do. Rochdale	2,400	..	..	..	161·0	2·58
Teak, dry	14,800	2,400,000	12,101	15,000	41·1	0·66
Tin, cast	..	4,600,000	..	5,300	455·7	7·30

CATALOGUE

OF

PRACTICAL AND SCIENTIFIC BOOKS,

PUBLISHED BY

HENRY CAREY BAIRD & CO.,

Industrial Publishers and Booksellers,

NO. 810 WALNUT STREET,

PHILADELPHIA.

☞ **Any of the Books comprised in this Catalogue will be sent by mail, free of postage, at the publication price.**

☞ **A Descriptive Catalogue, 96 pages, 8vo., will be sent, free of postage, to any one who will furnish the publisher with his address.**

ARLOT.—A Complete Guide for Coach Painters.

Translated from the French of M. ARLOT, Coach Painter; for eleven years Foreman of Painting to M. Eherler, Coach Maker, Paris. By A. A. FESQUET, Chemist and Engineer. To which is added an Appendix, containing Information respecting the Materials and the Practice of Coach and Car Painting and Varnishing in the United States and Great Britain. 12mo. $1.25

ARMENGAUD, AMOROUX, and JOHNSON.—The Practical Draughtsman's Book of Industrial Design, and Machinist's and Engineer's Drawing Companion:

Forming a Complete Course of Mechanical Engineering and Architectural Drawing. From the French of M. Armengaud the elder, Prof. of Design in the Conservatoire of Arts and Industry, Paris, and MM. Armengaud the younger, and Amoroux, Civil Engineers. Rewritten and arranged with additional matter and plates, selections from and examples of the most useful and generally employed mechanism of the day. By WILLIAM JOHNSON, Assoc. Inst. C. E., Editor of "The Practical Mechanic's Journal." Illustrated by 50 folio steel plates, and 50 wood-cuts. A new edition, 4to. $10.00

ARROWSMITH.—Paper-Hanger's Companion:

A Treatise in which the Practical Operations of the Trade are Systematically laid down: with Copious Directions Preparatory to Papering; Preventives against the Effect of Damp on Walls; the Various Cements and Pastes Adapted to the Several Purposes of the Trade; Observations and Directions for the Panelling and Ornamenting of Rooms, etc. By JAMES ARROWSMITH, Author of "Analysis of Drapery," etc. 12mo., cloth. $1.25

ASHTON.—The Theory and Practice of the Art of Designing Fancy Cotton and Woollen Cloths from Sample:

Giving full Instructions for Reducing Drafts, as well as the Methods of Spooling and Making out Harness for Cross Drafts, and Finding any Required Reed, with Calculations and Tables of Yarn. By FREDERICK T. ASHTON, Designer, West Pittsfield, Mass. With 52 Illustrations. One volume, 4to. $10.00

BAIRD.—Letters on the Crisis, the Currency and the Credit System.

By HENRY CAREY BAIRD. Pamphlet. 05

BAIRD.—Protection of Home Labor and Home Productions necessary to the Prosperity of the American Farmer.

By HENRY CAREY BAIRD. 8vo., paper. 10

BAIRD.—Some of the Fallacies of British Free-Trade Revenue Reform.

Two Letters to Arthur Latham Perry, Professor of History and Political Economy in Williams College. By HENRY CAREY BAIRD. Pamphlet. 05

BAIRD.—The Rights of American Producers, and the Wrongs of British Free-Trade Revenue Reform.

By HENRY CAREY BAIRD. Pamphlet. 05

BAIRD.—Standard Wages Computing Tables:

An Improvement in all former Methods of Computation, so arranged that wages for days, hours, or fractions of hours, at a specified rate per day or hour, may be ascertained at a glance. By T. SPANGLER BAIRD. Oblong folio. $5.00

BAIRD.—The American Cotton Spinner, and Manager's and Carder's Guide:

A Practical Treatise on Cotton Spinning; giving the Dimensions and Speed of Machinery, Draught and Twist Calculations, etc.; with notices of recent Improvements: together with Rules and Examples for making changes in the sizes and numbers of Roving and Yarn. Compiled from the papers of the late ROBERT H. BAIRD. 12mo. $1.50

BAKER.—Long-Span Railway Bridges :

Comprising Investigations of the Comparative Theoretical and Practical Advantages of the various Adopted or Proposed Type Systems of Construction; with numerous Formulæ and Tables. By B. BAKER. 12mo. $2.00

BAUERMAN.—A Treatise on the Metallurgy of Iron :

Containing Outlines of the History of Iron Manufacture, Methods of Assay, and Analysis of Iron Ores, Processes of Manufacture of Iron and Steel, etc., etc. By H. BAUERMAN, F. G. S., Associate of the Royal School of Mines. First American Edition, Revised and Enlarged. With an Appendix on the Martin Process for Making Steel, from the Report of ABRAM S. HEWITT, U. S. Commissioner to the Universal Exposition at Paris, 1867. Illustrated. 12mo. . $2.00

BEANS.—A Treatise on Railway Curves and the Location of Railways.

By E. W. BEANS, C. E. Illustrated. 12mo. Tucks. . . $1.50

BELL.—Carpentry Made Easy :

Or, The Science and Art of Framing on a New and Improved System. With Specific Instructions for Building Balloon Frames, Barn Frames, Mill Frames, Warehouses, Church Spires, etc. Comprising also a System of Bridge Building, with Bills, Estimates of Cost, and valuable Tables. Illustrated by 38 plates, comprising nearly 200 figures. By WILLIAM E. BELL, Architect and Practical Builder. 8vo. . $5.00

BELL.—Chemical Phenomena of Iron Smelting :

An Experimental and Practical Examination of the Circumstances which determine the Capacity of the Blast Furnace, the Temperature of the Air, and the proper Condition of the Materials to be operated upon. By I. LOWTHIAN BELL. Illustrated. 8vo. . . $6.00

BEMROSE.—Manual of Wood Carving :

With Practical Illustrations for Learners of the Art, and Original and Selected Designs. By WILLIAM BEMROSE, Jr. With an Introduction by LLEWELLYN JEWITT, F. S. A., etc. With 128 Illustrations. 4to., cloth. $3.00

BICKNELL.—Village Builder, and Supplement :

Elevations and Plans for Cottages, Villas, Suburban Residences, Farm Houses, Stables and Carriage Houses, Store Fronts, School Houses, Churches, Court Houses, and a model Jail; also, Exterior and Interior details for Public and Private Buildings, with approved Forms of Contracts and Specifications, including Prices of Building Materials and Labor at Boston, Mass., and St. Louis, Mo. Containing 75 plates drawn to scale; showing the style and cost of building in different sections of the country, being an original work comprising the designs of twenty leading architects, representing the New England, Middle, Western, and Southwestern States. 4to. . $12.00

BLENKARN.—Practical Specifications of Works executed in Architecture, Civil and Mechanical Engineering, and in Road Making and Sewering:

To which are added a series of practically useful Agreements and Reports. By JOHN BLENKARN. Illustrated by 15 large folding plates. 8vo. $9.00

BLINN.—A Practical Workshop Companion for Tin, Sheet-Iron, and Copperplate Workers:

Containing Rules for describing various kinds of Patterns used by Tin, Sheet-Iron, and Copper-plate Workers; Practical Geometry; Mensuration of Surfaces and Solids; Tables of the Weights of Metals, Lead Pipe, etc.; Tables of Areas and Circumferences of Circles; Japan, Varnishes, Lackers, Cements, Compositions, etc., etc. By LEROY J. BLINN, Master Mechanic. With over 100 Illustrations. 12mo. $2.50

BOOTH.—Marble Worker's Manual:

Containing Practical Information respecting Marbles in general, their Cutting, Working, and Polishing; Veneering of Marble; Mosaics; Composition and Use of Artificial Marble, Stuccos, Cements, Receipts, Secrets, etc., etc. Translated from the French by M. L. BOOTH. With an Appendix concerning American Marbles. 12mo., cloth. $1.50

BOOTH AND MORFIT.—The Encyclopedia of Chemistry, Practical and Theoretical:

Embracing its application to the Arts, Metallurgy, Mineralogy, Geology, Medicine, and Pharmacy. By JAMES C. BOOTH, Melter and Refiner in the United States Mint, Professor of Applied Chemistry in the Franklin Institute, etc., assisted by CAMPBELL MORFIT, author of "Chemical Manipulations," etc. Seventh edition. Royal 8vo., 978 pages, with numerous wood-cuts and other illustrations. . $5.00

BOX.—A Practical Treatise on Heat:

As applied to the Useful Arts; for the Use of Engineers, Architects, etc. By THOMAS BOX, author of "Practical Hydraulics." Illustrated by 14 plates containing 114 figures. 12mo. $4.25

BOX.—Practical Hydraulics:

A Series of Rules and Tables for the use of Engineers, etc. By THOMAS BOX. 12mo. $2.50

BROWN.—Five Hundred and Seven Mechanical Movements:

Embracing all those which are most important in Dynamics, Hydraulics, Hydrostatics, Pneumatics, Steam Engines, Mill and other Gearing, Presses, Horology, and Miscellaneous Machinery; and including many movements never before published, and several of which have only recently come into use. By HENRY T. BROWN, Editor of the "American Artisan." In one volume, 12mo. $1.00

BUCKMASTER.—The Elements of Mechanical Physics:

By J. C. BUCKMASTER, late Student in the Government School of Mines; Certified Teacher of Science by the Department of Science and Art; Examiner in Chemistry and Physics in the Royal College of Preceptors; and late Lecturer in Chemistry and Physics of the Royal Polytechnic Institute. Illustrated with numerous engravings. In one volume, 12mo. $1.50

BULLOCK.—The American Cottage Builder:

A Series of Designs, Plans, and Specifications, from $200 to $20,000, for Homes for the People; together with Warming, Ventilation, Drainage, Painting, and Landscape Gardening. By JOHN BULLOCK, Architect, Civil Engineer, Mechanician, and Editor of "The Rudiments of Architecture and Building," etc., etc. Illustrated by 75 engravings. In one volume, 8vo. $3.50

BULLOCK.—The Rudiments of Architecture and Building:

For the use of Architects, Builders, Draughtsmen, Machinists, Engineers, and Mechanics. Edited by JOHN BULLOCK, author of "The American Cottage Builder." Illustrated by 250 engravings. In one volume, 8vo. $3.50

BURGH.—Practical Illustrations of Land and Marine Engines:

Showing in detail the Modern Improvements of High and Low Pressure, Surface Condensation, and Super-heating, together with Land and Marine Boilers. By N. P. BURGH, Engineer. Illustrated by 20 plates, double elephant folio, with text. $21.00

BURGH.—Practical Rules for the Proportions of Modern Engines and Boilers for Land and Marine Purposes.

By N. P. BURGH, Engineer. 12mo. $1.50

BURGH.—The Slide-Valve Practically Considered.

By N. P. BURGH, Engineer. Completely illustrated. 12mo. $2.00

BYLES.—Sophisms of Free Trade and Popular Political Economy Examined.

By a BARRISTER (Sir JOHN BARNARD BYLES, Judge of Common Pleas). First American from the Ninth English Edition, as published by the Manchester Reciprocity Association. In one volume, 12mo. Paper, 75 cts. Cloth $1.25

BYRN.—The Complete Practical Brewer:

Or Plain, Accurate, and Thorough Instructions in the Art of Brewing Beer, Ale, Porter, including the Process of making Bavarian Beer, all the Small Beers, such as Root-beer, Ginger-pop, Sarsaparilla-beer, Mead, Spruce Beer, etc., etc. Adapted to the use of Public Brewers and Private Families. By M. LA FAYETTE BYRN, M.D. With illustrations. 12mo. $1.25

BYRN.—The Complete Practical Distiller:

Comprising the most perfect and exact Theoretical and Practical Description of the Art of Distillation and Rectification; including all of the most recent improvements in distilling apparatus; instructions for preparing spirits from the numerous vegetables, fruits, etc.; directions for the distillation and preparation of all kinds of brandies and other spirits, spirituous and other compounds, etc., etc. By M. LA FAYETTE BYRN, M. D. Eighth Edition. To which are added, Practical Directions for Distilling, from the French of Th. Fling, Brewer and Distiller. 12mo. $1.50

BYRNE.—Handbook for the Artisan, Mechanic, and Engineer:

Comprising the Grinding and Sharpening of Cutting Tools, Abrasive Processes, Lapidary Work, Gem and Glass Engraving, Varnishing and Lackering, Apparatus, Materials and Processes for Grinding and Polishing, etc. By OLIVER BYRNE. Illustrated by 185 wood engravings. In one volume, 8vo. $5.00

BYRNE.—Pocket Book for Railroad and Civil Engineers:

Containing New, Exact, and Concise Methods for Laying out Railroad Curves, Switches, Frog Angles, and Crossings; the Staking out of work; Levelling; the Calculation of Cuttings; Embankments; Earth-work, etc. By OLIVER BYRNE. 18mo., full bound, pocket-book form. $1.75

BYRNE.—The Practical Model Calculator:

For the Engineer, Mechanic, Manufacturer of Engine Work, Naval Architect, Miner, and Millwright. By OLIVER BYRNE. 1 volume, 8vo., nearly 600 pages $4.50

BYRNE.—The Practical Metal-Worker's Assistant:

Comprising Metallurgic Chemistry; the Arts of Working all Metals and Alloys; Forging of Iron and Steel; Hardening and Tempering; Melting and Mixing; Casting and Founding; Works in Sheet Metal; The Processes Dependent on the Ductility of the Metals; Soldering; and the most Improved Processes and Tools employed by Metal-Workers. With the Application of the Art of Electro-Metallurgy to Manufacturing Processes; collected from Original Sources, and from the Works of Holtzapffel, Bergeron, Leupold, Plumier, Napier, Scoffern, Clay, Fairbairn, and others. By OLIVER BYRNE. A new, revised, and improved edition, to which is added An Appendix, containing THE MANUFACTURE OF RUSSIAN SHEET-IRON. By JOHN PERCY, M. D., F.R.S. THE MANUFACTURE OF MALLEABLE IRON CASTINGS, and IMPROVEMENTS IN BESSEMER STEEL. By A. A. FESQUET, Chemist and Engineer. With over 600 Engravings, illustrating every Branch of the Subject. 8vo. $7.00

Cabinet Maker's Album of Furniture:

Comprising a Collection of Designs for Furniture. Illustrated by 48 Large and Beautifully Engraved Plates. In one vol., oblong $5.00

CALLINGHAM.—Sign Writing and Glass Embossing:

A Complete Practical Illustrated Manual of the Art. By JAMES CALLINGHAM. In one volume, 12mo. $1.50

CAMPIN.—A Practical Treatise on Mechanical Engineering:

Comprising Metallurgy, Moulding, Casting, Forging, Tools, Workshop Machinery, Mechanical Manipulation, Manufacture of Steam-engines, etc., etc. With an Appendix on the Analysis of Iron and Iron Ores. By FRANCIS CAMPIN, C. E. To which are added, Observations on the Construction of Steam Boilers, and Remarks upon Furnaces used for Smoke Prevention; with a Chapter on Explosions. By R. Armstrong, C. E., and John Bourne. Rules for Calculating the Change Wheels for Screws on a Turning Lathe, and for a Wheel-cutting Machine. By J. LA NICCA. Management of Steel, Including Forging, Hardening, Tempering, Annealing, Shrinking, and Expansion. And the Case-hardening of Iron. By G. EDE. 8vo. Illustrated with 29 plates and 100 wood engravings . . . $6.00

CAMPIN.—The Practice of Hand-Turning in Wood, Ivory, Shell, etc.:

With Instructions for Turning such works in Metal as may be required in the Practice of Turning Wood, Ivory, etc. Also, an Appendix on Ornamental Turning. By FRANCIS CAMPIN; with Numerous Illustrations. 12mo., cloth $3.00

CAREY.—The Works of Henry C. Carey:

FINANCIAL CRISES, their Causes and Effects. 8vo. paper . 25

HARMONY OF INTERESTS: Agricultural, Manufacturing, and Commercial. 8vo., cloth $1.50

MANUAL OF SOCIAL SCIENCE. Condensed from Carey's "Principles of Social Science." By KATE MCKEAN. 1 vol. 12mo. $2.25

MISCELLANEOUS WORKS: comprising "Harmony of Interests," "Money," "Letters to the President," "Financial Crises," "The Way to Outdo England Without Fighting Her," "Resources of the Union," "The Public Debt," "Contraction or Expansion?" "Review of the Decade 1857–'67," "Reconstruction," etc., etc. Two vols., 8vo., cloth $10.00

PAST, PRESENT, AND FUTURE. 8vo. $2.50

PRINCIPLES OF SOCIAL SCIENCE. 3 vols., 8vo., cloth $10.00

THE SLAVE-TRADE, DOMESTIC AND FOREIGN; Why it Exists, and How it may be Extinguished (1853). 8vo., cloth . $2.00

LETTERS ON INTERNATIONAL COPYRIGHT (1867) . . 50

THE UNITY OF LAW: As Exhibited in the Relations of Physical, Social, Mental, and Moral Science (1872). In one volume, 8vo., pp. xxiii., 433. Cloth $3.50

CHAPMAN.—A Treatise on Ropemaking:

As Practised in private and public Rope yards, with a Description of the Manufacture, Rules, Tables of Weights, etc., adapted to the Trades, Shipping, Mining, Railways, Builders, etc. By ROBERT CHAPMAN. 24mo. $1.50

COLBURN.—The Locomotive Engine:

Including a Description of its Structure, Rules for Estimating its Capabilities, and Practical Observations on its Construction and Management. By ZERAH COLBURN. Illustrated. A new edition. 12mo. $1.25

CRAIK.—The Practical American Millwright and Miller.

By DAVID CRAIK, Millwright. Illustrated by numerous wood engravings, and two folding plates. 8vo. $5.00

DE GRAFF.—The Geometrical Stair Builders' Guide:

Being a Plain Practical System of Hand-Railing, embracing all its necessary Details, and Geometrically Illustrated by 22 Steel Engravings; together with the use of the most approved principles of Practical Geometry. By SIMON DE GRAFF, Architect. 4to. . $5.00

DE KONINCK.—DIETZ.—A Practical Manual of Chemical Analysis and Assaying:

As applied to the Manufacture of Iron from its Ores, and to Cast Iron, Wrought Iron, and Steel, as found in Commerce. By L. L. DE KONINCK, Dr. Sc., and E. DIETZ, Engineer. Edited with Notes, by ROBERT MALLET, F.R.S., F.S.G., M.I.C.E., etc. American Edition, Edited with Notes and an Appendix on Iron Ores, by A. A. FESQUET, Chemist and Engineer. One volume, 12mo. $2.50

DUNCAN.—Practical Surveyor's Guide:

Containing the necessary information to make any person, of common capacity, a finished land surveyor without the aid of a teacher. By ANDREW DUNCAN. Illustrated. 12mo., cloth. . . . $1.25

DUPLAIS.—A Treatise on the Manufacture and Distillation of Alcoholic Liquors:

Comprising Accurate and Complete Details in Regard to Alcohol from Wine, Molasses, Beets, Grain, Rice, Potatoes, Sorghum, Asphodel, Fruits, etc.; with the Distillation and Rectification of Brandy, Whiskey, Rum, Gin, Swiss, Absinthe, etc., the Preparation of Aromatic Waters, Volatile Oils or Essences, Sugars, Syrups, Aromatic Tinctures, Liqueurs, Cordial Wines, Effervescing Wines, etc., the Aging of Brandy and the Improvement of Spirits, with Copious Directions and Tables for Testing and Reducing Spirituous Liquors, etc., etc. Translated and Edited from the French of MM. DUPLAIS, Ainé et Jeune. By M. MCKENNIE, M.D. To which are added the United States Internal Revenue Regulations for the Assessment and Collection of Taxes on Distilled Spirits. Illustrated by fourteen folding plates and several wood engravings. 743 pp., 8vo. $10.00

DUSSAUCE.—A General Treatise on the Manufacture of Every Description of Soap:

Comprising the Chemistry of the Art, with Remarks on Alkalies, Saponifiable Fatty Bodies, the apparatus necessary in a Soap Factory, Practical Instructions in the manufacture of the various kinds of Soap, the assay of Soaps, etc., etc. Edited from Notes of Larmé, Fontenelle, Malapayre, Dufour, and others, with large and important additions by Prof. H. DUSSAUCE, Chemist. Illustrated. In one vol., 8vo. . $10.00

DUSSAUCE.—A General Treatise on the Manufacture of Vinegar:

Theoretical and Practical. Comprising the various Methods, by the Slow and the Quick Processes, with Alcohol, Wine, Grain, Malt, Cider, Molasses, and Beets; as well as the Fabrication of Wood Vinegar, etc., etc. By Prof. H. DUSSAUCE. In one volume, 8vo. . . $5.00

DUSSAUCE.—A New and Complete Treatise on the Arts of Tanning, Currying, and Leather Dressing:

Comprising all the Discoveries and Improvements made in France, Great Britain, and the United States. Edited from Notes and Documents of Messrs. Sallerou, Grouvelle, Duval, Dessables, Labarraque, Payen, René, De Fontenelle, Malapeyre, etc., etc. By Prof. H. DUSSAUCE, Chemist. Illustrated by 212 wood engravings. 8vo. $25.00

DUSSAUCE.—A Practical Guide for the Perfumer:

Being a New Treatise on Perfumery, the most favorable to the Beauty without being injurious to the Health, comprising a Description of the substances used in Perfumery, the Formulæ of more than 1000 Preparations, such as Cosmetics, Perfumed Oils, Tooth Powders, Waters, Extracts, Tinctures, Infusions, Spirits, Vinaigres, Essential Oils, Pastels, Creams, Soaps, and many new Hygienic Products not hitherto described. Edited from Notes and Documents of Messrs. Debay, Lunel, etc. With additions by Prof. H. DUSSAUCE, Chemist. 12mo. $3.00

DUSSAUCE.—Practical Treatise on the Fabrication of Matches, Gun Cotton, and Fulminating Powders.

By Prof. H. DUSSAUCE. 12mo. $3.00

Dyer and Color-maker's Companion:

Containing upwards of 200 Receipts for making Colors, on the most approved principles, for all the various styles and fabrics now in existence; with the Scouring Process, and plain Directions for Preparing, Washing-off, and Finishing the Goods. In one vol., 12mo. . $1.25

EASTON.—A Practical Treatise on Street or Horse-power Railways.

By ALEXANDER EASTON, C. E. Illustrated by 23 plates. 8vo., cloth. $2.00

ELDER.—Questions of the Day:

Economic and Social. By Dr. WILLIAM ELDER. 8vo. . $3.00

FAIRBAIRN.—The Principles of Mechanism and Machinery of Transmission:

Comprising the Principles of Mechanism, Wheels, and Pulleys, Strength and Proportions of Shafts, Coupling of Shafts, and Engaging and Disengaging Gear. By Sir WILLIAM FAIRBAIRN, C.E., LL.D., F.R.S., F.G.S. Beautifully illustrated by over 150 wood-cuts. In one volume, 12mo. $2.50

FORSYTH.—Book of Designs for Headstones, Mural, and other Monuments:

Containing 78 Designs. By JAMES FORSYTH. With an Introduction by CHARLES BOUTELL, M. A. 4to., cloth. $5.00

GIBSON.—The American Dyer:

A Practical Treatise on the Coloring of Wool, Cotton, Yarn and Cloth, in three parts. Part First gives a descriptive account of the Dye Stuffs; if of vegetable origin, where produced, how cultivated, and how prepared for use; if chemical, their composition, specific gravities, and general adaptability, how adulterated, and how to detect the adulterations, etc. Part Second is devoted to the Coloring of Wool, giving recipes for one hundred and twenty-nine different colors or shades, and is supplied with sixty colored samples of Wool. Part Third is devoted to the Coloring of Raw Cotton or Cotton Waste, for mixing with Wool Colors in the Manufacture of all kinds of Fabrics, gives recipes for thirty-eight different colors or shades, and is supplied with twenty-four colored samples of Cotton Waste. Also, recipes for Coloring Beavers, Doeskins, and Flannels, with remarks upon Anilines, giving recipes for fifteen different colors or shades, and nine samples of Aniline Colors that will stand both the Fulling and Scouring process. Also, recipes for Aniline Colors on Cotton Thread, and recipes for Common Colors on Cotton Yarns. Embracing in all over two hundred recipes for Colors and Shades, and ninety-four samples of Colored Wool and Cotton Waste, etc. By RICHARD H. GIBSON, Practical Dyer and Chemist. In one volume, 8vo. . . . $12.50

GILBART.—History and Principles of Banking:

A Practical Treatise. By JAMES W. GILBART, late Manager of the London and Westminster Bank. With additions. In one volume, 8vo., 600 pages, sheep $5.00

Gothic Album for Cabinet Makers:

Comprising a Collection of Designs for Gothic Furniture. Illustrated by 23 large and beautifully engraved plates. Oblong . . $3.00

GRANT.—Beet-root Sugar and Cultivation of the Beet.

By E. B. GRANT. 12mo. $1.25

GREGORY.—Mathematics for Practical Men:

Adapted to the Pursuits of Surveyors, Architects, Mechanics, and Civil Engineers. By OLINTHUS GREGORY. 8vo., plates, cloth $3.00

GRISWOLD.—Railroad Engineer's Pocket Companion for the Field:

Comprising Rules for Calculating Deflection Distances and Angles, Tangential Distances and Angles, and all Necessary Tables for Engineers; also the art of Levelling from Preliminary Survey to the Construction of Railroads, intended Expressly for the Young Engineer, together with Numerous Valuable Rules and Examples. By W. GRISWOLD. 12mo., tucks $1.75

GRUNER.—Studies of Blast Furnace Phenomena.

By M. L. GRUNER, President of the General Council of Mines of France, and lately Professor of Metallurgy at the Ecole des Mines. Translated, with the Author's sanction, with an Appendix, by L. D. B. Gordon, F. R. S. E., F. G. S. Illustrated. 8vo. $2.50

GUETTIER.—Metallic Alloys:

Being a Practical Guide to their Chemical and Physical Properties, their Preparation, Composition, and Uses. Translated from the French of A. GUETTIER, Engineer and Director of Foundries, author of "La Fouderie en France," etc., etc. By A. A. FESQUET, Chemist and Engineer. In one volume, 12mo. $3.00

HARRIS.—Gas Superintendent's Pocket Companion.

By HARRIS & BROTHER, Gas Meter Manufacturers, 1115 and 1117 Cherry Street, Philadelphia. Full bound in pocket-book form $2.00

Hats and Felting:

A Practical Treatise on their Manufacture. By a Practical Hatter. Illustrated by Drawings of Machinery, etc. 8vo. $1.25

HOFMANN.—A Practical Treatise on the Manufacture of Paper in all its Branches.

By CARL HOFMANN. Late Superintendent of paper mills in Germany and the United States; recently manager of the Public Ledger Paper Mills, near Elkton, Md. Illustrated by 110 wood engravings, and five large folding plates. In one volume, 4to., cloth; 398 pages $15.00

HUGHES.—American Miller and Millwright's Assistant.

By WM. CARTER HUGHES. A new edition. In one vol., 12mo. $1.50

HURST.—A Hand-Book for Architectural Surveyors and others engaged in Building:

Containing Formulæ useful in Designing Builder's work, Table of Weights, of the materials used in Building, Memoranda connected with Builders' work, Mensuration, the Practice of Builders' Measurement, Contracts of Labor, Valuation of Property, Summary of the Practice in Dilapidation, etc., etc. By J. F. HURST, C. E. Second edition, pocket-book form, full bound $2.50

JERVIS.—Railway Property:

A Treatise on the Construction and Management of Railways; designed to afford useful knowledge, in the popular style, to the holders of this class of property; as well as Railway Managers, Officers, and Agents. By JOHN B. JERVIS, late Chief Engineer of the Hudson River Railroad, Croton Aqueduct, etc. In one vol., 12mo., cloth $2.00

JOHNSTON.—Instructions for the Analysis of Soils, Limestones, and Manures.

By J. F. W. JOHNSTON. 12mo. 38

KEENE.—A Hand-Book of Practical Gauging:

For the Use of Beginners, to which is added, A Chapter on Distillation, describing the process in operation at the Custom House for ascertaining the strength of wines. By JAMES B. KEENE, of H. M. Customs. 8vo. $1.25

KELLEY.—Speeches, Addresses, and Letters on Industrial and Financial Questions.

By Hon. WILLIAM D. KELLEY, M. C. In one volume, 544 pages, 8vo. $3.00

KENTISH.—A Treatise on a Box of Instruments,

And the Slide Rule; with the Theory of Trigonometry and Logarithms, including Practical Geometry, Surveying, Measuring of Timber, Cask and Malt Gauging, Heights, and Distances. By THOMAS KENTISH. In one volume. 12mo. $1.25

KOBELL.—ERNI.—Mineralogy Simplified:

A short Method of Determining and Classifying Minerals, by means of simple Chemical Experiments in the Wet Way. Translated from the last German Edition of F. VON KOBELL, with an Introduction to Blow-pipe Analysis and other additions. By HENRI ERNI, M. D., late Chief Chemist, Department of Agriculture, author of "Coal Oil and Petroleum." In one volume, 12mo. $2.50

LANDRIN.—A Treatise on Steel:

Comprising its Theory, Metallurgy, Properties, Practical Working, and Use. By M. H. C. LANDRIN, Jr., Civil Engineer. Translated from the French, with Notes, by A. A. FESQUET, Chemist and Engineer. With an Appendix on the Bessemer and the Martin Processes for Manufacturing Steel, from the Report of Abram S. Hewitt, United States Commissioner to the Universal Exposition, Paris, 1867. In one volume, 12mo. $3.00

LARKIN.—The Practical Brass and Iron Founder's Guide:

A Concise Treatise on Brass Founding, Moulding, the Metals and their Alloys, etc.: to which are added Recent Improvements in the Manufacture of Iron, Steel by the Bessemer Process, etc., etc. By JAMES LARKIN, late Conductor of the Brass Foundry Department in Reany, Neafie & Co's. Penn Works, Philadelphia. Fifth edition, revised, with Extensive additions. In one volume, 12mo. . . . $2.25

LEAVITT.—Facts about Peat as an Article of Fuel:

With Remarks upon its Origin and Composition, the Localities in which it is found, the Methods of Preparation and Manufacture, and the various Uses to which it is applicable; together with many other matters of Practical and Scientific Interest. To which is added a chapter on the Utilization of Coal Dust with Peat for the Production of an Excellent Fuel at Moderate Cost, specially adapted for Steam Service. By T. H. LEAVITT. Third edition. 12mo. $1.75

LEROUX, C.—A Practical Treatise on the Manufacture of Worsteds and Carded Yarns:

Comprising Practical Mechanics, with Rules and Calculations applied to Spinning; Sorting, Cleaning, and Scouring Wools; the English and French methods of Combing, Drawing, and Spinning Worsteds and Manufacturing Carded Yarns. Translated from the French of CHARLES LEROUX, Mechanical Engineer, and Superintendent of a Spinning Mill, by HORATIO PAINE, M. D., and A. A. FESQUET, Chemist and Engineer. Illustrated by 12 large Plates. To which is added an Appendix, containing extracts from the Reports of the International Jury, and of the Artisans selected by the Committee appointed by the Council of the Society of Arts, London, on Woollen and Worsted Machinery and Fabrics, as exhibited in the Paris Universal Exposition, 1867. 8vo., cloth. $5.00

LESLIE (Miss).—Complete Cookery:

Directions for Cookery in its Various Branches. By MISS LESLIE. 60th thousand. Thoroughly revised, with the addition of New Receipts. In one volume, 12mo., cloth. $1.50

LESLIE (Miss).—Ladies' House Book:

A Manual of Domestic Economy. 20th revised edition. 12mo., cloth.

LESLIE (Miss).—Two Hundred Receipts in French Cookery.

Cloth, 12mo.

LIEBER.—Assayer's Guide:

Or, Practical Directions to Assayers, Miners, and Smelters, for the Tests and Assays, by Heat and by Wet Processes, for the Ores of all the principal Metals, of Gold and Silver Coins and Alloys, and of Coal, etc. By OSCAR M. LIEBER. 12mo., cloth. . . . $1.25

LOTH.—The Practical Stair Builder:

A Complete Treatise on the Art of Building Stairs and Hand-Rails, Designed for Carpenters, Builders, and Stair-Builders. Illustrated with Thirty Original Plates. By C. EDWARD LOTH, Professional Stair-Builder. One large 4to. volume. $10.00

LOVE.—The Art of Dyeing, Cleaning, Scouring, and Finishing, on the Most Approved English and French Methods:

Being Practical Instructions in Dyeing Silks, Woollens, and Cottons, Feathers, Chips, Straw, etc. Scouring and Cleaning Bed and Window Curtains, Carpets, Rugs, etc. French and English Cleaning, any Color or Fabric of Silk, Satin, or Damask. By THOMAS LOVE, a Working Dyer and Scourer. Second American Edition, to which are added General Instructions for the Use of Aniline Colors. In one volume, 8vo., 343 pages. $5.00

MAIN and BROWN.—Questions on Subjects Connected with the Marine Steam-Engine:

And Examination Papers; with Hints for their Solution. By THOMAS J. MAIN, Professor of Mathematics, Royal Naval College, and THOMAS BROWN, Chief Engineer, R. N. 12mo., cloth. . . . $1.50

MAIN and BROWN.—The Indicator and Dynamometer:

With their Practical Applications to the Steam-Engine. By THOMAS J. MAIN, M. A. F. R., Assistant Professor Royal Naval College, Portsmouth, and THOMAS BROWN, Assoc. Inst. C. E., Chief Engineer, R. N., attached to the Royal Naval College. Illustrated. From the Fourth London Edition. 8vo. $1.50

MAIN and BROWN.—The Marine Steam-Engine.

By THOMAS J. MAIN, F. R.; Assistant S. Mathematical Professor at the Royal Naval College, Portsmouth, and THOMAS BROWN, Assoc. Inst. C. E., Chief Engineer R. N. Attached to the Royal Naval College. Authors of "Questions connected with the Marine Steam-Engine," and the "Indicator and Dynamometer." With numerous Illustrations. In one volume, 8vo. $5.00

MARTIN.—Screw-Cutting Tables, for the Use of Mechanical Engineers:

Showing the Proper Arrangement of Wheels for Cutting the Threads of Screws of any required Pitch; with a Table for Making the Universal Gas-Pipe Thread and Taps. By W. A. MARTIN, Engineer. 8vo. 50

Mechanics' (Amateur) Workshop:

A treatise containing plain and concise directions for the manipulation of Wood and Metals, including Casting, Forging, Brazing, Soldering, and Carpentry. By the author of the "Lathe and its Uses." Third edition. Illustrated. 8vo. $3.00

MOLESWORTH.—Pocket-Book of Useful Formulæ and Memoranda for Civil and Mechanical Engineers.

By GUILFORD L. MOLESWORTH, Member of the Institution of Civil Engineers, Chief Resident Engineer of the Ceylon Railway. Second American, from the Tenth London Edition. In one volume, full bound in pocket-book form. $2.00

NAPIER.—A System of Chemistry Applied to Dyeing.

By JAMES NAPIER, F. C. S. A New and Thoroughly Revised Edition. Completely brought up to the present state of the Science, including the Chemistry of Coal Tar Colors, by A. A. FESQUET, Chemist and Engineer. With an Appendix on Dyeing and Calico Printing, as shown at the Universal Exposition, Paris, 1867. Illustrated. In one volume, 8vo., 422 pages. $5.00

NAPIER.—Manual of Electro-Metallurgy:

Including the Application of the Art to Manufacturing Processes. By JAMES NAPIER. Fourth American, from the Fourth London edition, revised and enlarged. Illustrated by engravings. In one vol., 8vo. $2.00

NASON.—Table of Reactions for Qualitative Chemical Analysis.

By HENRY B. NASON, Professor of Chemistry in the Rensselaer Polytechnic Institute, Troy, New York. Illustrated by Colors. . 63

NEWBERY.—Gleanings from Ornamental Art of every style:

Drawn from Examples in the British, South Kensington, Indian, Crystal Palace, and other Museums, the Exhibitions of 1851 and 1862, and the best English and Foreign works. In a series of one hundred exquisitely drawn Plates, containing many hundred examples. By ROBERT NEWBERY. 4to. $15.00

NICHOLSON.—A Manual of the Art of Bookbinding:

Containing full instructions in the different Branches of Forwarding, Gilding, and Finishing. Also, the Art of Marbling Book-edges and Paper. By JAMES B. NICHOLSON. Illustrated. 12mo., cloth. $2.25

NICHOLSON.—The Carpenter's New Guide:

A Complete Book of Lines for Carpenters and Joiners. By PETER NICHOLSON. The whole carefully and thoroughly revised by H. K. DAVIS, and containing numerous new and improved and original Designs for Roofs, Domes, etc. By SAMUEL SLOAN, Architect. Illustrated by 80 plates. 4to. $4.50

NORRIS.—A Hand-book for Locomotive Engineers and Machinists:

Comprising the Proportions and Calculations for Constructing Locomotives; Manner of Setting Valves; Tables of Squares, Cubes, Areas, etc., etc. By SEPTIMUS NORRIS, Civil and Mechanical Engineer. New edition. Illustrated. 12mo., cloth. $2.00

NYSTROM.—On Technological Education, and the Construction of Ships and Screw Propellers:

For Naval and Marine Engineers. By JOHN W. NYSTROM, late Acting Chief Engineer, U. S. N. Second edition, revised with additional matter. Illustrated by seven engravings. 12mo. . . $1.50

O'NEILL.—A Dictionary of Dyeing and Calico Printing:

Containing a brief account of all the Substances and Processes in use in the Art of Dyeing and Printing Textile Fabrics; with Practical Receipts and Scientific Information. By CHARLES O'NEILL, Analytical Chemist; Fellow of the Chemical Society of London; Member of the Literary and Philosophical Society of Manchester; Author of "Chemistry of Calico Printing and Dyeing." To which is added an Essay on Coal Tar Colors and their application to Dyeing and Calico Printing. By A. A. FESQUET, Chemist and Engineer. With an Appendix on Dyeing and Calico Printing, as shown at the Universal Exposition, Paris, 1867. In one volume, 8vo., 491 pages. . $6.00

ORTON.—Underground Treasures:

How and Where to Find Them. A Key for the Ready Determination of all the Useful Minerals within the United States. By JAMES ORTON, A. M. Illustrated, 12mo. $1.50

OSBORN.—American Mines and Mining:

Theoretically and Practically Considered. By Prof. H. S. OSBORN. Illustrated by numerous engravings. 8vo. (*In preparation.*)

OSBORN.—The Metallurgy of Iron and Steel:

Theoretical and Practical in all its Branches; with special reference to American Materials and Processes. By H. S. OSBORN, LL. D., Professor of Mining and Metallurgy in Lafayette College, Easton, Pennsylvania. Illustrated by numerous large folding plates and wood-engravings. 8vo. $15.00

OVERMAN.—The Manufacture of Steel:

Containing the Practice and Principles of Working and Making Steel. A Handbook for Blacksmiths and Workers in Steel and Iron, Wagon Makers, Die Sinkers, Cutlers, and Manufacturers of Files and Hardware, of Steel and Iron, and for Men of Science and Art. By FREDERICK OVERMAN, Mining Engineer, Author of the "Manufacture of Iron," etc. A new, enlarged, and revised Edition. By A. A. FESQUET, Chemist and Engineer. $1.50

OVERMAN.—The Moulder and Founder's Pocket Guide:

A Treatise on Moulding and Founding in Green-sand, Dry-sand, Loam, and Cement; the Moulding of Machine Frames, Mill-gear, Hollow-ware, Ornaments, Trinkets, Bells, and Statues; Description of Moulds for Iron, Bronze, Brass, and other Metals; Plaster of Paris, Sulphur, Wax, and other articles commonly used in Casting; the Construction of Melting Furnaces, the Melting and Founding of Metals; the Composition of Alloys and their Nature. With an Appendix containing Receipts for Alloys, Bronze, Varnishes and Colors for Castings; also, Tables on the Strength and other qualities of Cast Metals. By FREDERICK OVERMAN, Mining Engineer, Author of "The Manufacture of Iron." With 42 Illustrations. 12mo. $1.50

Painter, Gilder, and Varnisher's Companion:

Containing Rules and Regulations in everything relating to the Arts of Painting, Gilding, Varnishing, Glass-Staining, Graining, Marbling, Sign-Writing, Gilding on Glass, and Coach Painting and Varnishing; Tests for the Detection of Adulterations in Oils, Colors, etc.; and a Statement of the Diseases to which Painters are peculiarly liable, with the Simplest and Best Remedies. Sixteenth Edition. Revised, with an Appendix. Containing Colors and Coloring—Theoretical and Practical. Comprising descriptions of a great variety of Additional Pigments, their Qualities and Uses, to which are added, Dryers, and Modes and Operations of Painting, etc. Together with Chevreul's Principles of Harmony and Contrast of Colors. 12mo., cloth. $1.50

PALLETT.—The Miller's, Millwright's, and Engineer's Guide.

By HENRY PALLETT. Illustrated. In one volume, 12mo. $3.00

PERCY.—The Manufacture of Russian Sheet-Iron.

By JOHN PERCY, M.D., F.R.S., Lecturer on Metallurgy at the Royal School of Mines, and to The Advanced Class of Artillery Officers at the Royal Artillery Institution, Woolwich; Author of "Metallurgy." With Illustrations. 8vo., paper. . . . . 50 cts.

PERKINS.—Gas and Ventilation.

Practical Treatise on Gas and Ventilation. With Special Relation to Illuminating, Heating, and Cooking by Gas. Including Scientific Helps to Engineer-students and others. With Illustrated Diagrams. By E. E. PERKINS. 12mo., cloth. $1.25

PERKINS and STOWE.—A New Guide to the Sheet-iron and Boiler Plate Roller:

Containing a Series of Tables showing the Weight of Slabs and Piles to produce Boiler Plates, and of the Weight of Piles and the Sizes of Bars to produce Sheet-iron; the Thickness of the Bar Gauge in decimals; the Weight per foot, and the Thickness on the Bar or Wire Gauge of the fractional parts of an inch; the Weight per sheet, and the Thickness on the Wire Gauge of Sheet-iron of various dimensions to weigh 112 lbs. per bundle; and the conversion of Short Weight into Long Weight, and Long Weight into Short. Estimated and collected by G. H. PERKINS and J. G. STOWE. $2.50

PHILLIPS and DARLINGTON.—Records of Mining and Metallurgy;

Or Facts and Memoranda for the use of the Mine Agent and Smelter. By J. ARTHUR PHILLIPS, Mining Engineer, Graduate of the Imperial School of Mines, France, etc., and JOHN DARLINGTON. Illustrated by numerous engravings. In one volume, 12mo. . . $2.00

PROTEAUX.—Practical Guide for the Manufacture of Paper and Boards.

By A. PROTEAUX, Civil Engineer, and Graduate of the School of Arts and Manufactures, and Director of Thiers' Paper Mill, Puy-de-Dôme. With additions, by L. S. LE NORMAND. Translated from the French, with Notes, by HORATIO PAINE, A. B., M. D. To which is added a Chapter on the Manufacture of Paper from Wood in the United States, by HENRY T. BROWN, of the "American Artisan." Illustrated by six plates, containing Drawings of Raw Materials, Machinery, Plans of Paper-Mills, etc., etc. 8vo. $10.00

REGNAULT.—Elements of Chemistry.

By M. V. REGNAULT. Translated from the French by T. FORREST BETTON, M. D., and edited, with Notes, by JAMES C. BOOTH, Melter and Refiner U. S. Mint, and WM. L. FABER, Metallurgist and Mining Engineer. Illustrated by nearly 700 wood engravings. Comprising nearly 1500 pages. In two volumes, 8vo., cloth. $7.50

REID.—A Practical Treatise on the Manufacture of Portland Cement:

By HENRY REID, C. E. To which is added a Translation of M. A. Lipowitz's Work, describing a New Method adopted in Germany for Manufacturing that Cement, by W. F. REID. Illustrated by plates and wood engravings. 8vo. $6.00

RIFFAULT, VERGNAUD, and TOUSSAINT.—A Practical Treatise on the Manufacture of Varnishes.

By MM. RIFFAULT, VERGNAUD, and TOUSSAINT. Revised and Edited by M. F. MALEPEYRE and Dr. EMIL WINCKLER. Illustrated. In one volume, 8vo. (*In preparation.*)

RIFFAULT, VERGNAUD, and TOUSSAINT.—A Practical Treatise on the Manufacture of Colors for Painting:

Containing the best Formulæ and the Processes the Newest and in most General Use. By MM. RIFFAULT, VERGNAUD, and TOUSSAINT. Revised and Edited by M. F. MALEPEYRE and Dr. EMIL WINCKLER. Translated from the French by A. A. FESQUET, Chemist and Engineer. Illustrated by Engravings. In one volume, 650 pages, 8vo. $7.50

ROBINSON.—Explosions of Steam Boilers:

How they are Caused, and how they may be Prevented. By J. R. ROBINSON, Steam Engineer. 12mo. $1.25

ROPER.—A Catechism of High Pressure or Non-Condensing Steam-Engines:

Including the Modelling, Constructing, Running, and Management of Steam Engines and Steam Boilers. With Illustrations. By STEPHEN ROPER, Engineer. Full bound tucks $2.00

ROSELEUR.—Galvanoplastic Manipulations:

A Practical Guide for the Gold and Silver Electro-plater and the Galvanoplastic Operator. Translated from the French of ALFRED ROSELEUR, Chemist, Professor of the Galvanoplastic Art, Manufacturer of Chemicals, Gold and Silver Electro-plater. By A. A. FESQUET, Chemist and Engineer. Illustrated by over 127 Engravings on wood. 8vo., 495 pages. $6.00

☞ *This Treatise is the fullest and by far the best on this subject ever published in the United States.*

SCHINZ.—Researches on the Action of the Blast Furnace.

By CHARLES SCHINZ. Translated from the German with the special permission of the Author by WILLIAM H. MAW and MORITZ MULLER. With an Appendix written by the Author expressly for this edition. Illustrated by seven plates, containing 28 figures. In one volume, 12mo. $4.25

SHAW.—Civil Architecture:

Being a Complete Theoretical and Practical System of Building, containing the Fundamental Principles of the Art. By EDWARD SHAW, Architect. To which is added a Treatise on Gothic Architecture, etc. By THOMAS W. SILLOWAY and GEORGE M. HARDING, Architects. The whole illustrated by One Hundred and Two quarto plates finely engraved on copper. Eleventh Edition. 4to., cloth. . $10.00

SHUNK.—A Practical Treatise on Railway Curves and Location, for Young Engineers.

By WILLIAM F. SHUNK, Civil Engineer. 12mo. . . $2.00

SLOAN.—American Houses:

A variety of Original Designs for Rural Buildings. Illustrated by 26 colored Engravings, with Descriptive References. By SAMUEL SLOAN, Architect, author of the "Model Architect," etc., etc. 8vo. $2.50

SMEATON.—Builder's Pocket Companion:

Containing the Elements of Building, Surveying, and Architecture; with Practical Rules and Instructions connected with the subject. By A. C. SMEATON, Civil Engineer, etc. In one volume, 12mo. $1.50

SMITH.—A Manual of Political Economy.

By E. PESHINE SMITH. A new Edition, to which is added a full Index. 12mo., cloth. $1.25

SMITH.—Parks and Pleasure Grounds:

Or Practical Notes on Country Residences, Villas, Public Parks, and Gardens. By CHARLES H. J. SMITH, Landscape Gardener and Garden Architect, etc., etc. 12mo. $2.25

SMITH.—The Dyer's Instructor:

Comprising Practical Instructions in the Art of Dyeing Silk, Cotton, Wool, and Worsted, and Woollen Goods: containing nearly 800 Receipts. To which is added a Treatise on the Art of Padding; and the Printing of Silk Warps, Skeins, and Handkerchiefs, and the various Mordants and Colors for the different styles of such work. By DAVID SMITH, Pattern Dyer. 12mo., cloth. . . . $3.00

SMITH.—The Practical Dyer's Guide:

Comprising Practical Instructions in the Dyeing of Shot Cobourgs, Silk Striped Orleans, Colored Orleans from Black Warps, Ditto from White Warps, Colored Cobourgs from White Warps, Merinos, Yarns, Woollen Cloths, etc. Containing nearly 300 Receipts, to most of which a Dyed Pattern is annexed. Also, A Treatise on the Art of Padding. By DAVID SMITH. In one volume, 8vo. Price. . . $25.00

STEWART.—The American System.

Speeches on the Tariff Question, and on Internal Improvements, principally delivered in the House of Representatives of the United States. By ANDREW STEWART, late M. C. from Pennsylvania. With a Portrait, and a Biographical Sketch. In one volume, 8vo., 407 pages. $3.00

STOKES.—Cabinet-maker's and Upholsterer's Companion:

Comprising the Rudiments and Principles of Cabinet-making and Upholstery, with Familiar Instructions, illustrated by Examples for attaining a Proficiency in the Art of Drawing, as applicable to Cabinet-work; the Processes of Veneering, Inlaying, and Buhl-work; the Art of Dyeing and Staining Wood, Bone, Tortoise Shell, etc. Directions for Lackering, Japanning, and Varnishing; to make French Polish; to prepare the Best Glues, Cements, and Compositions, and a number of Receipts particularly useful for workmen generally. By J. STOKES. In one volume, 12mo. With Illustrations. . $1.25

Strength and other Properties of Metals:

Reports of Experiments on the Strength and other Properties of Metals for Cannon. With a Description of the Machines for testing Metals, and of the Classification of Cannon in service. By Officers of the Ordnance Department U. S. Army. By authority of the Secretary of War. Illustrated by 25 large steel plates. In one volume, 4to. . $10.00

SULLIVAN.—Protection to Native Industry.

By Sir EDWARD SULLIVAN, Baronet, author of "Ten Chapters on Social Reforms." In one volume, 8vo. $1.50

Tables Showing the Weight of Round, Square, and Flat Bar Iron, Steel, etc.,

By Measurement. Cloth. 63

TAYLOR.—Statistics of Coal:

Including Mineral Bituminous Substances employed in Arts and Manufactures; with their Geographical, Geological, and Commercial Distribution and Amount of Production and Consumption on the American Continent. With Incidental Statistics of the Iron Manufacture. By R. C. TAYLOR. Second edition, revised by S. S. HALDEMAN. Illustrated by five Maps and many wood engravings. 8vo., cloth. $10.00

TEMPLETON.—The Practical Examinator on Steam and the Steam-Engine:

With Instructive References relative thereto, arranged for the Use of Engineers, Students, and others. By WM. TEMPLETON, Engineer. 12mo. $1.25

THOMAS.—The Modern Practice of Photography.

By R. W. THOMAS, F. C. S. 8vo., cloth. 75

THOMSON.—Freight Charges Calculator.

By ANDREW THOMSON, Freight Agent. 24mo. $1.25

TURNING: Specimens of Fancy Turning Executed on the Hand or Foot Lathe:

With Geometric, Oval, and Eccentric Chucks, and Elliptical Cutting Frame. By an Amateur. Illustrated by 30 exquisite Photographs. 4to. $3.00

Turner's (The) Companion:

Containing Instructions in Concentric, Elliptic, and Eccentric Turning: also various Plates of Chucks, Tools, and Instruments; and Directions for using the Eccentric Cutter, Drill, Vertical Cutter, and Circular Rest; with Patterns and Instructions for working them. A new edition in one volume, 12mo. $1.50

URBIN.—BRULL.—A Practical Guide for Puddling Iron and Steel.

By ED. URBIN, Engineer of Arts and Manufactures. A Prize Essay read before the Association of Engineers, Graduate of the School of Mines, of Liege, Belgium, at the Meeting of 1865–6. To which is added A COMPARISON OF THE RESISTING PROPERTIES OF IRON AND STEEL. By A. BRULL. Translated from the French by A. A. FESQUET, Chemist and Engineer. In one volume, 8vo. $1.00

VAILE.—Galvanized Iron Cornice-Worker's Manual:

Containing Instructions in Laying out the Different Mitres, and Making Patterns for all kinds of Plain and Circular Work. Also, Tables of Weights, Areas and Circumferences of Circles, and other Matter calculated to Benefit the Trade. By CHARLES A. VAILE, Superintendent "Richmond Cornice Works," Richmond, Indiana. Illustrated by 21 Plates. In one volume, 4to. $5.00

VILLE.—The School of Chemical Manures:

Or, Elementary Principles in the Use of Fertilizing Agents. From the French of M. GEORGE VILLE, by A. A. FESQUET, Chemist and Engineer. With Illustrations. In one volume, 12 mo. . . . $1.25

VOGDES.—The Architect's and Builder's Pocket Companion and Price Book:

Consisting of a Short but Comprehensive Epitome of Decimals, Duodecimals, Geometry and Mensuration; with Tables of U. S. Measures, Sizes, Weights, Strengths, etc., of Iron, Wood, Stone, and various other Materials, Quantities of Materials in Given Sizes, and Dimensions of Wood, Brick, and Stone; and a full and complete Bill of Prices for Carpenter's Work; also, Rules for Computing and Valuing Brick and Brick Work, Stone Work, Painting, Plastering, etc. By FRANK W. VOGDES, Architect. Illustrated. Full bound in pocket-book form. $2.00
Bound in cloth. 1.50

WARN.—The Sheet-Metal Worker's Instructor:

For Zinc, Sheet-Iron, Copper, and Tin-Plate Workers, etc. Containing a selection of Geometrical Problems; also, Practical and Simple Rules for describing the various Patterns required in the different branches of the above Trades. By REUBEN H. WARN, Practical Tin-plate Worker. To which is added an Appendix, containing Instructions for Boiler Making, Mensuration of Surfaces and Solids, Rules for Calculating the Weights of different Figures of Iron and Steel, Tables of the Weights of Iron, Steel, etc. Illustrated by 32 Plates and 37 Wood Engravings. 8vo. $3.00

WARNER.—New Theorems, Tables, and Diagrams for the Computation of Earth-Work:

Designed for the use of Engineers in Preliminary and Final Estimates, of Students in Engineering, and of Contractors and other non-professional Computers. In Two Parts, with an Appendix. Part I.—A Practical Treatise; Part II.—A Theoretical Treatise; and the Appendix. Containing Notes to the Rules and Examples of Part I.; Explanations of the Construction of Scales, Tables, and Diagrams, and a Treatise upon Equivalent Square Bases and Equivalent Level Heights. The whole illustrated by numerous original Engravings, comprising Explanatory Cuts for Definitions and Problems, Stereometric Scales and Diagrams, and a Series of Lithographic Drawings from Models, showing all the Combinations of Solid Forms which occur in Railroad Excavations and Embankments. By JOHN WARNER, A. M., Mining and Mechanical Engineer. 8vo. $5.00

WATSON.—A Manual of the Hand-Lathe:

Comprising Concise Directions for working Metals of all kinds, Ivory, Bone and Precious Woods; Dyeing, Coloring, and French Polishing; Inlaying by Veneers, and various methods practised to produce Elaborate work with Dispatch, and at Small Expense. By EGBERT P. WATSON, late of "The Scientific American," Author of "The Modern Practice of American Machinists and Engineers." Illustrated by 78 Engravings. $1.50

WATSON.—The Modern Practice of American Machinists and Engineers:

Including the Construction, Application, and Use of Drills, Lathe Tools, Cutters for Boring Cylinders, and Hollow Work Generally, with the most Economical Speed for the same; the Results verified by Actual Practice at the Lathe, the Vice, and on the Floor. Together with Workshop Management, Economy of Manufacture, the Steam-Engine, Boilers, Gears, Belting, etc., etc. By EGBERT P. WATSON, late of the "Scientific American." Illustrated by 86 Engravings. In one volume, 12mo. $2.50

WATSON.—The Theory and Practice of the Art of Weaving by Hand and Power:

With Calculations and Tables for the use of those connected with the Trade. By JOHN WATSON, Manufacturer and Practical Machine Maker. Illustrated by large Drawings of the best Power Looms. 8vo. $10.00

WEATHERLY.—Treatise on the Art of Boiling Sugar, Crystallizing, Lozenge-making, Comfits, Gum Goods.

12mo. $2.00

WEDDING.—The Metallurgy of Iron;

Theoretically and Practically Considered. By Dr. HERMANN WEDDING, Professor of the Metallurgy of Iron at the Royal Mining Academy, Berlin. Translated by JULIUS DU MONT, Bethlehem, Pa. Illustrated by 207 Engravings on Wood, and three Plates. In one volume, 8vo. (*In press.*)

WILL.—Tables for Qualitative Chemical Analysis.

By Professor Heinrich Will, of Giessen, Germany. Seventh edition. Translated by Charles F. Himes, Ph. D., Professor of Natural Science, Dickinson College, Carlisle, Pa. $1.50

WILLIAMS.—On Heat and Steam:

Embracing New Views of Vaporization, Condensation, and Explosions. By Charles Wye Williams, A. I. C. E. Illustrated. 8vo. $3.50

WOHLER.—A Hand-Book of Mineral Analysis.

By F. Wohler, Professor of Chemistry in the University of Göttingen. Edited by Henry B. Nason, Professor of Chemistry in the Rensselaer Polytechnic Institute, Troy, New York. Illustrated. In one volume, 12mo. $3 00

WORSSAM.—On Mechanical Saws:

From the Transactions of the Society of Engineers, 1869. By S. W. Worssam, Jr. Illustrated by 18 large plates. 8vo. . . $5.00